CDS Auctions *

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Abstract We analyze credit default swap settlement auctions theoretically and evaluate them empirically. In our theoretical analysis we show that the current auction design may not give rise to the fair bond price, and we suggest modifications to minimize this mispricing. In our empirical study, we find that auctions undervalue bonds by an average of 10% on the auction day, and link the undervaluation to the number of bonds exchanged. We also find that underlying bond prices follow a V pattern around the day of the auction: in the preceding 10 days prices decrease by 30% on average, and in the subsequent 10 days they revert to their pre-auction levels.

JEL Classification Codes: G10, G13, D44

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Introduction

Credit Default Swaps (CDS) have been one of the most significant financial innovations in the last 20 years. They have become very popular among investment and commercial banks, insurance companies, pension fund managers and many other economic agents. As a result, the market has experienced enormous growth. According to the Bank of International Settlements (BIS), the notional amount of single-name CDS contracts grew from $5.1 trillion in December 2004 to $33.4 trillion in June 2008, and was still $18.4 trillion in June 2010 following a decline in the aftermath of the credit crisis.

The recent crisis put CDS in the spotlight, with policymakers now assigning them a central role in many reforms. The success of these reforms depends on the efficient functioning of the CDS market and on a thorough understanding of how it operates. Recognizing this, much research has been dedicated to the valuation of CDS contracts, econometric analysis of CDS premia, violations of the law of one price in the context of basis trades, search frictions, counterparty risk, private information, and moral hazard problems associated with holding both bonds issued by a particular entity and CDS protection on this entity.[1]

In this paper we focus on another aspect of CDS. We study how the payoff of a CDS contract is determined when a credit event occurs. Our theoretical analysis of the unusual auction-based procedure reveals that this mechanism may lead to deviations from fundamental value. The mispricing is attributable, in large part, to strategic bidding on the part of investors holding CDS. Empirically, we find that CDS auctions undervalue the underlying securities, by 10% on average. Because the magnitude of this mispricing is economically large, our findings may have implications for how CDS are valued, used and analyzed.

In a nutshell, a CDS is a contract that protects a buyer against the loss of a bond's principal in the case of a credit event (e.g., default, liquidation, debt restructuring, etc.). Initially, CDS were settled physically with the cheapest-to-deliver option. Under such settlement, the protection buyer was required to deliver any bond issued

[1]This work includes, but is not limited to, Acharya and Johnson (2007), Arora, Gandhi, and Longstaff (2009), Bolton and Oehmke (2011), Duffie (1999), Duffie and Zhu (2011), Garleanu and Pedersen (2011), Longstaff, Mithal, and Neis (2005), Pan and Singleton (2008), and Parlour and Winton (2010).

by the reference entity to the protection seller in exchange for the bond's par value. But as a result of the rapid development of the CDS market, the notional amount of outstanding CDS contracts came to exceed the notional amount of deliverable bonds many times over. This made physical settlement impractical and led the industry to develop a cash settlement mechanism. This mechanism is the object of our study.

While many derivatives are settled in cash, the settlement of CDS in this way is challenging for two reasons. First, the underlying bond market is opaque and illiquid, which makes establishing a benchmark bond price difficult. Second, parties with both CDS and bond positions face recovery basis risk if their positions are not closed simultaneously.[2] The presence of this risk renders it necessary that the settlement procedure include an option to replicate an outcome of the physical settlement.

In response to these challenges, the industry has developed a novel two-stage auction. In the first stage of the auction, parties that wish to replicate the outcome of the physical settlement submit their requests for physical delivery via dealers. These requests for physical delivery are aggregated into the net open interest (NOI). Dealers also submit bid and offer prices with a commitment to transact in a predetermined minimal amount at the quoted prices. These quotations are used to construct the initial market midpoint price (IMM). The IMM is used to derive a limit on the final auction price, which is imposed to avoid potential price manipulation. The limit is referred to as the price cap. The NOI and the IMM are announced to all participants.

In the second stage a uniform divisible good auction is implemented, in which the net open interest is cleared. Each participant may submit limit bids that are combined with the bids of the dealers from the first stage. The bid that clears the net open interest is declared to be the final auction price, which is then used to settle the CDS contracts in cash.

