Debt Financing: Bond Valuation & Forward Interest Rates

Gestão Financeira II

Undergraduate Courses

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Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA

Outline

- Bond Cash Flows, Prices, and Yields;
- Dynamic Behavior of Bond Prices;
- The Yield Curve and Bond Arbitrage;
- Corporate Bonds.

- Forward Interest Rates

Bond Terminology

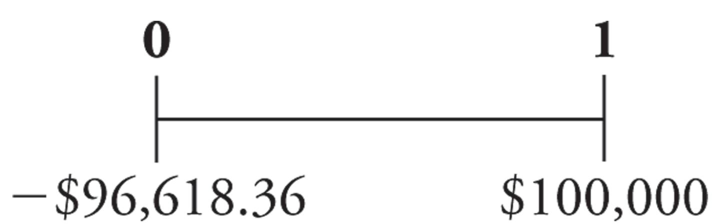
- Bond Certificate
  - States the terms of the bond
- Maturity Date
  - Final repayment date
- Term
  - The time remaining until the repayment date
- Coupon
  - Promised interest payments

- Face Value (*FV*)
  - Notional amount used to compute the interest payments
- Coupon Rate
  - Determines the amount of each coupon payment, expressed as an APR
- Coupon Payment (*CPN*)

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

Zero-Coupon Bonds

- A **Zero-Coupon Bond**:
  - Does not make coupon payments
  - Always sells at a **discount** (a price lower than face value), so they are also called **pure discount bonds**
  - **Treasury Bills** are U.S. government zero-coupon bonds with a maturity of up to one year.
- Example: Suppose that a one-year, risk-free, zero-coupon bond with a $100,000 face value has an initial price of $96,618.36. The cash flows would be:



Zero-Coupon Bonds (cont.)

- Yield to Maturity (*YTM* or *y*):
  - The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.
- Price of a Zero-Coupon bond: $$P = \frac{FV}{(1 + YTM_n)^n}$$
- Example: For the previous example's one-year zero coupon bond, we have $$96{,}618.36 = \frac{100{,}000}{(1 + YTM_1)}$$
$$1 + YTM_1 = \frac{100{,}000}{96{,}618.36} = 1.035$$
- Thus, the Yield to Maturity is YTM= 3.5%.

Zero-Coupon Bonds (cont.)

- The Yield to Maturity of an *n*-Year Zero-Coupon Bond is

$$\boxed{YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1}$$

- Example: Suppose that the following zero-coupon bonds are selling at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

Maturity	1 year	2 years	3 years	4 years
Price	$98.04	$95.18	$91.51	$87.14

$$\boxed{\begin{aligned} &YTM = (100/98.04) - 1 = 0.02 = 2\% \\ &YTM = (100/95.18)^{1/2} - 1 = 0.025 = 2.5\% \\ &YTM = (100/91.51)^{1/3} - 1 = 0.03 = 3\% \\ &YTM = (100/87.14)^{1/4} - 1 = 0.035 = 3.5\% \end{aligned}}$$

Zero-Coupon Bonds (cont.)

