

4. The greater an asset's covariance with the other assets in the portfolio, the more it contributes to portfolio variance. An asset that is perfectly negatively correlated with a portfolio can serve as a perfect hedge. The perfect hedge asset can reduce the portfolio variance to zero.
5. The efficient frontier is the graphical representation of a set of portfolios that maximize expected return for each level of portfolio risk. Rational investors will choose a portfolio on the efficient frontier.
6. A portfolio manager identifies the efficient frontier by first establishing estimates for asset expected returns and the covariance matrix. This input list is then fed into an optimization program that reports as outputs the investment proportions, expected returns, and standard deviations of the portfolios on the efficient frontier.
7. In general, portfolio managers will arrive at different efficient portfolios because of differences in methods and quality of security analysis. Managers compete on the quality of their security analysis relative to their management fees.
8. If a risk-free asset is available and input lists are identical, all investors will choose the same portfolio on the efficient frontier of risky assets: the portfolio tangent to the CAL. All investors with identical input lists will hold an identical risky portfolio, differing only in how much each allocates to this optimal portfolio and to the risk-free asset. This result is characterized as the separation principle of portfolio construction.
9. Diversification is based on the allocation of a *fixed* portfolio across several assets, limiting the exposure to any one source of risk. Adding additional risky assets to a portfolio, thereby increasing the total amounts invested, does not reduce dollar risk, even if it makes the rate of return more predictable. This is because that uncertainty is applied to a larger investment base. Nor does investing over longer horizons reduce risk. Increasing the investment horizon is analogous to investing in more assets. It increases total risk. Analogously, the key to the insurance industry is risk sharing—the spreading of risk across many investors, each of whom takes on only a small exposure to any given source of risk. Risk pooling—the assumption of ever-more sources of risk—may increase rate of return predictability, but not the predictability of total dollar returns.

Related Web sites for this chapter are available at [www.mhhe.com/bkm](http://www.mhhe.com/bkm)

## KEY TERMS

diversification	firm-specific risk	optimal risky portfolio
insurance principle	nonsystematic risk	minimum-variance frontier
market risk	diversifiable risk	efficient frontier of risky assets
systematic risk	minimum-variance portfolio	input list
nondiversifiable risk	portfolio opportunity set	separation property
unique risk	reward-to-volatility ratio	

## PROBLEM SETS

### Quiz

1. Which of the following factors reflect *pure* market risk for a given corporation?
  - a. Increased short-term interest rates.
  - b. Fire in the corporate warehouse.
  - c. Increased insurance costs.
  - d. Death of the CEO.
  - e. Increased labor costs.
2. When adding real estate to an asset allocation program that currently includes only stocks, bonds, and cash, which of the properties of real estate returns affect portfolio *risk*? Explain.
  - a. Standard deviation.
  - b. Expected return.
  - c. Correlation with returns of the other asset classes.
3. Which of the following statements about the minimum variance portfolio of all risky securities are valid? (Assume short sales are allowed.) Explain.
  - a. Its variance must be lower than those of all other securities or portfolios.
  - b. Its expected return can be lower than the risk-free rate.

- c. It may be the optimal risky portfolio.  
d. It must include all individual securities.

The following data apply to Problems 4 through 10: A pension fund manager is considering three mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund that yields a rate of 8%. The probability distribution of the risky funds is as follows:

### Problems

	Expected Return	Standard Deviation
Stock fund (S)	20%	30%
Bond fund (B)	12	15

The correlation between the fund returns is .10.

- What are the investment proportions in the minimum-variance portfolio of the two risky funds, and what is the expected value and standard deviation of its rate of return?
- Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of zero to 100% in increments of 20%.
- Draw a tangent from the risk-free rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal portfolio?
- Solve numerically for the proportions of each asset and for the expected return and standard deviation of the optimal risky portfolio.
- What is the reward-to-volatility ratio of the best feasible CAL?
- You require that your portfolio yield an expected return of 14%, and that it be efficient, on the best feasible CAL.
  - What is the standard deviation of your portfolio?
  - What is the proportion invested in the T-bill fund and each of the two risky funds?
- If you were to use only the two risky funds, and still require an expected return of 14%, what would be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in Problem 9. What do you conclude?
- Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.
  - In light of the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
  - Given the data above, reanswer (a) with the additional assumption that the correlation coefficient between gold and stocks equals 1. Draw a graph illustrating why one would or would not hold gold in one's portfolio. Could this set of assumptions for expected returns, standard deviations, and correlation represent an equilibrium for the security market?
- Suppose that there are many stocks in the security market and that the characteristics of Stocks A and B are given as follows:

Stock	Expected Return	Standard Deviation
A	10%	5%
B	15	10

Correlation = -1

Suppose that it is possible to borrow at the risk-free rate,  $r_f$ . What must be the value of the risk-free rate? (*Hint*: Think about constructing a risk-free portfolio from stocks A and B.)

