GESTÃO FINANCEIRA II

PROBLEM SET 4: Solutions
Chapters 10 and 11
Risk and Return
Optimal Portfolio Choice and the CAPM

(FROM BERK AND DEMARZO'S "CORPORATE FINANCE")

LICENCIATURA – UNDERGRADUATE COURSE

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Chapter 10
Capital Markets and the Pricing of Risk

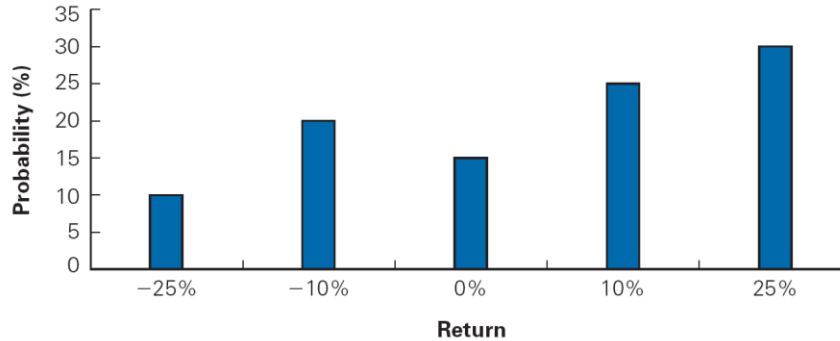**10-1.** **The figure below shows the one-year return distribution for RCS stock. Calculate**

    a. **The expected return.**

$$E[R] = -0.25(0.1) - 0.1(0.2) + 0.1(0.25) + 0.25(0.3) = 5.5\%$$

    b. **The standard deviation of the return.**

$$Variance(R) = 0.1 \times (-0.25 - 0.055)^2 + 0.2 \times (-0.1 - 0.055)^2 + 0.15 \times (-0.055)^2 + 0.25 \times (0.1 - 0.055)^2 +$$
$$0.3 \times (0.25 - 0.055)^2 = 0.026$$

$$\text{Standard Deviation} = \sqrt{0.026} = 16.13\%$$

10-6. **Using the data in the following table, calculate the return for investing in Boeing stock from January 2, 2003, to January 2, 2004, and also from January 2, 2008, to January 2, 2009, assuming all dividends are reinvested in the stock immediately.**

Historical Stock and Dividend Data for Boeing

Date	Price	Dividend	Date	Price	Dividend
1/2/03	33.88		1/2/08	86.62	
2/5/03	30.67	0.17	2/6/08	79.91	0.40
5/14/03	29.49	0.17	5/7/08	84.55	0.40
8/13/03	32.38	0.17	8/6/08	65.40	0.40
11/12/03	39.07	0.17	11/5/08	49.55	0.40
1/2/04	41.99		1/2/09	45.25	

Date	Price	Dividend	R	1+R
1/2/2003	33.88			
2/5/2003	30.67	0.17	-8.97%	0.910272
5/14/2003	29.49	0.17	-3.29%	0.967069
8/13/2003	32.38	0.17	10.38%	1.103764
11/12/2003	39.07	0.17	21.19%	1.211859
1/2/2004	41.99		7.47%	1.074738
			26.55%	1.265491
Date	Price	Dividend	R	1+R
1/2/2008	86.62			
2/6/2008	79.91	0.4	-7.28%	0.927153
5/7/2008	84.55	0.4	6.31%	1.063071
8/6/2008	65.4	0.4	-22.18%	0.778238
11/5/2008	49.55	0.4	-23.62%	0.763761
1/2/2009	45.25		-8.68%	0.913219
			-46.50%	0.535006

10-9. **Consider an investment with the following returns over four years:**

1	2	3	4
10%	20%	−5%	15%

a. **What is the compound annual growth rate (CAGR) for this investment over the four years?**

1	2	3	4		Ave
10%	20%	-5%	15%		10.00%
					CAGR
1.10	1.20	0.95	1.15		9.58%

b. **What is the average annual return of the investment over the four years?**

see table above.

c. **Which is a better measure of the investment's past performance?**

CAGR.

d. **If the investment's returns are independent and identically distributed, which is a better measure of the investment's expected return next year?**

Arithmetic Average.