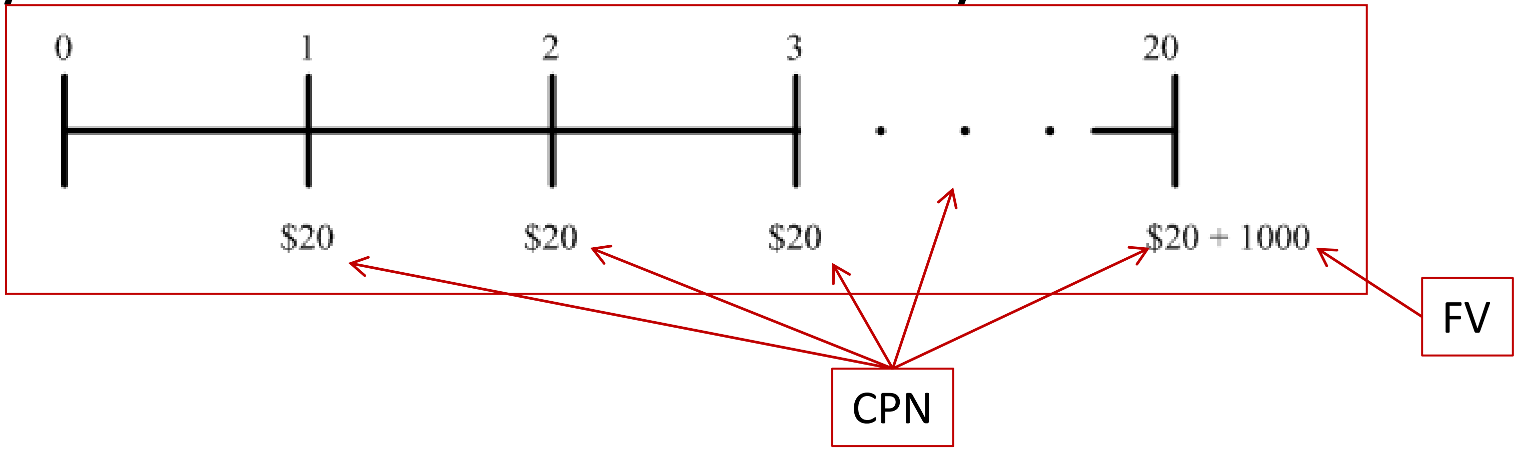
- Risk-Free Interest Rates: A default-free zero-coupon bond that matures on date *n* provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.
  - Risk-Free Interest Rate with Maturity *n*: $\boxed{r_n = YTM_n}$
- Spot Interest Rate
  - Another term for a default-free, zero-coupon yield
- Zero-Coupon Yield Curve
  - A plot of the yield of risk-free zero-coupon bonds as a function of the bond's maturity date.

Coupon Bonds

- A **Coupon Bond**:
  - Pays face value at maturity
  - Pays regular coupon interest payments
  - Examples:
    - Treasury Notes: U.S. Treasury coupon security with original maturities of 1–10 years
    - Treasury Bonds: U.S. Treasury coupon security with original maturities over 10 years

Coupon Bonds (cont.)

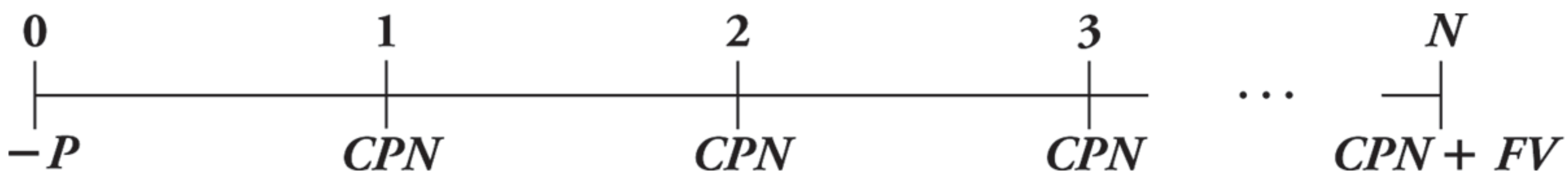
- Example: The U.S. Treasury has just issued a ten-year, $1000 bond with a 4% coupon and semi-annual coupon payments. What cash flows will you receive if you hold the bond until maturity?



- Note that the coupon rate is an APR, and that coupon payment is semi-annual: $CPN = \frac{0.04 * \$1000}{2} = \$20$

Coupon Bonds (cont.)

- Yield to Maturity: The YTM is the *single* discount rate that equates the present value of the bond's remaining cash flows to its current price.



- Yield to Maturity of a Coupon Bond:

$$\boxed{P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}}$$

Coupon Bonds (cont.)

