Part II Portfolio Theory

2 (b). Risk and Asset Allocation

Financial Markets and Investments

Adapted from Bodie, Kane and Marcus' slides, McGraw-Hill/Irwin

1 Risk and Risk Aversion

- Speculation
  - Considerable business risk (sufficient to affect the decision) in getting a
  - Commensurate gain (positive risk premium, relative to a risk-free alternative)
- Gamble
  - Bet or wager on an uncertain outcome (zero risk premium is like a “fair game”), with no purpose but risk enjoyment.

2. Risk and Asset Allocation

1 Risk and Risk Aversion

- Risk averse investors reject investment portfolios that are fair games or worse
- These investors are willing to consider only risk-free or speculative prospects with positive risk premiums
- Intuitively one would rank those portfolios as more attractive with higher expected returns

1 Risk and Risk Aversion

TABLE 6.1

Available risky portfolios (Risk-free rate = 5%)

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

1 Risk and Risk Aversion

Utility Function

$$U = E(r) - \frac{1}{2} A\sigma^2$$

Where,

U = utility

E (r) = expected return on the asset or portfolio

A = coefficient of risk aversion

σ^2 = variance of returns

1 Risk and Risk Aversion

Investor Risk Aversion (A)	Utility Score of Portfolio L $[E(r) = .07; \sigma = .05]$	Utility Score of Portfolio M $[E(r) = .09; \sigma = .10]$	Utility Score of Portfolio H $[E(r) = .13; \sigma = .20]$
2.0	$.07 - ½ \times 2 \times .05^2 = .0675$	$.09 - ½ \times 2 \times .1^2 = .0800$	$\mathbf{.13 - ½ \times 2 \times .2^2 = .09}$
3.5	$.07 - ½ \times 3.5 \times .05^2 = .0656$	$\mathbf{.09 - ½ \times 3.5 \times .1^2 = .0725}$	$.13 - ½ \times 3.5 \times .2^2 = .06$
5.0	$.07 - ½ \times 5 \times .05^2 = .0638$	$\mathbf{.09 - ½ \times 5 \times .1^2 = .0650}$	$.13 - ½ \times 5 \times .2^2 = .03$

TABLE 6.2

Utility scores of alternative portfolios for investors with varying degrees of risk aversion

1 Risk and Risk Aversion

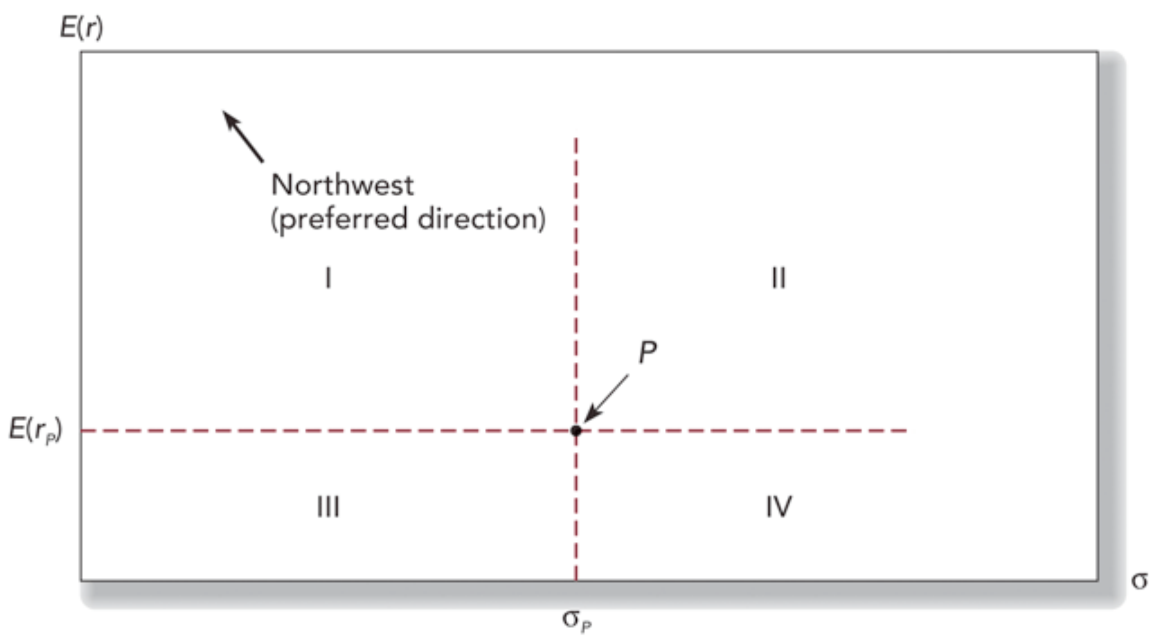

FIGURE 6.1 The trade-off between risk and return of a potential investment portfolio, *P*

