



Valuation with Corporate Taxes: WACC, APV and FTE methods

Gestão Financeira II
Undergraduate Courses
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Valuation of Firms and Projects

- We consider the following assumptions to start with, and introduce **the three methods**:
 1. The project has average risk (same risk as the firm);
 2. The firm's **D/E ratio is constant** (r_E and r_{wacc} will be constant);
 3. Corporate Taxes are the only imperfection.
 - **WACC: Weighted Average Cost of Capital;**
 - **APV: Adjusted Present Value;**
 - **FTE: Flow to Equity.**
- We then consider **alternative Leverage Policies**, for which the **APV method** is more convenient to use:
 - **Constant Interest Coverage Ratio;**
 - **Predetermined Debt Levels.**

Constant D/E ratio: WACC Method

- Remember what the weighted average cost of capital is:

$$r_{wacc} = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D (1 - \tau_c)$$

- An investment's initial levered value is given by the present value of the FCFs discounted at the rate r_{wacc} :

$$V_0^L = \frac{FCF_1}{1 + r_{wacc}} + \frac{FCF_2}{(1 + r_{wacc})^2} + \frac{FCF_3}{(1 + r_{wacc})^3} + \dots$$

Constant D/E ratio: WACC Method

- Example:** Avco, Inc. Is a manufacturer of custom packaging products, and is considering introducing a new line of packaging (the RFX series). The spreadsheet forecasts the project's expected FCFs:

	Year	0	1	2	3	4
Incremental Earnings Forecast (\$ million)						
1	Sales	—	60.00	60.00	60.00	60.00
2	Cost of Goods Sold	—	(25.00)	(25.00)	(25.00)	(25.00)
3	Gross Profit	—	35.00	35.00	35.00	35.00
4	Operating Expenses	(6.67)	(9.00)	(9.00)	(9.00)	(9.00)
5	Depreciation	—	(6.00)	(6.00)	(6.00)	(6.00)
6	EBIT	(6.67)	20.00	20.00	20.00	20.00
7	Income Tax at 40%	2.67	(8.00)	(8.00)	(8.00)	(8.00)
8	Unlevered Net Income	(4.00)	12.00	12.00	12.00	12.00
Free Cash Flow						
9	Plus: Depreciation	—	6.00	6.00	6.00	6.00
10	Less: Capital Expenditures	(24.00)	—	—	—	—
11	Less: Increases in NWC	—	—	—	—	—
12	Free Cash Flow	(28.00)	18.00	18.00	18.00	18.00

Constant D/E ratio: WACC Method

- To determine the firm's r_{wacc} we need the market values (when possible) of Equity and of net Debt, as well as the cost of equity (r_E), the cost of debt (r_D) and the corporate tax rate (τ_c):

Assets		Liabilities		Cost of Capital	
Cash	20	Debt	320	Debt	6%
Existing Assets	600	Equity	300	Equity	10%
Total Assets	620	Total Liabilities and Equity	620		

$$r_{wacc} = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D (1 - \tau_c) = \frac{300}{600} (10\%) + \frac{300}{600} (6\%) (1 - 0.40) = 6.8\%$$

- Note that net debt $D=320-20$; and that $D/(D+E)=0.5$, or $D/E=1$.

Constant D/E ratio: WACC Method

- The value of the project, including the tax shield from debt, is calculated as the present value of its future free cash flows.
- We are assuming that the project uses the same capital structure, target D/E=1.

$$V_0^L = \frac{18}{1.068} + \frac{18}{1.068^2} + \frac{18}{1.068^3} + \frac{18}{1.068^4} = \$61.25 \text{ million}$$

– The NPV of the project is \$33.25 million

- \$61.25 million – \$28 million = \$33.25 million

Constant D/E ratio: WACC Method Summary

1. Determine the free cash flow of the investment.
2. Compute the weighted average cost of capital.
3. Compute the value of the investment, including the tax benefit of leverage, by discounting the free cash flow of the investment using the WACC.

Constant D/E ratio: Implementing the constant D/E ratio

- In the example just seen we considered a constant ratio $D/E=1$ or $D/(D+E)=0.5$.
- By this we mean that – every year – the value of Debt is 50% of the (present) value of the project. **Debt Capacity** is a fixed proportion of V_L .
- So we know that in the beginning (time 0) the new debt that the firm must raise for the RFX project is:

$$D_0 = 0.5 \times \$61.25 = \$30.62$$

Constant D/E ratio: Implementing the constant D/E ratio

- For each year, as time passes, we can recompute V_t^L and D_t .