- Assume that expected returns and standard deviations for all securities (including the risk-free rate for borrowing and lending) are known. In this case all investors will have the same optimal risky portfolio. (True or false?)
- The standard deviation of the portfolio is always equal to the weighted average of the standard deviations of the assets in the portfolio. (True or false?)

15. Suppose you have a project that has a .7 chance of doubling your investment in a year and a .3 chance of halving your investment in a year. What is the standard deviation of the rate of return on this investment?
16. Suppose that you have \$1 million and the following two opportunities from which to construct a portfolio:
- Risk-free asset earning 12% per year.
  - Risky asset with expected return of 30% per year and standard deviation of 40%.

If you construct a portfolio with a standard deviation of 30%, what is its expected rate of return?

**The following data are for Problems 17 through 19:** The correlation coefficients between pairs of stocks are as follows:  $\text{Corr}(A,B) = .85$ ;  $\text{Corr}(A,C) = .60$ ;  $\text{Corr}(A,D) = .45$ . Each stock has an expected return of 8% and a standard deviation of 20%.

17. If your entire portfolio is now composed of stock A and you can add some of only one stock to your portfolio, would you choose (explain your choice):
- B.
  - C.
  - D.
  - Need more data.
18. Would the answer to Problem 17 change for more risk-averse or risk-tolerant investors? Explain.
19. Suppose that in addition to investing in one more stock you can invest in T-bills as well. Would you change your answers to Problems 17 and 18 if the T-bill rate is 8%?

### Challenge Problems

**The following table of compound annual returns by decade applies to Challenge Problems 20 and 21.**

	1920s*	1930s	1940s	1950s	1960s	1970s	1980s	1990s
Small-company stocks	-3.72%	7.28%	20.63%	19.01%	13.72%	8.75%	12.46%	13.84%
Large-company stocks	18.36	-1.25	9.11	19.41	7.84	5.90	17.60	18.20
Long-term government	3.98	4.60	3.59	0.25	1.14	6.63	11.50	8.60
Intermediate-term government	3.77	3.91	1.70	1.11	3.41	6.11	12.01	7.74
Treasury bills	3.56	0.30	0.37	1.87	3.89	6.29	9.00	5.02
Inflation	-1.00	-2.04	5.36	2.22	2.52	7.36	5.10	2.93

\*Based on the period 1926-1929.

20. Input the data from the table into a spreadsheet. Compute the serial correlation in decade returns for each asset class and for inflation. Also find the correlation between the returns of various asset classes. What do the data indicate?
21. Convert the asset returns by decade presented in the table into real rates. Repeat the analysis of Challenge Problem 20 for the real rates of return.



**The following data apply to CFA Problems 1 through 3:** Hennessy & Associates manages a \$30 million equity portfolio for the multimanager Wilstead Pension Fund. Jason Jones, financial vice president of Wilstead, noted that Hennessy had rather consistently achieved the best record among the Wilstead's six equity managers. Performance of the Hennessy portfolio had been clearly superior to that of the S&P 500 in 4 of the past 5 years. In the one less-favorable year, the shortfall was trivial.

Hennessy is a "bottom-up" manager. The firm largely avoids any attempt to "time the market." It also focuses on selection of individual stocks, rather than the weighting of favored industries.

There is no apparent conformity of style among the six equity managers. The five managers, other than Hennessy, manage portfolios aggregating \$250 million made up of more than 150 individual issues.

Jones is convinced that Hennessy is able to apply superior skill to stock selection, but the favorable returns are limited by the high degree of diversification in the portfolio. Over the years, the portfolio generally held 40–50 stocks, with about 2%–3% of total funds committed to each issue. The reason Hennessy seemed to do well most years was that the firm was able to identify each year 10 or 12 issues that registered particularly large gains.

Based on this overview, Jones outlined the following plan to the Wilstead pension committee:

Let's tell Hennessy to limit the portfolio to no more than 20 stocks. Hennessy will double the commitments to the stocks that it really favors, and eliminate the remainder. Except for this one new restriction, Hennessy should be free to manage the portfolio exactly as before.

All the members of the pension committee generally supported Jones's proposal because all agreed that Hennessy had seemed to demonstrate superior skill in selecting stocks. Yet the proposal was a considerable departure from previous practice, and several committee members raised questions. Respond to each of the following questions.

