



# Capital Structure in a Perfect World

(Modigliani-Miller, 1958)

Gestão Financeira II  
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# Capital Structure in a Perfect World

- The **Capital Structure** of a firm (or project) is the **mix of Equity and Debt** that the firm uses.
- **In a Perfect World**, as presented first by Modigliani and Miller in 1958, we will see that **the capital structure choice is irrelevant**.
- Why? The total value of the firm depends on the value of its assets, independently of them being financed with equity or debt. (**MM Proposition I**)
- Even if the cost of debt is lower than the cost of equity, **on average the cost of capital of the firm is always the same ( $r_{WACC}$  does not change with the capital structure)**. This happens because when a firm increases its cheaper source of financing – debt – it also increases the risk for equity-holders, and hence the cost of equity goes up.

# Capital Structure in a Perfect World: Compare Different Structures

- Example:** Consider the following investment opportunity, with equal probability for the date 1 scenarios.

Date 0	Date 1	
	Strong Economy	Weak Economy
-\$800	\$1400	\$900

- Capital Structure #1: 100% Equity Financing (Unlevered Equity)**
  - Consider a  $R_f$  interest rate of 5%, and that you require a 10% risk premium.
  - The project's NPV is: \_\_\_\_\_
  - You can actually raise \$1000 in equity and gain \$200 on top of your required return.
  - The cash flows and returns for Unlevered Equity-holders are:

	Date 0	Date 1: Cash Flows		Date 1: Returns	
	Initial Value	Strong Economy	Weak Economy	Strong Economy	Weak Economy
Unlevered equity	\$1000	\$1400	\$900	40%	-10%

$$\text{Expected Return} = 0.5(40\%) + 0.5(-10\%) = 15\%$$

# Capital Structure in a Perfect World: Compare Different Structures

- **Capital Structure #2: 50% Debt / 50% Equity (Levered equity)**
  - Borrow \$500. At rate  $R_f=5\%$ , because debt will be riskless (look at cash flows)
  - Get the remainder as equity. What's the value of equity?
  - Look at the Cash Flows:

	Date 0	Date 1: Cash Flows	
	Initial Value	Strong Economy	Weak Economy
Debt	\$500	\$525	\$525
Levered equity	$E = ?$	\$875	\$375
Firm	\$1000	\$1400	\$900

- In equilibrium the value of equity must be  $E=\$500$ .
- How can we check this?

# Capital Structure in a Perfect World: Compare Different Structures

- **Levered Equity has higher risk!** The variability of returns is higher. Therefore, the expected return of levered equity is higher than the expected return of unlevered equity.
  - Check it:

	Date 0	Date 1: Cash Flows		Date 1: Returns		Expected Return
	Initial Value	Strong Economy	Weak Economy	Strong Economy	Weak Economy	
Debt	\$500	\$525	\$525	5%	5%	5%
Levered equity	\$500	\$875	\$375	75%	-25%	25%
Unlevered equity	\$1000	\$1400	\$900	40%	-10%	15%

- While debt might be cheaper on its own, it increases the cost of equity.
- Therefore the cash flows for equity-holders cannot be discounted at rate 15% anymore, but rather at 25%!

# Modigliani-Miller: Proposition I (MM I)

- **Modigliani and Miller (MM)** showed in 1958 that this result holds more generally under a set of conditions referred to as **perfect capital markets**:
  - Investors and firms can trade the same set of securities at **competitive market prices** equal to the present value of their future cash flows.
  - There are **no taxes, transaction costs, or issuance costs** associated with security trading.
  - A firm's **financing decisions do not change the cash flows generated by its investments, nor do they reveal new information about them.**

# Modigliani-Miller: Proposition I (MM I)

- MM Proposition I:

- *In a perfect capital market, the total value of a firm is equal to the market value of the total cash flows generated by its assets and is not affected by its choice of capital structure.*

$$E + D = U$$

- This is also known as the Irrelevance result: the choice of capital structure does not affect the value of the firm.

# MM I: Homemade Leverage

- **Example:** Turning to our previous example we can understand MM's result by considering *homemade* leverage made by the investors themselves in case the firm is unlevered.
  - Suppose the firm is unlevered, and the investor creates his own leverage by borrowing \$500 (or getting a margin loan) at  $R_f=5\%$ .

	Date 0	Date 1: Cash Flows	
	Initial Cost	Strong Economy	Weak Economy
Unlevered equity	\$1000	\$1400	\$900
Margin loan	-\$500	-\$525	-\$525
Levered equity	\$500	\$875	\$375

- Likewise, if the firm is levered (with 50% Debt and 50% equity) an investor who wants to invest in the firm as if it were unlevered, can buy both the equity and the debt of the firm.