- Example: Consider the following semi-annual bond:
  - \$1000 par value
  - 7 years until maturity
  - 9% coupon rate
  - Price is \$1,080.55

– **What is the bond's yield to maturity?**

$$\$1{,}080.55 = \left(\frac{0.09}{2}\times \$1{,}000\right) * \frac{1}{y}\left[1-\frac{1}{(1+y)^{14}}\right]+\frac{\$1{,}000}{(1+y)^{14}}$$

Microsoft Office
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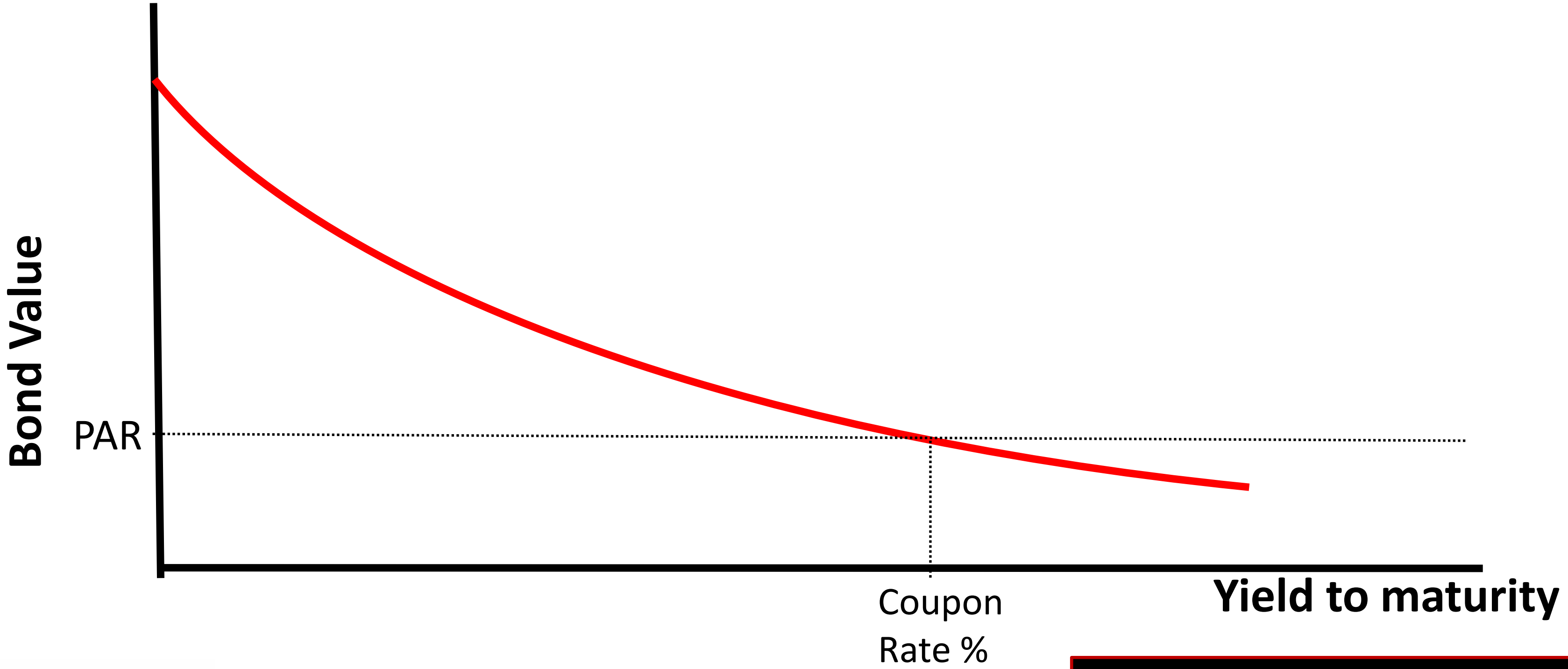
- With a financial calculator, or with excel:
  - y=3.75% (in semi-annual compounding);
  - So the annual Yield to maturity (APR, with semiannual compounding) is y=7.50%.

Dynamic Behavior of Bond Prices

- A bond may be selling at:
  - a Discount (below par): A bond is selling at a **discount** if the price is less than the face value.
  - Par: A bond is selling at **par** if the price is equal to the face value.
  - a Premium (above par): A bond is selling at a **premium** if the price is greater than the face value.

Dynamic Behavior of Bond Prices

When the bond price is …	greater than the face value	equal to the face value	less than the face value
We say the bond trades	"above par" or "at a premium"	"at par"	"below par" or "at a discount"
This occurs when	Coupon Rate > Yield to Maturity	Coupon Rate = Yield to Maturity	Coupon Rate < Yield to Maturity



Dynamic Behavior of Bond Prices (cont.)