Estimating Risk Aversion

- Observe individuals' decisions when confronted with risk
- Observe how much people are willing to pay to avoid risk
  - – Insurance against large losses

1 Risk and Risk Aversion

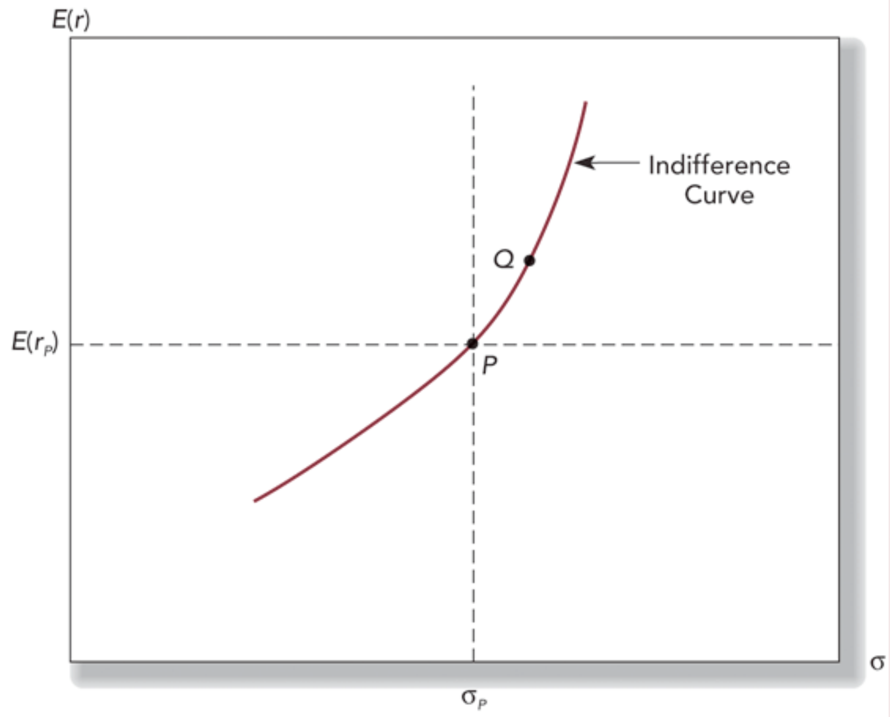

FIGURE 6.2 The indifference curve

1 Risk and Risk Aversion

TABLE 6.3

Utility values of possible portfolios for investor with risk aversion, $A = 4$

Expected Return, $E(r)$	Standard Deviation, σ	Utility = $E(r) - ½ A\sigma^2$
.10	.200	$.10 - .5 \times 4 \times .04 = .02$
.15	.255	$.15 - .5 \times 4 \times .065 = .02$
.20	.300	$.20 - .5 \times 4 \times .09 = .02$
.25	.339	$.25 - .5 \times 4 \times .115 = .02$

1 Risk and Risk Aversion

	Expected Rate of Loss, $p = .0001$	Expected Rate of Loss, $p = .01$
Investor Risk Aversion, *A*	Maximum Premium, *v*, as a Multiple of Expected Loss, *p*	Maximum Premium, *v*, as a Multiple of Expected Loss, *p*
0	1.0000	1.0000
1	1.5000	1.4950
2	1.9999	1.9900
3	2.4999	2.4850
4	2.9998	2.9800
5	3.4998	3.4750

TABLE 6.4

Investor's willingness to pay for catastrophe insurance

2 Capital Allocation (Risky and Risk-Free Portfolios)

- Control risk
  - Asset allocation choice
    - Fraction of the portfolio invested in Treasury bills or other safe money market securities

The Risky Asset Example

Total portfolio value	= \$300,000
Risk-free value	= 90,000
Risky (Vanguard & Fidelity)	= 210,000