	Year	0	1	2	3	4
Project Debt Capacity (\$ million)						
1	Free Cash Flow	(28.00)	18.00	18.00	18.00	18.00
2	Levered Value, V^L (at $r_{wacc} = 6.8\%$)	61.25	47.41	32.63	16.85	—
3	Debt Capacity (at $d = 50\%$)	30.62	23.71	16.32	8.43	—

- For example: after the first year passes,

$$V_1^L = \frac{18}{(1+0.068)} + \frac{18}{(1+0.068)^2} + \frac{18}{(1+0.068)^3} = 47.41$$

$$D_1 = 0.5 \times 47.41 = 23.71$$

Constant D/E ratio: APV Method

- The **Adjusted Present Value (APV)** method is an alternative to the WACC method.
- It first values the project as if it were unlevered: V^U
- And separately adds the present value of the interest tax shield.

$$V^L = V^U + PV(\text{Interest Tax Shield})$$

Constant D/E ratio: APV Method

- In the **first step**, the **APV method** determines the unlevered value of the firm, by discounting the FCFs at the **unlevered cost of capital r_U or Pre-Tax WACC**.

$$r_U = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D = \text{Pretax WACC}$$

- In Avco's RFX project **Example**:

$$r_U = 0.50 \times 10.0\% + 0.50 \times 6.0\% = 8.0\%$$

$$V^U = \frac{18}{1.08} + \frac{18}{1.08^2} + \frac{18}{1.08^3} + \frac{18}{1.08^4} = \$59.62 \text{ million}$$

Constant D/E ratio: APV Method

- In the **second step of the APV method** we compute the **present value of the interest tax shield**.
- Note this is easy (possible!) after knowing the Debt Capacity in each year...
- From the Debt Capacity we estimate the annual interest payments:

$$\text{Interest paid in year } t = r_D \times D_{t-1}$$

- The **interest tax shield** is equal to the interest paid multiplied by the corporate tax rate.
- Note: With a target D/E ratio the WACC method is more convenient)

Constant D/E ratio: APV Method

- We compute the **Annual Interest Tax Shields**:

	Year	0	1	2	3	4
Interest Tax Shield (\$ million)						
1	Debt Capacity, D_t	30.62	23.71	16.32	8.43	—
2	Interest Paid (at $r_D = 6\%$)		1.84	1.42	0.98	0.51
3	Interest Tax Shield (at $\tau_c = 40\%$)		0.73	0.57	0.39	0.20

- And the **present value of the interest tax shields**, by discounting them at the unlevered cost of capital r_u (or **Pre-Tax WACC**).

$$PV(\text{interest tax shield}) = \frac{0.73}{1.08} + \frac{0.57}{1.08^2} + \frac{0.39}{1.08^3} + \frac{0.20}{1.08^4} = \$1.63 \text{ million}$$

*Note: When the firm maintains a **target leverage ratio**, its future **interest tax shields** have similar risk to the project's cash flows, so they should be **discounted at the project's unlevered cost of capital**.*

Constant D/E ratio: APV Method

- Finally we get the same Levered Value for the project, as with the WACCC method:

$$V^L = V^U + PV(\text{interest tax shield}) = 59.62 + 1.63 = \$61.25 \text{ million}$$

- The difficulty with applying the APV method is when the debt capacity is not known, but just the target D/E ratio. In that case you would need to determine simultaneously V_t^L and D_t .

Constant D/E ratio: APV Method Summary

1. Determine the investment's value without leverage.
2. Determine the present value of the interest tax shield.
 - a. Determine the expected interest tax shield.
 - b. Discount the interest tax shield.
3. Add the unlevered value to the present value of the interest tax shield to determine the value of the investment with leverage.

Constant D/E ratio: FTE Method

- The **Flow-to-Equity method** is:
 - A valuation method that calculates the **free cash flow available to equity holders, FCFE**, *taking into account all payments to and from debt holders*
 - The cash flows to equity holders are then **discounted using the equity cost of capital r_E** .
 - Gives you the value of Equity – same as $V_L - D$.