We analyze the auction outcomes from both theoretical and empirical perspectives.

[2]Recovery basis risk can be illustrated as follows. Imagine a party that wishes to hedge a long position in a bond by buying a CDS with the same notional amount. The final physically-settled position is known in advance: the protection buyer delivers a bond in exchange for a predetermined cash payment equal to par value. However, the cash-settled position is uncertain before the auction: the protection buyer keeps the bond, pays the auction-determined bond value (unknown at the outset) to the protection seller, and receives par value in exchange. The difference between the market value of the bond held by the protection buyer and the auction-determined value is the recovery basis.

To study price formation, we follow Wilson (1979) and Back and Zender (1993). We formalize the auction using an idealized setup in which all auction participants are risk-neutral and have identical expected valuations of the bond, v. This case is not only tractable, but also provides a useful benchmark against which to test whether CDS auctions lead to the fair-value price. While Wilson (1979) shows that a standard uniform divisible good auction can result in underpricing specifically, we demonstrate that the current auction design can yield a final price either above or below v.

Our conclusion differs because participants of CDS auctions can have prior positions in derivatives on the asset being auctioned. If a participant chooses to settle her entire CDS position physically, her final payoff is not affected by the auction outcome. However, in the case of cash settlement, buyers of protection benefit if the auction price is set below fair value, while sellers benefit if it is set above. Therefore, an auction outcome depends on the size of the net CDS positions; that is, positions that remain after participants submit their physical settlement requests.

To be specific, consider the case of positive NOI: a second-stage auction in which the agents buy bonds. When the net CDS positions of protection sellers are less than the NOI, the Wilson (1979) argument still holds. Underpricing occurs if the auction participants choose not to bid aggressively. The current auction rule is such that bids above the final price are guaranteed to be fully filled, so participants are not sufficiently rewarded for raising their bids. On the other hand, when the net CDS positions of sellers are larger than the NOI, bidding above the fair value and realizing a loss from buying NOI units of bonds is counterbalanced by a reduction in the net payoff of the existing CDS contracts. In the absence of a cap, the auction price would be at least v.

Our theory delivers a rich set of testable predictions. Full implementation of such tests requires data on individual CDS positions and bids, which are not available. Nonetheless, we are able to analyse some aspects of the auction data and find evidence that is consistent with our theoretical predictions. We use TRACE bond data to construct the reference bond price. Using the reference bond price on the day before the auction as a proxy for v, we find that the auction price is set at the price cap whenever there is overpricing. Furthermore, the extent of overpricing does not exceed the spread between the price cap and IMM. When the final auction price is uncapped and the NOI is positive (a typical situation), the bonds are undervalued and the

degree of undervaluation increases with the NOI. In addition, underlying bond prices follow a V pattern around the auction day. In the 10 days before the auction, prices decrease by 30% on average. They reach their lowest level on the day of the auction (average underpricing of 10%), before reverting to their pre-auction levels over the next 10 days. This evidence suggests that our conclusions are robust to the choice of the reference bond price.

Our findings prompt us to consider ways to mitigate the observed mispricing. In a standard setting, in which agents have no prior positions in derivative contracts written on the asset being auctioned, Kremer and Nyborg (2004) suggest a likely source of underpricing equilibria. They show that a simple change of allocation rule from pro-rata on the margin to pro-rata destroys all underpricing equilibria. We show that the same change of allocation rule would be beneficial in our setting. In addition, we suggest that imposing an auction price cap conditional on the outcome of the first stage could further reduce mispricing in equilibrium outcomes.

To our knowledge there are four other papers that examine CDS auctions, two of which were carried out independently and contemporaneously with our work. Three of the papers analyse the auctions empirically. Helwege, Maurer, Sarkar, and Wang (2009) find no evidence of mispricing in an early sample of 10 auctions, of which only four used the current auction format. Coudert and Gex (2010) study a somewhat different sample of auctions, using Bloomberg data for reference bond prices. They document a large gap between a bond's price on the auction date and the final auction price. However, they do not link the gap to the net open interest, nor do they provide any theoretical explanations for their findings. Gupta and Sundaram (2011) also document a V pattern in bond prices around the auction day. Under a simplifying assumption that bidders in the second stage of the auction have zero CDS positions, they find that a discriminatory auction format could reduce the mispricing. Finally, Du and Zhu (2011) examine the outcome types that are possible in CDS auctions. Their paper considers a special case of our model, in which they implicitly assume that all market participants can buy and short-sell bonds of distressed companies at the fair value v without any restrictions. This setup implies that only overpricing equilibria can exist. Further, they treat physical settlement requests as given. We show that this setup results in fair pricing if agents choose physical settlement optimally. We allow for a more realistic setup, where there are constraints on short selling, and

where some participants cannot hold distressed debt. We show that there can be substantial underpricing in this case.