Vanguard (V) = 54%

Fidelity (F) = 46%

2 Capital Allocation (Risky and Risk-Free Portfolios)

Vanguard	113,400/300,000 = 0.378
Fidelity	96,600/300,000 = 0.322
Portfolio *P*	210,000/300,000 = **0.700**
Risk-Free Assets *F*	90,000/300,000 = **0.300**
Portfolio *C*	300,000/300,000 = 1.000

The Risk-Free Asset

- Only the government can issue default-free bonds
  - Guaranteed real rate only if the duration of the bond is identical to the investor's desire holding period
- T-bills viewed as the risk-free asset
  - Less sensitive to interest rate fluctuations

Spread Between 3-Month CD and T-bill Rates

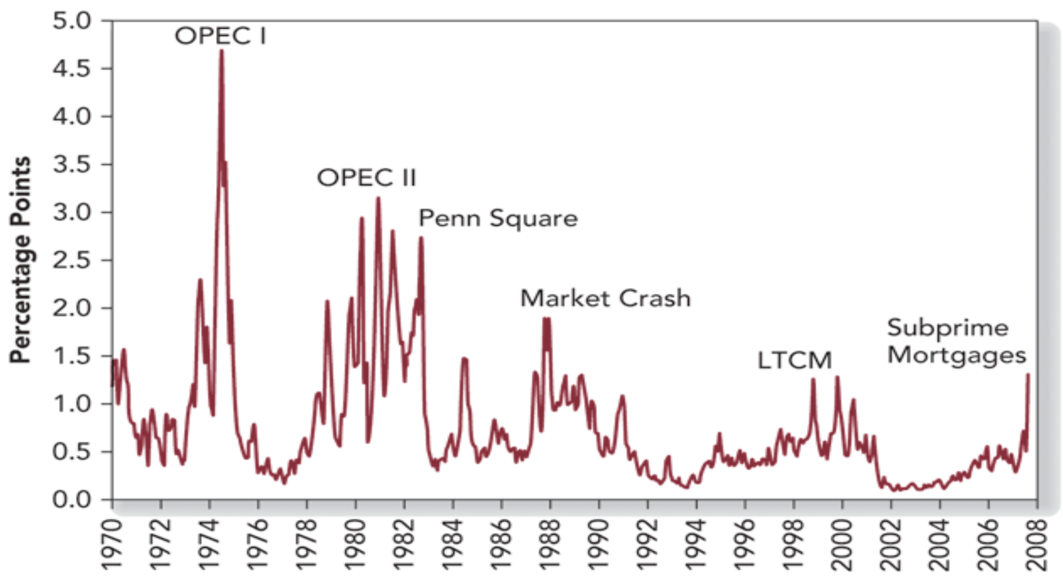

FIGURE 6.3 Spread between 3-month CD and T-bill rates

Portfolios of One Risky Asset and a Risk-Free Asset

- It’s possible to split investment funds between safe and risky assets.
- Risk free asset: proxy; T-bills
- Risky asset: stock (or a portfolio)

Example

$r_f = 7\%$ $\sigma_{rf} = 0\%$

$E(r_p) = 15\%$ $\sigma_p = 22\%$

$y = \%$ in p $(1-y) = \%$ in r_f

Expected Returns for Combinations

$$E(r_c) = yE(r_p) + (1 - y)r_f$$

$$\sigma_C^2 = y^2\sigma_P^2$$

r_c = complete or combined portfolio

2.2 Capital Allocation (Risky and Risk-Free Portfolios)

y	$E(r_c)$	σ_c
0	7%	0%
1	15%	22%
0.75	13%	16.5%

Borrow at the Risk-Free Rate and invest in stock, using 50% leverage,

y	$E(r_c)$	σ_c
1.5	19%	33%

2.2 Capital Allocation (Risky and Risk-Free Portfolios)

The Investment Opportunity Set with a Risky Asset and a Risk-free Asset in the Expected Return-Standard Deviation Plane