- Interpretation:
  - If a coupon bond trades *at a discount*, an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond.
  - If a coupon bond trades *at a premium* it will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond.

Dynamic Behavior of Bond Prices (cont.)

- Example: Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity is 5%:
  - What is the price of each bond per $100 face value?
  - Which bond trades at a premium, which trades at a discount, and which trades at par?

$$P(10\%\text{ coupon}) = (0.1 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$176.86$$ Trades at a premium

$$P(5\%\text{ coupon}) = (0.05 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$100.00$$ Trades at par

$$P(3\%\text{ coupon}) = (0.03 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$69.26$$ Trades at a discount

The effect of Time on Bond Prices

- Holding all other things constant, a bond's yield to maturity will not change over time.
- Holding all other things constant, the price of a discount or of a premium bond will move towards par value over time.
- If a bond's yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity even if you sell the bond early.

The effect of Time on Bond Prices (cont.)

- Example: Consider a 30-year bond with
  - a 10% coupon rate
  - Annual payments
  - \$100 face value

– What is the initial price of this bond if it has a 5% yield to maturity?

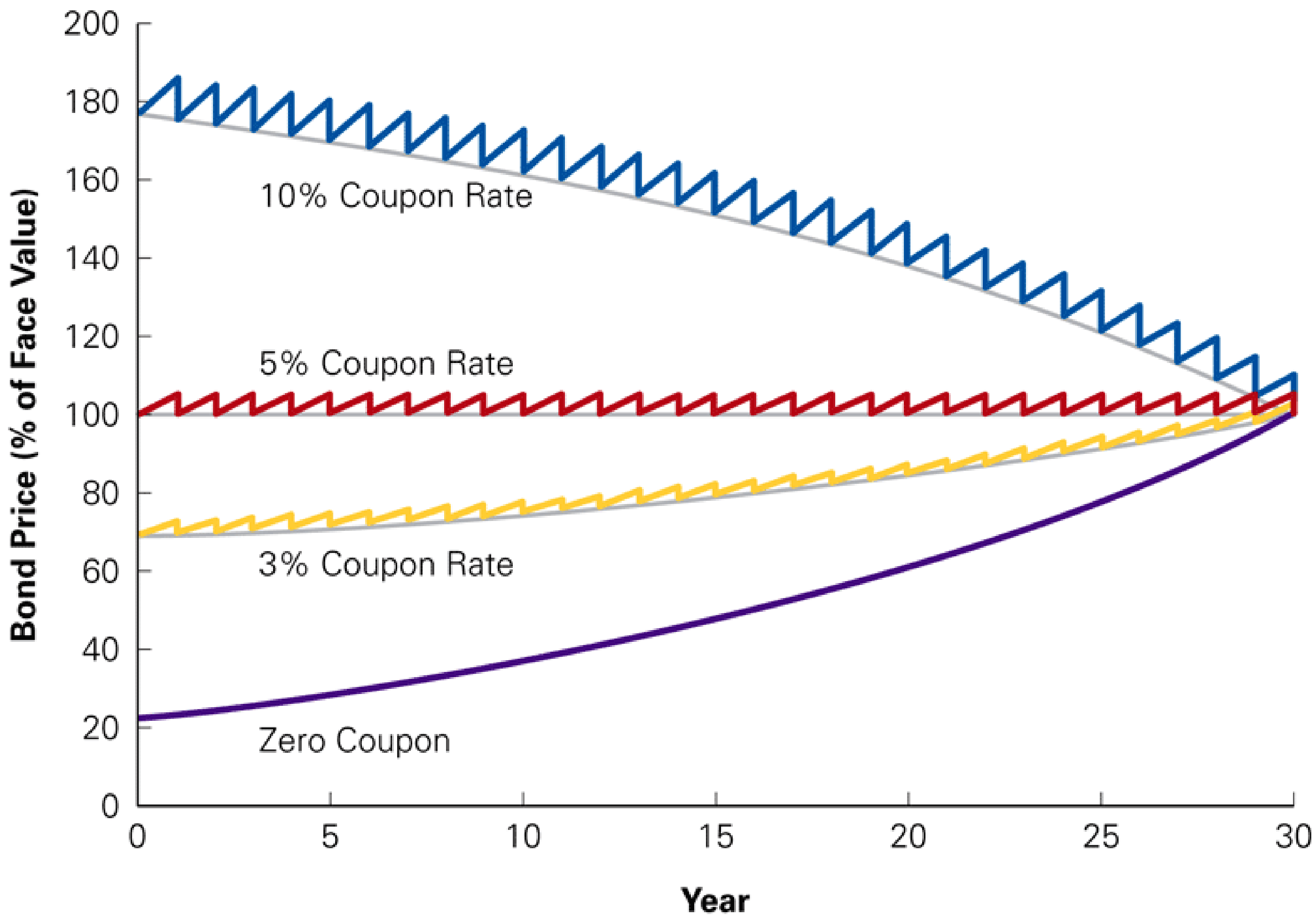
$$P = 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$$