10-15. Download the spreadsheet from MyFinanceLab containing the data for Figure 10.1.

   a. Compute the average return for each of the assets from 1929 to 1940 (The Great Depression).

   b. Compute the variance and standard deviation for each of the assets from 1929 to 1940.

	S&P 500	Small Stocks	Corp Bonds	World Portfolio	Treasury Bills	CPI
Average	2.553%	16.550%	5.351%	2.940%	0.859%	1.491%
Variance:	0.1018	0.6115	0.0013	0.0697	0.0002	0.0022
Stand. deviation:	31.904%	78.195%	3.589%	26.398%	1.310%	4.644%

   c. Which asset was riskiest during the Great Depression? How does that fit with your intuition?

   The riskiest assets were the small stocks. Intuition tells us that this asset class would be the riskiest.

10-16. Using the data from Problem 15, repeat your analysis over the 1990s.

   a. Which asset was riskiest?

   Using Excel:

		S&P 500	Small Stocks	Corp Bonds	World Portfolio	Treasury Bills	CPI
	Average	18.990%	14.482%	9.229%	12.819%	4.961%	2.935%
	Variance:	0.0201	0.0460	0.0062	0.0194	0.0002	0.0002
Standard	deviation:	14.161%	21.451%	7.858%	13.938%	1.267%	1.239%

   The riskiest asset class was small stocks.

   b. Compare the standard deviations of the assets in the 1990s to their standard deviations in the Great Depression. Which had the greatest difference between the two periods?

   The greatest absolute difference in standard deviation is in the small stocks asset class, which saw standard deviation fall 56.7%. But in relative terms, the riskiness of corporate bonds rose 118% (relative to 1940), while the riskiness of small stocks fell only 72.6% (relative to 1940 levels). Inflation is now much less risky as well, falling in relative riskiness by 73.3%.

   c. If you only had information about the 1990s, what would you conclude about the relative risk of investing in small stocks?

   If you were only looking at the 1990s, you would conclude that small stocks are relatively less risky than they actually are.
   The results that one can derive from analyzing data from a particular time period can change depending on the time period analyzed. These differences can be large if the time periods being analyzed are short.

Chapter 11
Optimal Portfolio Choice and the Capital Asset Pricing Model

11-1. **You are considering how to invest part of your retirement savings. You have decided to put $200,000 into three stocks: 50% of the money in GoldFinger (currently $25/share), 25% of the money in Moosehead (currently $80/share), and the remainder in Venture Associates (currently $2/share). If GoldFinger stock goes up to $30/share, Moosehead stock drops to $60/share, and Venture Associates stock rises to $3 per share,**

   a. **What is the new value of the portfolio?**

   Let n_i be the number of share in stock I, then

   $$n_G = \frac{200,000 \times 0.5}{25} = 4,000$$

   $$n_M = \frac{200,000 \times 0.25}{80} = 625$$

   $$n_V = \frac{200,000 \times 0.25}{2} = 25,000.$$

   The new value of the portfolio is

   $$p = 30n_G + 60n_M + 3n_v$$

   $$= \$232,500.$$

   b. **What return did the portfolio earn?**

   $$\text{Return} = \frac{232,500}{200,000} - 1 = 16.25\%$$

   c. **If you don't buy or sell shares after the price change, what are your new portfolio weights?**

   The portfolio weights are the fraction of value invested in each stock.

   GoldFinger: $\dfrac{n_G \times 30}{232,500} = 51.61\%$

   Moosehead: $\dfrac{n_M \times 60}{232,500} = 16.13\%$

   Venture: $\dfrac{n_V \times 3}{232,500} = 32.26\%$

11-2. You own three stocks: 1000 shares of Apple Computer, 10,000 shares of Cisco Systems, and 5000 shares of Goldman Sachs Group. The current share prices and expected returns of Apple, Cisco, and Goldman are, respectively, $125, $19, $120 and 12%, 10%, 10.5%.

a. What are the portfolio weights of the three stocks in your portfolio?

b. What is the expected return of your portfolio?

c. Assume that both Apple and Cisco go up by $5 and Goldman goes down by $10. What are the new portfolio weights?

d. Assuming the stocks' expected returns remain the same, what is the expected return of the portfolio at the new prices?