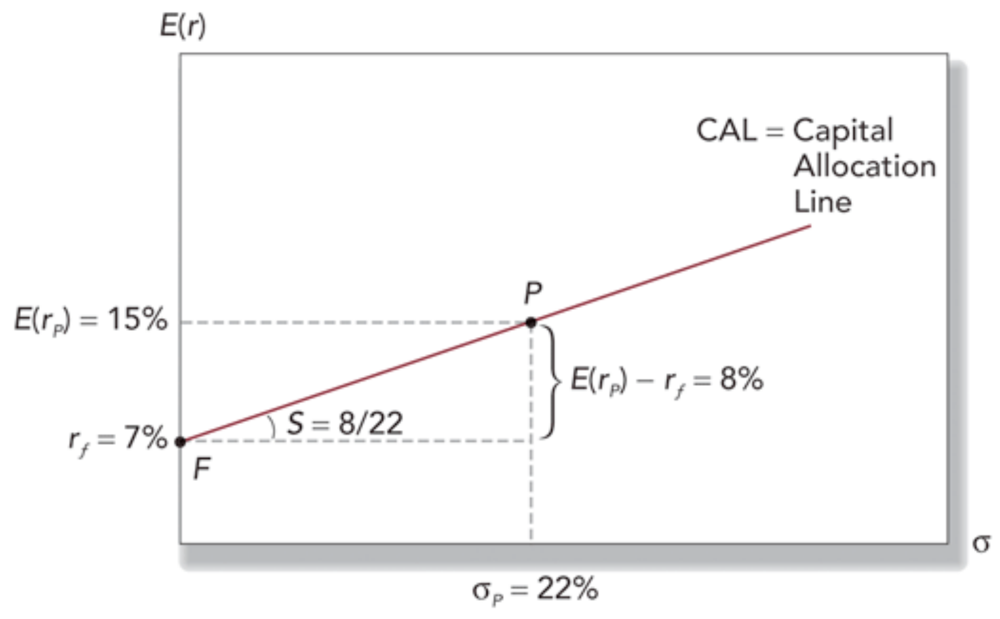

FIGURE 6.4 The investment opportunity set with a risky asset and a risk-free asset in the expected return–standard deviation plane

2.2 Capital Allocation (Risky and Risk-Free Portfolios)

$$
\begin{cases}
E(r_c) = yE(r_p) + (1 - y)r_f \\
\sigma_C^2 = y^2 \sigma_P^2
\end{cases}
$$

$$
\Rightarrow \quad E(r_c) = r_f + \frac{E(r_p) - r_f}{\sigma_p} \sigma_c
$$

CAPITAL ALLOCATION LINE

The Opportunity Set with Differential Borrowing and Lending Rates

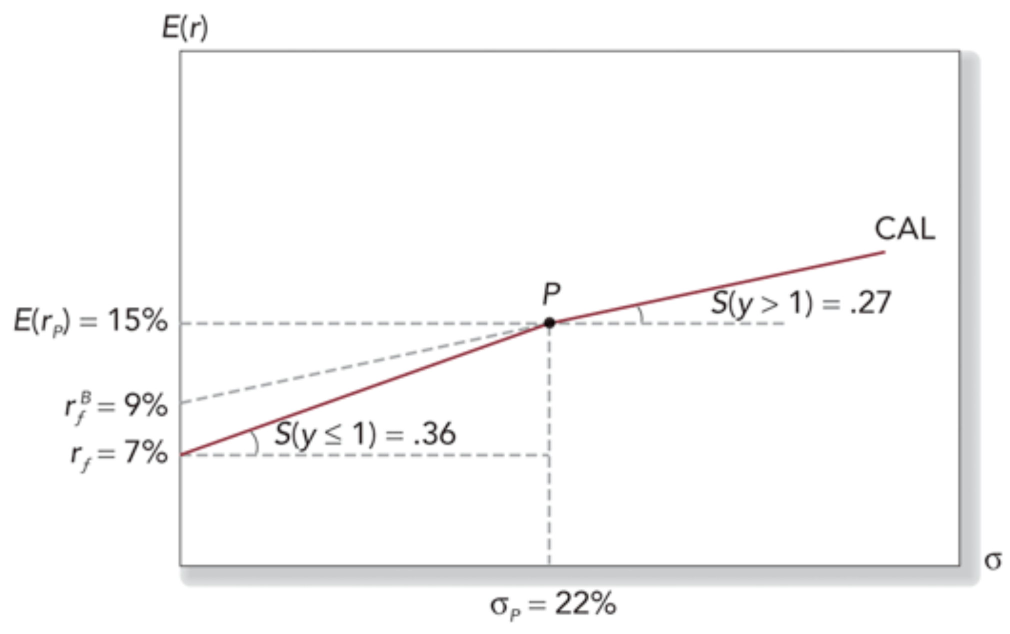

FIGURE 6.5 The opportunity set with differential borrowing and lending rates

Risk Tolerance and Asset Allocation

- The investor must choose one optimal portfolio, *C,* from the set of feasible choices
  - Trade-off between risk and return