– If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?

$$P(\text{just before first coupon}) = 10 + 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$185.71$$

$$P(\text{just after first coupon}) = 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$175.71$$

The effect of Time on Bond Prices (cont.)



The effect of Interest Rate Changes on Bond Prices

- There is an inverse relationship between interest rates and bond prices.
  - As interest rates and bond yields rise, bond prices fall.
  - As interest rates and bond yields fall, bond prices rise.
- The sensitivity of a bond's price to changes in interest rates is measured by the bond's **duration**.
  - Bonds with high durations are highly sensitive to interest rate changes.
  - Bonds with low durations are less sensitive to interest rate changes.

The effect of Interest Rate Changes on Bond Prices (cont.)

- Example: Consider two bonds
  - A 15-year zero-coupon bond;
  - A 30-year coupon bond with annual coupons of 10%.
  - By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?

Yield to maturity	Price of 15-yr zero-coupon bond	Price of 30-yr 10% annual coupon bond
5%	$\frac{100}{1.05^{15}} = \$48.10$	$10 \times \frac{1}{1.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$
6%	$\frac{100}{1.06^{15}} = \$41.73$	$10 \times \frac{1}{1.06}\left(1 - \frac{1}{1.06^{30}}\right) + \frac{100}{1.06^{30}} = \$155.06$
% Change in price due to 1% change in ytm	$\frac{(41.73 - 48.10)}{48.10} = -13.2\%$	$\frac{(155.06 - 176.86)}{176.86} = -12.3\%$

- Even though the 30-year bond has a longer maturity, the fact that it pays coupons reduces its sensitivity to changes in the interest rate, when compared to a zero-coupon bond.

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The Yield Curve and Bond Arbitrage

- Using the Law of One Price and the yields of default-free zero-coupon bonds, one can determine the price and yield of any other default-free bond.
  - The yield curve provides sufficient information to evaluate all such bonds.
- Example: Replicating a three-year $1000 bond that pays 10% annual coupon using three zero-coupon bonds:

	0	1	2	3
Coupon bond:		\$100	\$100	\$1100
1-year zero:		\$100		
2-year zero:			\$100	
3-year zero:				\$1100
Zero-coupon Bond portfolio:		\$100	\$100	\$1100

The Yield Curve and Bond Arbitrage (cont.)

- Yields and Prices (per $100 Face Value) for default free Zero Coupon Bonds:
  - Example: Assume additionally that we know

Maturity	1 year	2 years	3 years	4 years
YTM	3.50%	4.00%	4.50%	4.75%
Price	$96.62	$92.45	$87.63	$83.06

  - By the Law of One Price, the three-year default free 10% annual coupon bond must trade for a price of $1153.