				Value	a.	b.	New Price	New Value	c.	d.
Apple	1000	125	12	125000	0.136612022	1.639344262	130	130000	0.141304	1.6957
Cisco	10000	19	10	190000	0.207650273	2.076502732	24	240000	0.26087	2.6087
Goldman	5000	120	10.5	600000	0.655737705	6.885245902	110	550000	0.597826	6.2772
		Total		915000		10.60109%		920000		10.5815%

11-5. Using the data in the following table, estimate (a) the average return and volatility for each stock, (b) the covariance between the stocks, and (c) the correlation between these two stocks.

Year	2004	2005	2006	2007	2008	2009
Stock A	−10%	20%	5%	−5%	2%	9%
Stock B	21%	7%	30%	−3%	−8%	25%

a. $\overline{R}_A = \dfrac{-10+20+5-5+2+9}{6} = 3.5\%$

$\overline{R}_B = \dfrac{21+30+7-3-8+25}{6}$

$= 12\%$

$Var(R_A) = \dfrac{1}{5}\left[\begin{array}{l}(0.1-0.035)^2 + (0.2-0.035)^2 + (0.05-0.035)^2 + (0.05-0.035)^2 + \\ (0.02-0.035)^2 + (0.09-0.035)^2\end{array}\right] = 0.01123$

$Volatility(R_A) = SD(R_A) = \sqrt{0.01123} = 10.60\%$

$Var(R_B) = 0.02448$

$Volatility(R_B) = SD(R_B) = 15.65\%$

b. $Cov(R_A, R_B) = \frac{1}{5}\begin{bmatrix} (-0.1-0.035)(0.21-0.12) + \\ (0.2-0.035)(0.07-0.12) + \\ (0.05-0.035)(0.3-0.12) + \\ (-0.05-0.035)(-0.03-0.12) + \\ (0.02-0.035)(-0.08-0.12) + \\ (0.09-0.035)(0.25-0.12) \end{bmatrix} =$

0.00104

c. $Correlation = \dfrac{0.00104}{0.106 \times 0.1565} = 0.0627$

11-6. **Use the data in Problem 5, consider a portfolio that maintains a 50% weight on stock A and a 50% weight on stock B.**

    a. **What is the return each year of this portfolio?**

    b. **Based on your results from part a, compute the average return and volatility of the portfolio.**

    c. **Show that (i) the average return of the portfolio is equal to the average of the average returns of the two stocks, and (ii) the volatility of the portfolio equals the same result as from the calculation in Eq. 11.9.**

Year	2004	2005	2006	2007	2008	2009
A&B	5.5%	13.5%	17.5%	-4.0%	-3.0%	17.0%
Ave	7.75%					
Vol	9.72%					

    d. **Explain why the portfolio has a lower volatility than the average volatility of the two stocks.**

The portfolio has a lower volatility than the average volatility of the two stocks because some of the idiosyncratic risk of the stocks in the portfolio is diversified away.

For Problems 22–25, suppose Johnson & Johnson and the Walgreen Company have expected returns and volatilities shown below, with a correlation of 22%.

	E[R]	SD[R]
Johnson & Johnson	7%	16%
Walgreen Company	10%	20%

11-22. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that is equally invested in Johnson & Johnson's and Walgreen's stock.

In this case, the portfolio weights are $x_j = x_w = 0.50$.

$$E[R_P] = x_j E[R_j] + x_w E[R_w]$$
$$= 0.50(7\%) + 0.50(10\%)$$
$$= 8.5\%.$$

We can use

$$SD(R_P) = \sqrt{x_j{}^2 SD(R_j)^2 + x_w{}^2 SD(R_w)^2 + 2x_j x_w Corr(R_j, R_w)SD(R_j)SD(R_w)}$$
$$= \sqrt{.50^2(.16^2) + .50^2(.20)^2 + 2(.50)(.50)(.22)(.16)(.20)}$$
$$= 14.1\%$$

11-23. For the portfolio in Problem 22, if the correlation between Johnson & Johnson's and Walgreen's stock were to increase,

    a. Would the expected return of the portfolio rise or fall?