Constant D/E ratio: FTE Method

- The method starts by **calculating the FCFE**, which can be obtained by adjusting the FCF:

$$FCFE_t = FCF_t - (1 - \tau_c) \times \text{Interest Payments}_t + \text{Net Borrowing}_t$$

or

$$FCFE_t = \text{Net Income}_t + \text{Depreciation}_t - \text{Capital Expenditures}_t - \text{Increases in NWC}_t + \text{Net Borrowing}_t$$

- The Net Borrowing of a certain year is the change in the level of debt from the previous year:

$$\text{Net Borrowing at Date } t = D_t - D_{t-1}$$

Constant D/E ratio: FTE Method

- In the Avco **Example**, compute the FCFE:

**TABLE 18.7
SPREADSHEET**

Computing FCFE from FCF for Avco's RFX Project

	Year	0	1	2	3	4
Free Cash Flow to Equity (\$ million)						
1	Free Cash Flow	(28.00)	18.00	18.00	18.00	18.00
2	After-tax Interest Expense	—	(1.10)	(0.85)	(0.59)	(0.30)
3	Net Borrowing	30.62	(6.92)	(7.39)	(7.89)	(8.43)
4	Free Cash Flow to Equity	2.62	9.98	9.76	9.52	9.27

Year	0	1	2	3	4
EBIT	-6,67	20,00	20,00	20,00	20,00
Interest Expenses	0	1,84	1,42	0,98	0,51
PreTax Income	-6,67	18,16	18,58	19,02	19,49
Income Tax (40%)	-2,67	7,26	7,43	7,61	7,80
Net Income	-4,00	10,90	11,15	11,41	11,70
+ Depreciation	0,00	6,00	6,00	6,00	6,00
- Capital Expenditures	-24,00	0,00	0,00	0,00	0,00
- Increases in NWC	0,00	0,00	0,00	0,00	0,00
+ Net Borrowing	30,62	-6,92	-7,39	-7,89	-8,43
Free Cash Flow to Equity	2,62	9,98	9,76	9,52	9,27

Constant D/E ratio: FTE Method

- Because the FCFE is for equity-holders only, we compute the net present value of the Equity invested in the project as the **present value of the FCFEs, discounted at the equity cost of capital, r_E .**

$$NPV(FCFE) = 2.62 + \frac{9.98}{1.10} + \frac{9.76}{1.10^2} + \frac{9.52}{1.10^3} + \frac{9.27}{1.10^4} = \$33.25 \text{ million}$$

- This is the same NPV we saw with the WACC method and with the APV method.
- Note: The FTE method is only easy to apply when we know the interest payments each year.

Constant D/E ratio: FTE Method Summary

1. Determine the free cash flow to equity of the investment.
2. Determine the equity cost of capital.
3. Compute the equity value by discounting the free cash flow to equity using the equity cost of capital.

Project-Based Cost of Capital

- It is quite possible that firms use a different capital structure when financing a new project, or that a new project is not in the same line of business.
- In this case the “old” discount rates of the firm should not be used for the project.
- How to calculate the cost of capital for the project’s cash flows when a project’s risk and leverage differ from the firm?

Project-Based Cost of Capital:

(i) Estimating the Unlevered Cost of Capital

- **Example:** Suppose Avco launches a new plastics manufacturing division.
- We can estimate r_U for the plastics division by looking at other single-division plastics firms that have similar business risks (**Comparables**).

Firm	Equity Cost of Capital	Debt Cost of Capital	Debt-to-Value Ratio, $D/(E + D)$
Comparable #1	12.0%	6.0%	40%
Comparable #2	10.7%	5.5%	25%

- For each competitor we get:

$$\text{Competitor 1: } r_U = 0.60 \times 12.0\% + 0.40 \times 6.0\% = 9.6\%$$

$$\text{Competitor 2: } r_U = 0.75 \times 10.7\% + 0.25 \times 5.5\% = 9.4\%$$

- For the average **project in the plastics industry:**

$$r_U = 9.5\%$$

Project-Based Cost of Capital:

(ii) Project Leverage and the Equity Cost of Capital

- Knowing r_U allows us to use the **APV method**.
- To use the **WACC or the FTE methods**, we need to assess the cost of equity r_E for the project.
- With a **target D/E ratio** we use **MMII**:

$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

- In the **Avco plastics division Example** (D/E=1, $r_U=9.5\%$, and $r_D=6\%$ for Avco):

$$r_E = 9.5\% + \frac{0.50}{0.50}(9.5\% - 6\%) = 13.0\%$$

Project-Based Cost of Capital:

(iii) Project Leverage and the WACC

- The **weighted average cost of capital** for the project in the plastics division would be

$$r_{WACC} = 0.50 \times 13.0\% + 0.50 \times 6.0\% \times (1 - 0.40) = 8.3\%$$

- If the **target D/E ratio were different** for this project, we would use the unlevered rate $r_U=9.5\%$ of the project's industry, and compute r_E and r_{WACC} based on the new D/E ratio.

$$\text{Suppose } \frac{D}{E} = 0.75$$

$$r_E = 9.5\% + 0.75(9.5\% - 6\%) = 12.125\%$$

$$r_{WACC} = \frac{1}{1+0.75} 12.125\% + \frac{0.75}{1+0.75} 6\%(1 - 40\%) = 8.47\%$$

Project-Based Cost of Capital:

(iii) Project Leverage and the WACC

- Indeed, knowing a project's (industry's) unlevered cost of capital, is crucial to determine the WACC rate under different target leverage ratios.
- An **alternative way of re-estimating rate WACC when target ratio D/E changes is:**

$$r_{wacc} = r_U - \frac{D}{E + D} \tau_c r_D$$

- In the last example of slide 24 we can confirm:

$$r_{wacc} = 9.5\% - \frac{0.75}{1 + 0.75} 40\% \times 6\% = 8.47\%$$

Project-Based Cost of Capital:

(iv) Determining the Incremental Leverage of a Project

- For capital budgeting purposes, the project's financing is the incremental financing that results from the firm taking the project.
- Things to remember:
 - Cash is Negative Debt: if an investment reduces the firm's cash holdings, it's equivalent to the firm adding debt;
 - A Fixed Equity Payout policy implies 100% Debt Financing: if the payout policy is not affected by a new project, then that project must be financed with $D/(E+D)=1$.

APV Method with Other Leverage Policies

- Besides the policy of keeping a target ratio D/E , there are other common leverage policies.
- We will have a look at 2 cases, which are well captured by the APV method:
 - Constant Interest Coverage Ratio;
 - Predetermined Debt Levels.

APV Method: Constant Interest Coverage Ratio

- When a firm keeps its interest payments equal to a target fraction of its free cash flows we say it has a constant interest coverage ratio.
 - If the target fraction is k , then:

$$\text{Interest Paid in Year } t = k \times FCF_t$$

- To implement the APV approach, the present value of the tax shield under this policy needs to be computed:

$$\begin{aligned} PV(\text{Interest Tax Shield}) &= PV(\tau_c k \times FCF) = \tau_c k \times PV(FCF) \\ &= \tau_c k \times V^U \end{aligned}$$

- With a constant interest coverage policy, the value of the interest tax shield is proportional to the project's unlevered value.

APV Method: Constant Interest Coverage Ratio

- The value of the levered project, using the APV method, is:

- Levered Value with a Constant Interest Coverage Ratio

$$\begin{aligned}V^L &= V^U + PV(\text{interest tax shield}) = V^U + \tau_c k \times V^U \\ &= (1 + \tau_c k)V^U\end{aligned}$$

- **Example:** In the Avco RFX project, if the firm plans to use debt such that interest is always 20% of the FCF, the value of the levered project is:

$$V^L = (1 + 0.4 \times 20\%) \$59.62 = \$64.39 \text{ million}$$

APV Method: Predetermined Debt Levels

- A firm may **adjust its debt according to a fixed schedule that is known in advance.**
- **Example:** For the RFX project, assume now that Avco plans to borrow \$30.62 million and then will reduce the debt on a fixed schedule:
 - to \$20 million after one year, to \$10 million after two years, and to zero after three years.

APV Method: Predetermined Debt Levels

	Year	0	1	2	3	4
Interest Tax Shield (\$ million)						
1	Debt Capacity, D_t	30.62	20.00	10.00	—	—
2	Interest Paid (at $r_D = 6\%$)		1.84	1.20	0.60	—
3	Interest Tax Shield (at $\tau_c = 40\%$)		0.73	0.48	0.24	—

- *When debt levels are set according to a fixed schedule, we can discount the predetermined interest tax shields using the debt cost of capital.*

$$PV(\text{interest tax shield}) = \frac{0.73}{1.06} + \frac{0.48}{1.06^2} + \frac{0.24}{1.06^3} = \$1.32 \text{ million}$$

APV Method: Predetermined Debt Levels

- If debt had a constant predetermined level D forever, we would actually get:

$$V^L = V^U + \tau_C D$$