Zero-Coupon Bond	Face Value Required	Cost
1 year	100	96.62
2 years	100	92.45
3 years	1100	$11 \times 87.63 = 963.93$
	Total Cost:	$1153.00

Valuing a Default-free Coupon Bond using zero-coupon yields

- The price of a coupon bond must equal the present value of its coupon payments and face value.

$$PV = PV(\text{Bond Cash Flows}) = \frac{CPN}{1+YTM_1} + \frac{CPN}{(1+YTM_2)^2} + \cdots + \frac{CPN+FV}{(1+YTM_n)^n}$$

- Example:

$$P = \frac{100}{1.035} + \frac{100}{1.04^2} + \frac{100+1000}{1.045^3} = \$1153$$

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Computing the Yield to Maturity of a Default-free Coupon Bond

- Given the yields for default free zero-coupon bonds, we can price a default free coupon bond.
- Once we have the price of a coupon bond, we can compute its yield to maturity.
- Example:

$$P = 1153 = \frac{100}{(1+y)} + \frac{100}{(1+y)^2} + \frac{100 + 1000}{(1+y)^3}$$

- Using a calculator or excel we can determine the yield to maturity, *y=4.44%*.

Treasury Yield Curves

- Treasury Coupon-Paying Yield Curve
  - Often referred to as “the yield curve”
- On-the-Run Bonds
  - Most recently issued bonds
  - The yield curve is often a plot of the yields on these bonds.

Corporate Bonds

- **Corporate Bonds** are bonds issued by corporations;
  - These bonds involve Risk of default, also known as Credit Risk.
  - The yield of bonds with credit risk will be higher than that of otherwise identical default-free bonds.
  - A bond's expected return will be less than the yield to maturity if there is a risk of default.
    - A higher yield to maturity does not necessarily imply that a bond's expected return is higher.

Corporate Bonds: Yield to Maturity

- Example:

  – *No Default Risk Bond*: Consider a 1-year, zero coupon Treasury Bill with a YTM of 4%. What is its price?

$$\boxed{P = \frac{1000}{1 + YTM_1} = \frac{1000}{1.04} = \$961.54}$$

  – *Risk of Default*: Consider a one-year, $1000, zero-coupon bond issued.

    - There is a 50% chance that the bond will repay its face value in full and a 50% chance that the bond will default and you will receive only $900.
    - Because of the uncertainty, the discount rate (expected return) is 5.1%.
    - The price of the bond will be: $\boxed{P = \frac{950}{1.051} = \$903.90}$
    - The yield to maturity of the bond will be:

$$\boxed{YTM = \frac{FV}{P} - 1 = \frac{1000}{903.90} - 1 = .1063}$$

Corporate Bonds: Bond Ratings

- Several rating agencies (Moody's S&Ps, Fitch) classify bond issues of firms according to their risks.
- They make a clear distinction between
  - Investment Grade Bonds, and
  - Speculative Bonds
    - Also known as Junk Bonds or High-Yield Bonds

Corporate Bonds: Bond Ratings (cont.)

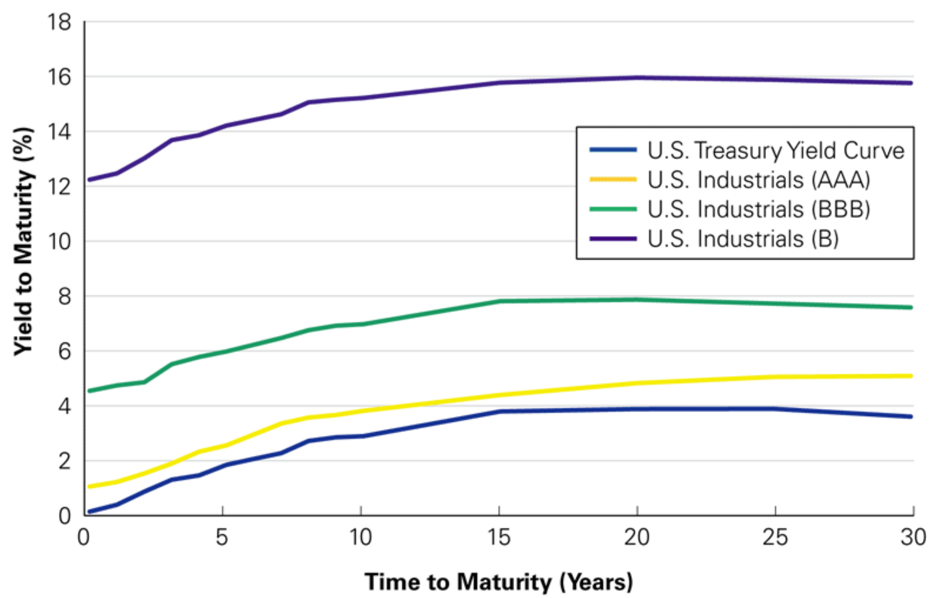
Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

*Ratings: Moody's/Standard & Poor's
Source: www.moodys.com

Corporate Bonds: Corporate Yield Curves

- The Default Spread, also known as Credit Spread, is the difference between the yield on corporate bonds and Treasury yields.