1. *a.* Will the limitation to 20 stocks likely increase or decrease the risk of the portfolio? Explain.
- b.* Is there any way Hennessy could reduce the number of issues from 40 to 20 without significantly affecting risk? Explain.
2. One committee member was particularly enthusiastic concerning Jones's proposal. He suggested that Hennessy's performance might benefit further from reduction in the number of issues to 10. If the reduction to 20 could be expected to be advantageous, explain why reduction to 10 might be less likely to be advantageous. (Assume that Wilstead will evaluate the Hennessy portfolio independently of the other portfolios in the fund.)
3. Another committee member suggested that, rather than evaluate each managed portfolio independently of other portfolios, it might be better to consider the effects of a change in the Hennessy portfolio on the total fund. Explain how this broader point of view could affect the committee decision to limit the holdings in the Hennessy portfolio to either 10 or 20 issues.
4. Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz?

	Portfolio	Expected Return (%)	Standard Deviation (%)
<i>a.</i>	W	15	36
<i>b.</i>	X	12	15
<i>c.</i>	Z	5	7
<i>d.</i>	Y	9	21

5. Which statement about portfolio diversification is correct?
  - a.* Proper diversification can reduce or eliminate systematic risk.
  - b.* Diversification reduces the portfolio's expected return because it reduces a portfolio's total risk.
  - c.* As more securities are added to a portfolio, total risk typically would be expected to fall at a decreasing rate.
  - d.* The risk-reducing benefits of diversification do not occur meaningfully until at least 30 individual securities are included in the portfolio.
6. The measure of risk for a security held in a diversified portfolio is:
  - a.* Specific risk.
  - b.* Standard deviation of returns.
  - c.* Reinvestment risk.
  - d.* Covariance.
7. Portfolio theory as described by Markowitz is most concerned with:
  - a.* The elimination of systematic risk.
  - b.* The effect of diversification on portfolio risk.

- c. The identification of unsystematic risk.  
 d. Active portfolio management to enhance return.
8. Assume that a risk-averse investor owning stock in Miller Corporation decides to add the stock of either Mac or Green Corporation to her portfolio. All three stocks offer the same expected return and total variability. The covariance of return between Miller and Mac is  $-.05$  and between Miller and Green is  $+.05$ . Portfolio risk is expected to:
- a. Decline more when the investor buys Mac.  
 b. Decline more when the investor buys Green.  
 c. Increase when either Mac or Green is bought.  
 d. Decline or increase, depending on other factors.
9. Stocks A, B, and C have the same expected return and standard deviation. The following table shows the correlations between the returns on these stocks.

	Stock A	Stock B	Stock C
Stock A	+1.0		
Stock B	+0.9	+1.0	
Stock C	+0.1	-0.4	+1.0

Given these correlations, the portfolio constructed from these stocks having the lowest risk is a portfolio:

- a. Equally invested in stocks A and B.  
 b. Equally invested in stocks A and C.  
 c. Equally invested in stocks B and C.  
 d. Totally invested in stock C.
10. Statistics for three stocks, A, B, and C, are shown in the following tables.

**Standard Deviations of Returns**

Stock:	A	B	C
Standard deviation (%)	40	20	40

**Correlations of Returns**

Stock	A	B	C
A	1.00	0.90	0.50
B		1.00	0.10
C			1.00

Based *only* on the information provided in the tables, and given a choice between a portfolio made up of equal amounts of stocks A and B or a portfolio made up of equal amounts of stocks B and C, which portfolio would you recommend? Justify your choice.

11. George Stephenson's current portfolio of \$2 million is invested as follows:

**Summary of Stephenson's Current Portfolio**

	Value	Percent of Total	Expected Annual Return	Annual Standard Deviation
Short-term bonds	\$ 200,000	10%	4.6%	1.6%
Domestic large-cap equities	600,000	30%	12.4%	19.5%
Domestic small-cap equities	1,200,000	60%	16.0%	29.9%
Total portfolio	\$2,000,000	100%	13.8%	23.1%

Stephenson soon expects to receive an additional \$2 million and plans to invest the entire amount in an index fund that best complements the current portfolio. Stephanie Coppa, CFA, is evaluating

the four index funds shown in the following table for their ability to produce a portfolio that will meet two criteria relative to the current portfolio: (1) maintain or enhance expected return and (2) maintain or reduce volatility.

Each fund is invested in an asset class that is not substantially represented in the current portfolio.

#### Index Fund Characteristics

Index Fund	Expected Annual Return	Expected Annual Standard Deviation	Correlation of Returns with Current Portfolio
Fund A	15%	25%	+0.80
Fund B	11	22	+0.60
Fund C	16	25	+0.90
Fund D	14	22	+0.65

State which fund Coppa should recommend to Stephenson. Justify your choice by describing how your chosen fund *best* meets both of Stephenson's criteria. No calculations are required.

12. Abigail Grace has a \$900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth \$100,000. Her financial adviser provided her with the following forecast information:

#### Risk and Return Characteristics

	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

The correlation coefficient of ABC stock returns with the original portfolio returns is .40.