2.3 Optimum Portfolio

TABLE 6.5

Utility levels for various positions in risky assets (y) for an investor with risk aversion $A = 4$

(1) y	(2) $E(r_C)$	(3) σ_C	(4) $U = E(r) - \frac{1}{2}A\sigma^2$
0	.070	0	.0700
0.1	.078	.022	.0770
0.2	.086	.044	.0821
0.3	.094	.066	.0853
0.4	.102	.088	.0865
0.5	.110	.110	.0858
0.6	.118	.132	.0832
0.7	.126	.154	.0786
0.8	.134	.176	.0720
0.9	.142	.198	.0636
1.0	.150	.220	.0532

2. Risk and Asset Allocation

2.3 Optimum Portfolio

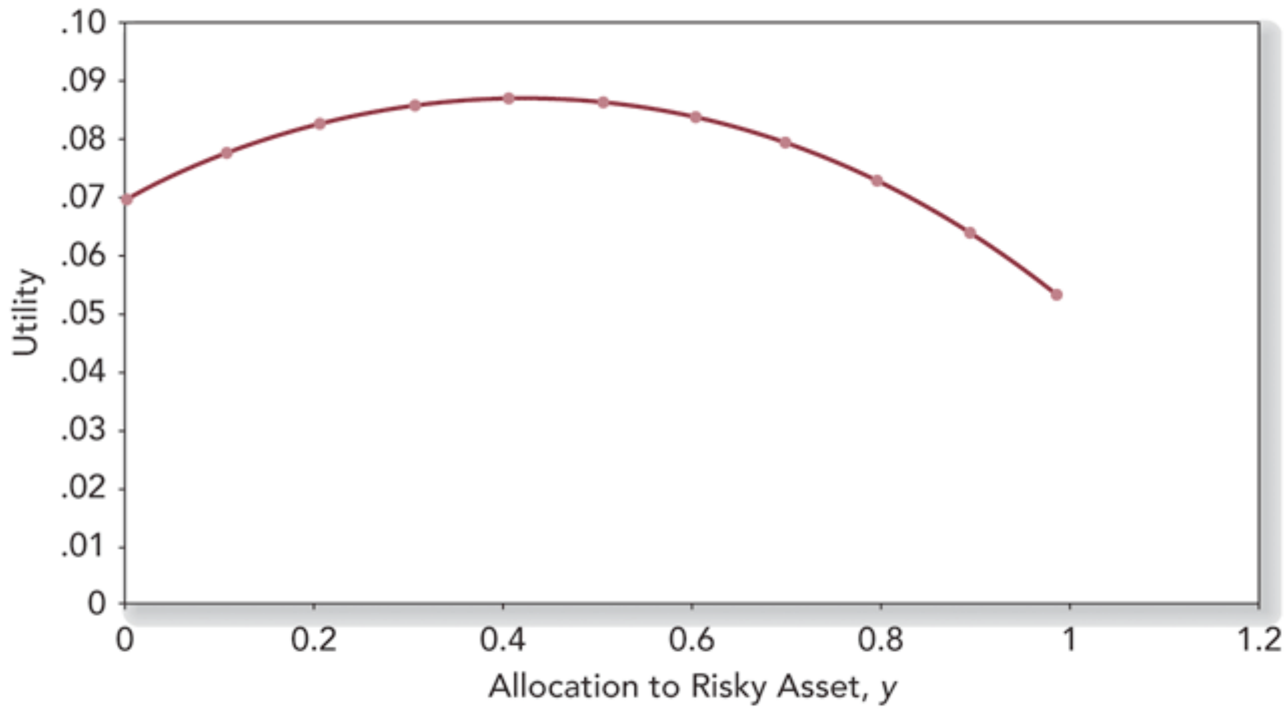

FIGURE 6.6 Utility as a function of allocation to the risky asset, *y*

2.3 Optimum Portfolio

How to reach the optimum portfolio?