- Example:
  - February 2009;
  - Source: Reuters.



Forward Interest Rates

– A forward interest rate (or forward rate) is an interest rate that we can guarantee today for a loan or investment that will occur in the future.

– We consider interest rate forward contracts for one-year investments:
  - so the forward rate for year 5 means the rate available today on a one-year investment that begins four years from today.

Computing Forward Rates

- By the Law of one price, the forward rate for year 1 is equivalent to an investment in a one-year, zero-coupon bond: $f_1 = YTM_1$

- What about for year 2, or for year 3, etc?

Computing Forward Rates (cont.)

- Example: Consider a two-year forward rate.
  - Suppose the one-year, zero-coupon yield is 5.5% and the two-year, zero-coupon yield is 7.0%.

1. We can invest in the two-year, zero-coupon bond at 7.0% and earn $\$(1.07)^2$ after two years.
2. Or, we can invest in the one-year bond and earn \$1.055 at the end of the first year. We can simultaneously enter into a one-year interest rate forward contract for year 2 at a rate of f_2. At the end of two years, we will have $\$(1.055)(1+f_2)$.

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Computing Forward Rates (cont.)

- Since both strategies are risk free, by the Law of One Price they should have the same return:

$$(1.07)^2 = (1.055)(1+f_2)$$

- Rearranging, we have: $(1+f_2) = \frac{1.07^2}{1.055} = 1.0852$
- The forward rate for year 2 is f_2=8.52%.

- In general: $\boxed{(1+YTM_n)^n = (1+YTM_{n-1})^{n-1}(1+f_n)}$
- Rearranging, we get the general formula for the forward interest rate:

$$\boxed{f_n = \frac{(1+YTM_n)^n}{(1+YTM_{n-1})^{n-1}} - 1}$$

Computing Forward Rates: Example

Computing Forward Rates

Problem

Calculate the forward rates for years 1 through 5 from the following zero-coupon yields:

Maturity	1	2	3	4
YTM	5.00%	6.00%	6.00%	5.75%

Solution

Using Eqs. 8A.1 and 8A.2:

$$f_1 = YTM_1 = 5.00\%$$

$$f_2 = \frac{(1+YTM_2)^2}{(1+YTM_1)} - 1 = \frac{1.06^2}{1.05} - 1 = 7.01\%$$

$$f_3 = \frac{(1+YTM_3)^3}{(1+YTM_2)^2} - 1 = \frac{1.06^3}{1.06^2} - 1 = 6.00\%$$

$$f_4 = \frac{(1+YTM_4)^4}{(1+YTM_3)^3} - 1 = \frac{1.0575^4}{1.06^3} - 1 = 5.00\%$$

Computing Bond Yields from Forward Rates

- It is also possible to compute the zero-coupon yields from the forward interest rates:

$$\boxed{(1+f_1)\times(1+f_2)\times\ldots\times(1+f_n)=(1+YTM_n)^n}$$

- For example, using the forward rates from the previous example, the four-year zero-coupon yield is:

$$\begin{aligned}1+YTM_4 &= \left[(1+f_1)(1+f_2)(1+f_3)(1+f_4)\right]^{1/4}\\ &= \left[(1.05)(1.0701)(1.06)(1.05)\right]^{1/4}\\ &=1.0575\end{aligned}$$

Forward Rates and future Interest Rates

- How does the forward rate compare to the interest rate that will actually prevail in the future?
  - It is a good predictor only when investors do not care about risk.
  - We can think of the forward rate as a break-even rate.
  - Since investors do care about risk:

Expected Future Spot Interest Rate =

Forward Interest Rate + Risk Premium