	Date 0	Date 1: Cash Flows	
	Initial Cost	Strong Economy	Weak Economy
Debt	\$500	\$525	\$525
Levered equity	\$500	\$875	\$375
Unlevered equity	\$1000	\$1400	\$900



# Modigliani-Miller: Proposition II (MM II)

- In their Proposition II, MM give the justification for Proposition I.
  - The reason why the value of the firm is unaltered by the capital structure is that higher leverage causes higher risk for equity-holders, and hence a higher cost of equity.
- **Proposition II:**
  - *The cost of capital of levered equity is equal to the cost of capital of unlevered equity plus a premium that is proportional to the market value debt-equity ratio.*

The diagram illustrates the Modigliani-Miller Proposition II equation,  $r_E = r_U + \frac{D}{E}(r_U - r_D)$ , with labels for each component:

- Cost of Unlevered Equity**: Points to  $r_U$ .
- Market Value of Debt**: Points to  $D$ .
- Cost of Debt**: Points to  $r_D$ .
- Market Value of Equity**: Points to  $E$ .
- Cost of Levered Equity**: Points to  $r_E$ .

The equation is: 
$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

# Modigliani-Miller: Proposition II (MM II)

- If we want we can visualize MM's reasoning by looking at a **Market Value Balance-Sheet**:

U	D
	E

- Therefore when we compute the average cost of capital of the firm, we find that it's always the same in a perfect world. It equals the unlevered cost of capital ( $R_U$ ):

$$\frac{E}{E+D} R_E + \frac{D}{E+D} R_D = R_U$$

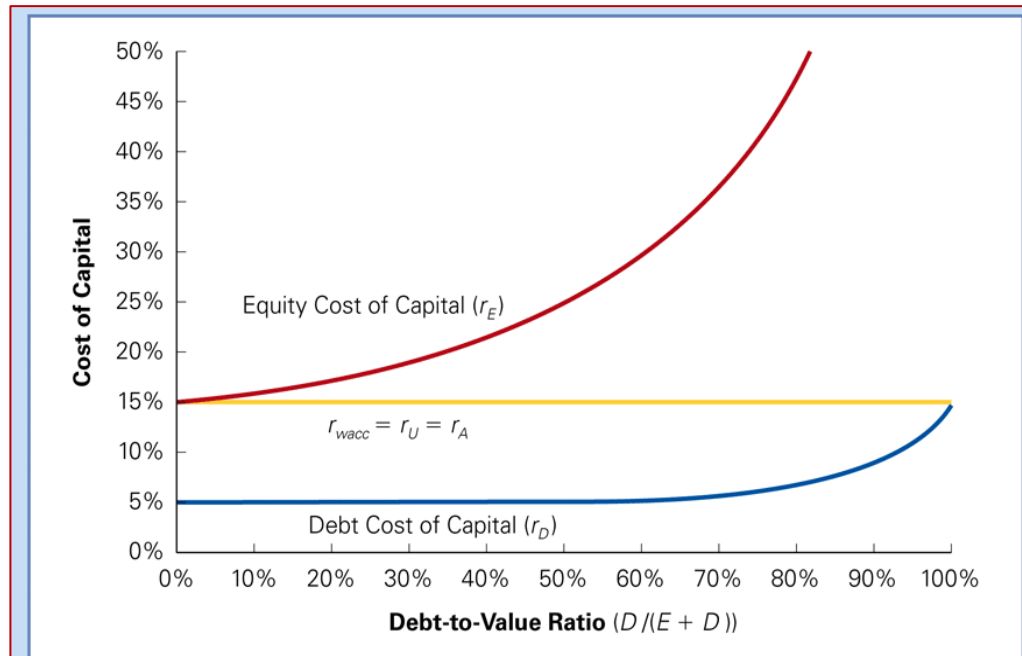
# Cost of Capital in a perfect world

- We define the **Unlevered Cost of Capital (or pre-tax WACC)** as:

$$r_U = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D$$

- And we know that in the MM world, the WACC equals the Unlevered Cost of Equity, which is the Cost of Capital of the firm's Assets:

$$r_{wacc} = r_U = r_A$$



(a)

$E$	$D$	$r_E$	$r_D$	$\frac{E}{E+D} r_E + \frac{D}{E+D} r_D$	$= r_{wacc}$
1000	0	15.0%	5.0%	$1.0 \times 15.0\% + 0.0 \times 5.0\%$	= 15%
800	200	17.5%	5.0%	$0.8 \times 17.5\% + 0.2 \times 5.0\%$	= 15%
500	500	25.0%	5.0%	$0.5 \times 25.0\% + 0.5 \times 5.0\%$	= 15%
100	900	75.0%	8.3% <sup>4</sup>	$0.1 \times 75.0\% + 0.9 \times 8.3\%$	= 15%

(b)

# Additional Comments

- **Computing the WACC with multiple securities:**
  - It is possible to compute the WACC rate if we want to consider more securities than just debt and common stock.
- **Levered betas ( $\beta_E$ ) and Unlevered Betas ( $\beta_U$ ):**
  - The Beta of a stock will vary according to the debt-equity ratio of the firm. We can apply the same logic of MM II to betas:

$$\beta_U = \frac{E}{E+D} \beta_E + \frac{D}{E+D} \beta_D$$
$$\beta_E = \beta_U + \frac{D}{E} (\beta_U - \beta_D)$$

- **Holding Cash:** Holding cash has the opposite effect of having debt.
  - When calculating  $D$ , we use **Net Debt**.
  - **Example:**  $E = \$114.8$  billion;  $\text{Debt} = \$10.3$  billion; and Cash and Short term investments = \$ 33.6 billion.

$$\gg D = 10.3 - 33.6 = - \$ 23.33 \text{ billion}$$