TABLE 6.6

Spreadsheet calculations of indifference curves (Entries in columns 2–4 are expected returns necessary to provide specified utility value.)

	A = 2		A = 4	
σ	U = .05	U = .09	U = .05	U = .09
0	.0500	.0900	.050	.090
.05	.0525	.0925	.055	.095
.10	.0600	.1000	.070	.110
.15	.0725	.1125	.095	.135
.20	.0900	.1300	.130	.170
.25	.1125	.1525	.175	.215
.30	.1400	.1800	.230	.270
.35	.1725	.2125	.295	.335
.40	.2100	.2500	.370	.410
.45	.2525	.2925	.455	.495
.50	.3000	.3400	.550	.590

2. Risk and Asset Allocation

2.3 Optimum Portfolio

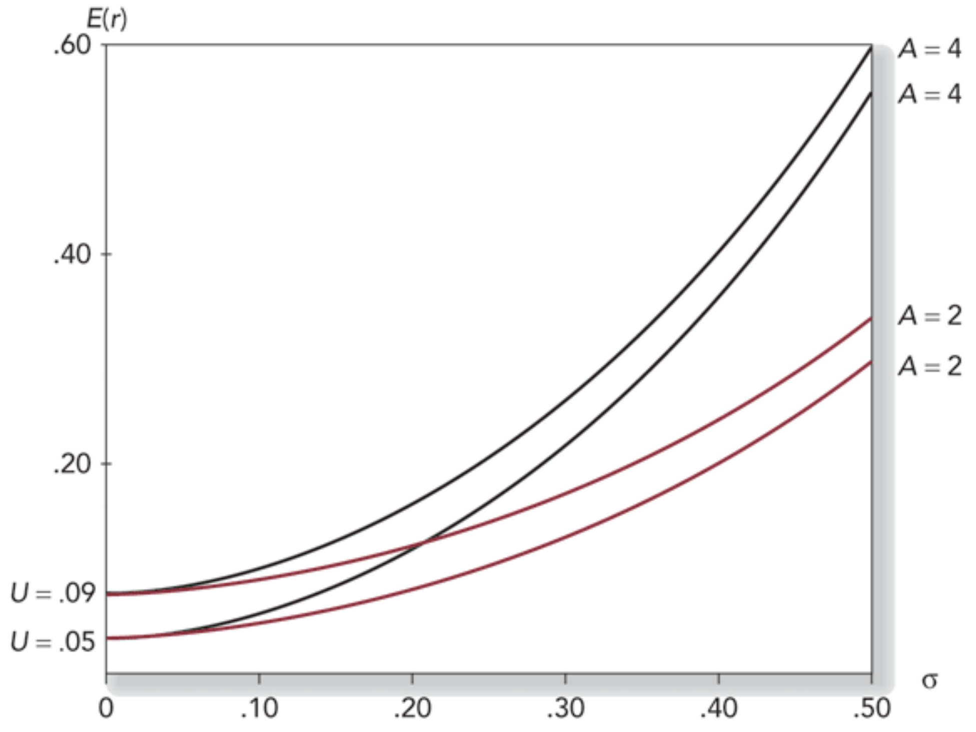

FIGURE 6.7 Indifference curves for $U = .05$ and $U = .09$ with $A = 2$ and $A = 4$