- The inheritance changes Grace's overall portfolio and she is deciding whether to keep the ABC stock. Assuming Grace keeps the ABC stock, calculate the:
  - Expected return of her new portfolio which includes the ABC stock.
  - Covariance of ABC stock returns with the original portfolio returns.
  - Standard deviation of her new portfolio which includes the ABC stock.
- If Grace sells the ABC stock, she will invest the proceeds in risk-free government securities yielding .42% monthly. Assuming Grace sells the ABC stock and replaces it with the government securities, calculate the
  - Expected return of her new portfolio, which includes the government securities.
  - Covariance of the government security returns with the original portfolio returns.
  - Standard deviation of her new portfolio, which includes the government securities.
- Determine whether the systematic risk of her new portfolio, which includes the government securities, will be higher or lower than that of her original portfolio.
- Based on conversations with her husband, Grace is considering selling the \$100,000 of ABC stock and acquiring \$100,000 of XYZ Company common stock instead. XYZ stock has the same expected return and standard deviation as ABC stock. Her husband comments, "It doesn't matter whether you keep all of the ABC stock or replace it with \$100,000 of XYZ stock." State whether her husband's comment is correct or incorrect. Justify your response.
- In a recent discussion with her financial adviser, Grace commented, "If I just don't lose money in my portfolio, I will be satisfied." She went on to say, "I am more afraid of losing money than I am concerned about achieving high returns."
  - Describe *one* weakness of using standard deviation of returns as a risk measure for Grace.
  - Identify an alternate risk measure that is more appropriate under the circumstances.

13. Dudley Trudy, CFA, recently met with one of his clients. Trudy typically invests in a master list of 30 equities drawn from several industries. As the meeting concluded, the client made the following statement: "I trust your stock-picking ability and believe that you should invest my funds in your five best ideas. Why invest in 30 companies when you obviously have stronger opinions on a few of them?" Trudy plans to respond to his client within the context of Modern Portfolio Theory.
- Contrast the concepts of systematic risk and firm-specific risk, and give an example of each type of risk.
  - Critique the client's suggestion. Discuss how both systematic and firm-specific risk change as the number of securities in a portfolio is increased.

## E-Investments

**Diversification**

Go to the [www.investopedia.com/articles/basics/03/050203.asp](http://www.investopedia.com/articles/basics/03/050203.asp) Web site to learn more about diversification, the factors that influence investors' risk preferences, and the types of investments that fit into each of the risk categories. Then check out [www.investopedia.com/articles/pf/05/061505.asp](http://www.investopedia.com/articles/pf/05/061505.asp) for asset allocation guidelines for various types of portfolios from conservative to very aggressive. What do you conclude about your own risk preferences and the best portfolio type for you? What would you expect to happen to your attitude toward risk as you get older? How might your portfolio composition change?

**SOLUTIONS TO CONCEPT CHECKS**

- The first term will be  $w_D \times w_D \times \sigma_D^2$ , because this is the element in the top corner of the matrix ( $\sigma_D^2$ ) times the term on the column border ( $w_D$ ) times the term on the row border ( $w_D$ ). Applying this rule to each term of the covariance matrix results in the sum  $w_D^2 \sigma_D^2 + w_D w_E \text{Cov}(r_E, r_D) + w_E w_D \text{Cov}(r_D, r_E) + w_E^2 \sigma_E^2$ , which is the same as Equation 7.3, because  $\text{Cov}(r_E, r_D) = \text{Cov}(r_D, r_E)$ .
  - The bordered covariance matrix is

	$w_X$	$w_Y$	$w_Z$
$w_X$	$\sigma_X^2$	$\text{Cov}(r_X, r_Y)$	$\text{Cov}(r_X, r_Z)$
$w_Y$	$\text{Cov}(r_Y, r_X)$	$\sigma_Y^2$	$\text{Cov}(r_Y, r_Z)$
$w_Z$	$\text{Cov}(r_Z, r_X)$	$\text{Cov}(r_Z, r_Y)$	$\sigma_Z^2$

There are nine terms in the covariance matrix. Portfolio variance is calculated from these nine terms:

$$\begin{aligned}
 \sigma_p^2 &= w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + w_Z^2 \sigma_Z^2 \\
 &\quad + w_X w_Y \text{Cov}(r_X, r_Y) + w_Y w_X \text{Cov}(r_Y, r_X) \\
 &\quad + w_X w_Z \text{Cov}(r_X, r_Z) + w_Z w_X \text{Cov}(r_Z, r_X) \\
 &\quad + w_Y w_Z \text{Cov}(r_Y, r_Z) + w_Z w_Y \text{Cov}(r_Z, r_Y) \\
 &= w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + w_Z^2 \sigma_Z^2 \\
 &\quad + 2w_X w_Y \text{Cov}(r_X, r_Y) + 2w_X w_Z \text{Cov}(r_X, r_Z) + 2w_Y w_Z \text{Cov}(r_Y, r_Z)
 \end{aligned}$$