        The expected return would remain constant, assuming only the correlation changes, $0.5 \times 0.07 + 0.5 \times 0.10 = 0.085$.

    b. Would the volatility of the portfolio rise or fall?

        The volatility of the portfolio would increase (due to the correlation term in the equation for the volatility of a portfolio).

11-24. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that consists of a long position of $10,000 in Johnson & Johnson and a short position of $2000 in Walgreen's.

In this case, the total investment is $10,000 – 2,000 = $8,000, so the portfolio weights are $x_j = 10,000/8,000 = 1.25$, $x_w = -2,000/8,000 = -0.25$.

$$E[R_P] = x_j E[R_j] + x_w E[R_w]$$
$$= 1.25(7\%) - 0.25(10\%)$$
$$= 6.25\%.$$

We can use

$$SD(R_P) = \sqrt{x_j{}^2 SD(R_j)^2 + x_w{}^2 SD(R_w)^2 + 2x_j x_w Corr(R_j, R_w)SD(R_j)SD(R_w)}$$
$$= \sqrt{1.25^2(.16^2) + (-0.25)^2(.20)^2 + 2(1.25)(-0.25)(.22)(.16)(.20)}$$
$$= 19.5\%.$$

11-25. Using the same data as for Problem 22, calculate the expected return and the volatility (standard deviation) of a portfolio consisting of Johnson & Johnson's and Walgreen's stocks using a wide range of portfolio weights. Plot the expected return as a function of the portfolio volatility. Using your graph, identify the range of Johnson & Johnson's portfolio weights that yield efficient combinations of the two stocks, rounded to the nearest percentage point.

The set of efficient portfolios is approximately those portfolios with no more than 65% invested in J&J (this is the portfolio with the lowest possible volatility).

x(J&J)	x(Walgreen)	SD	ER
-50%	150%	29.30%	11.50%
-40%	140%	27.32%	11.20%
-30%	130%	25.38%	10.90%
-20%	120%	23.50%	10.60%
-10%	110%	21.70%	10.30%
0%	100%	20.00%	10.00%
10%	90%	18.42%	9.70%
20%	80%	16.99%	9.40%
30%	70%	15.77%	9.10%
40%	60%	14.79%	8.80%
50%	50%	14.11%	8.50%
60%	40%	13.78%	8.20%
65%	**35%**	**13.75%**	**8.05%**
70%	30%	13.82%	7.90%
80%	20%	14.23%	7.60%
90%	10%	14.97%	7.30%
100%	0%	16.00%	7.00%
110%	-10%	17.27%	6.70%
120%	-20%	18.73%	6.40%
130%	-30%	20.34%	6.10%
140%	-40%	22.07%	5.80%
150%	-50%	23.88%	5.50%

11-47. Consider a portfolio consisting of the following three stocks:

	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.25	12%	0.4
Green Midget	0.35	25%	0.6
AliveAndWell	0.4	13%	0.5

The volatility of the market portfolio is 10% and it has an expected return of 8%. The risk-free rate is 3%.

a. **Compute the beta and expected return of each stock.**

b. **Using your answer from part a, calculate the expected return of the portfolio.**

c. **What is the beta of the portfolio?**

d. **Using your answer from part c, calculate the expected return of the portfolio and verify that it matches your answer to part b.**

	Portfolio Weight	Volatility	Correlation with the Market Portfolio	Beta (Part a answer)	Expected Return (Part a answer)
HEC Corp	0.25	12%	0.4	0.48	5.4
Green Midget	0.35	25%	0.6	1.5	10.5
AliveAndWell	0.4	13%	0.5	0.65	6.25
				Part c answer:	Part b answer
			Porfolio	0.905	7.525
					Part d answer
			Expected Return calculated from porfolio beta		7.525

11-48. **Suppose Intel stock has a beta of 2.16, whereas Boeing stock has a beta of 0.69. If the risk-free interest rate is 4% and the expected return of the market portfolio is 10%, what is the expected return of a portfolio that consists of 60% Intel stock and 40% Boeing stock, according to the CAPM?**

$$\beta = 0.6 \ 2.16 + 0.4 \ 0.69 = 1.572$$

$$E\ R = 4 + 1.572 \ 10 - 4 = 13.432\%$$