The remainder of the paper is organized as follows. Section 1 describes the CDS auction methodology as it is currently employed. Section 2 describes the auction model. Section 3 provides the main theoretical analysis. Section 4 relates the predictions of the theoretical model to empirical data from CDS auctions. Section 5 discusses modifications that could potentially improve the efficiency of the auction. Section 6 concludes. The appendix contains proofs that are not provided in the main text.

1 The Auction Format

This discussion is based on a reading of the auction protocols available from the ISDA website. Initially, CDS auctions were designed for cash settlement of contracts on credit indexes. The first auction that allowed single-name CDS to be settled in cash was the Dura auction, conducted on November 28, 2006. The auction design used in this case, and for all subsequent credit events, consists of two stages.

In the first stage, participants in the auction submit their requests for physical settlement. Each request for physical settlement is an order to buy or sell bonds at the auction price. To the best of the relevant party's knowledge, the order must be in the same direction as – and not in excess of – the party's market position, which allows the participants to replicate traditional physical settlement of the contracts. For example, if a party is long one unit of protection and submits a request to physically deliver one bond, the resulting cash flow is 100 and is identical to that of physical settlement.

In addition, a designated group of agents (dealers) makes a two-way market in the defaulted assets by submitting bids and offers with a predefined maximum spread and associated quotation size. The spread and quotation sizes are subject to specification prior to each auction and may vary depending on the liquidity of the defaulted assets.[3]

The first stage inputs are then used to calculate the net open interest (NOI) and an 'initial market midpoint' (IMM), which are carried through to the second part

[3]The most common value of the spread is 2% of par. Quotation sizes range from \$2 to \$10 million; \$2 million is the most common amount.

of the auction. The NOI is computed as the difference of the physical-settlement buy and sell requests. The IMM is set by discarding crossing/touching bids and offers, taking the 'best half' of each, and calculating the average. The best halves would be, respectively, the highest bids and the lowest offers. If a dealer's quotation is crossed and is on the wrong side of the IMM, she must make a payment, called an adjustment amount, to the ISDA. That is, she pays the adjustment amount if her bid is higher than the IMM and the NOI is to sell, or if an offer is lower than the IMM and the NOI is to buy. The adjustment amount itself is a product of the quotation amount and the difference between the quotation and the IMM.

As an example, consider the Nortel Limited auction of February 10, 2009. Table 1 lists the market quotes submitted by participating dealers. Once these quotes have been received, the bids are sorted in descending order and the offers in ascending order. The highest bid is then matched with the lowest offer, the second highest bid with the second lowest offer, and so on. Figure 1 displays the quotes from Table 1 which are organized in this way. For example, the Citibank bid of 10.5 and the Barclays offer of 6.0 create a tradeable market.

The IMM is computed from the non-tradeable quotes. First, the 'best half' of the non-tradeable quotes is selected (i.e., the first five pairs). Second, the IMM is computed as an average of bid and offer quotes in the best half, rounded to the nearest one-eighth of a percentage point. In our example there are nine pairs of such quotes. The relevant bids are: three times 7.0 and two times 6.5. The relevant offers are: two times 8.0; two times 8.5; and 9. The average is 7.6 and the rounded average is 7.625.

Given the established IMM and the direction of open interest, dealers whose quotes have resulted in tradeable markets pay the adjustment amount to the ISDA. In the case of Nortel, the open interest was to sell. Thus, dealers whose bids crossed the markets were required to pay an amount equal to (Bid-IMM) times the quotation amount, which was \$2 MM. Citigroup had to pay $(10.5 - 7.625)/100 \times \$2MM =$ \$57500 and Banc of America Securities had to pay $(9.5 - 7.625)/100 \times \$2MM =$ \$37500.