2.3 Optimum Portfolio

Finding the Optimal Portfolio

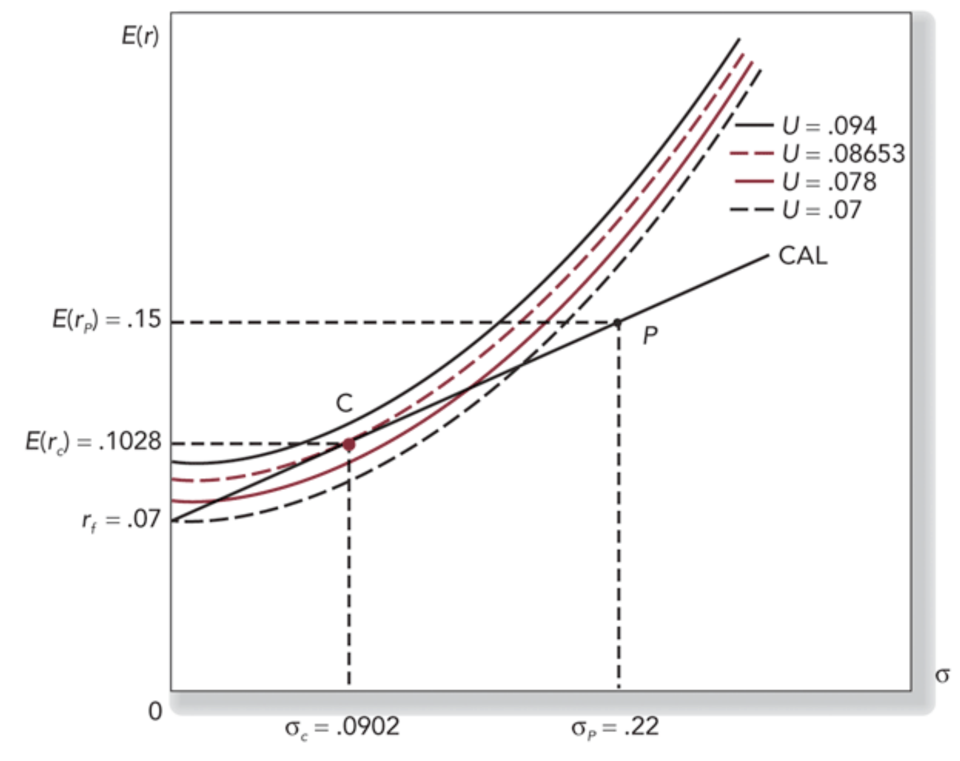

FIGURE 6.8 Finding the optimal complete portfolio by using indifference curves

2.3 Optimum Portfolio

σ	U = .07	U = .078	U = .08653	U = .094	CAL
0	.0700	.0780	.0865	.0940	.0700
.02	.0708	.0788	.0873	.0948	.0773
.04	.0732	.0812	.0897	.0972	.0845
.06	.0772	.0852	.0937	.1012	.0918
.08	.0828	.0908	.0993	.1068	.0991
.0902	.0863	.0943	.1028	.1103	.1028
.10	.0900	.0980	.1065	.1140	.1064
.12	.0988	.1068	.1153	.1228	.1136
.14	.1092	.1172	.1257	.1332	.1209
.18	.1348	.1428	.1513	.1588	.1355
.22	.1668	.1748	.1833	.1908	.1500
.26	.2052	.2132	.2217	.2292	.1645
.30	.2500	.2580	.2665	.2740	.1791

TABLE 6.7

Expected returns on four indifference curves and the CAL. Investor's risk aversion is $A = 4$.

2.4 Passive Strategies: The Capital Market Line

- Passive strategy involves a decision that avoids any direct or indirect security analysis

- Supply and demand forces may make such a strategy a reasonable choice for many investors

2.4 Passive Strategies: The Capital Market Line

- A natural candidate for a passively held risky asset would be a well-diversified portfolio of common stocks
- Because a passive strategy requires devoting no resources to acquiring information on any individual stock or group we must follow a "neutral" diversification strategy

2. Risk and Asset Allocation

2.4 Passive Strategies: The Capital Market Line

Period	Average Annual Returns: S&P 500 Portfolio	Average Annual Returns: 1-Month T-bills	S&P 500 Portfolio: Risk Premium	S&P 500 Portfolio: Standard Deviation	S&P 500 Portfolio: Sharpe Ratio (Reward-to-Volatility)	Probability of Observing this Subperiod Estimate*
1926–2005	12.15	3.75	8.39	20.54	.41	
1986–2005	13.16	4.56	8.60	16.24	.53	.63
1966–1985	10.12	7.41	2.72	17.83	.15	.30
1946–1965	14.97	1.97	13.00	17.65	.74	.20
1926–1945	10.33	1.07	9.26	27.95	.33	.73

TABLE 6.8

Average annual return on large stocks and 1-month T-bills; standard deviation, and reward-to-volatility ratio of large stocks over time

*The probability that the estimate of 1926–2005 is true and we observe the reported (or an even more different) value for the subperiod.