Finally, the direction of open interest determines the cap on the final price, where the price itself is set in the second part of the auction. In the Nortel example the open interest was to sell, which meant the final price could exceed the IMM by a

maximum of 1.0. Thus the price cap was 8.625, as depicted in Figure 1.

After the publication of the IMM, the NOI, and the adjustment amounts, the second stage of the auction begins. If the NOI is zero, the final price is set equal to the IMM. If the NOI is non-zero, dealers may submit corresponding limit orders on behalf of their customers (including those without CDS positions) – and for their own account – to offset the NOI. Agents submit 'buy' limit orders if the NOI is greater than zero and 'sell' limit orders if it is less than zero. In practice, it is unlikely that all agents involved in the first stage will participate in the second stage as well. Participants in the CDS market are diverse in terms of their investment objectives and institutional constraints. For example, many mutual and pension funds may not be allowed to hold any of the defaulted bonds.

Upon submission of the limit orders, if the NOI is to buy, the auction administrators match the open interest against the market bids from the first stage of the auction, and against the limit bids from the second stage of the auction. They start with the highest bid, proceeding through the second highest bid, third highest bid, and so on, until either the entire net open interest or all of the bids have been matched. If the NOI is cleared, the final price is set equal to the lowest bid corresponding to the last matched limit order. However, if this bid exceeds the IMM by more than the cap amount (typically half of the bid-offer spread), the final price is simply set equal to the IMM plus the cap amount. If all bids are matched before the NOI clears, the final price will be zero and all bids will be filled on a pro-rata basis. The procedure is similar if the NOI is to sell. If there are not enough offers to match the net open interest, the final price is set to par.

2 The Auction Model

The main question we wish to address in this paper is whether the current auction format may result in mispricing. Our approach is motivated by the classic work of Wilson (1979) and Back and Zender (1993) who show how this can happen in a standard divisible-good auction. As in Wilson (1979), we assume that all agents are risk-neutral and have identical expectations about the value of the bonds. This case is not only tractable, but also provides a useful benchmark from which to judge whether the auction leads to the fair-value price. This approach is popular in the

auction literature because if equilibria that result in mispricing can be found in this admittedly basic setup, it is likely they will also be possible in more realistic scenarios.

The goal of this section is to formalize the auction process described in Section 1. There are two dates: $t = 0$ and $t = 1$. There is a set $\mathcal{N}$ of strategic players and the total number of agents is $|\mathcal{N}| = N$. A set of dealers $\mathcal{N}_d$ constitutes a subset of all players, $\mathcal{N}_d \subseteq \mathcal{N}$, $|\mathcal{N}_d| = N_d$. Each agent $i \in \mathcal{N}$ is endowed with $n_i \in \mathbb{R}$ units of CDS contracts and $b_i \in \mathbb{R}$ units of bonds. Agents with positive (negative) n_i are called protection buyers (sellers). Because a CDS is a derivative contract, it is in zero net supply, $\sum_i n_i = 0$. One unit of bond pays $\tilde{v} \in [0, 100]$ at time $t = 1$. The auction takes place at time $t = 0$ and consists of two stages.

2.1 First Stage

In the first stage, the auction initial market midpoint (IMM) and the net open interest (NOI) are determined. Agent i may submit a request to sell y_i (or buy if $y_i < 0$) units of bonds at par (100). Each protection buyer, $n_i > 0$, is only allowed to submit a request to sell $y_i \in [0, n_i]$ units of bonds, while each protection seller, $n_i < 0$, may only submit a request to buy $y_i \in [n_i, 0]$ units of bonds. Given these requests, the NOI is determined as follows:

$$NOI = \sum_{i=1}^{N} y_i. \tag{1}$$

In addition, all dealers from the set $\mathcal{N}_d$ are asked to provide a price quote π_i. Given π_i, dealer i must stand ready to sell or buy L units of bonds at bid and offer prices $\pi_i + s$ and $\pi_i - s$, $s > 0$. Quotes from dealers whose bids and offers cross are discarded. The IMM, denoted by p^M, is then set equal to the average of the remaining mid-quotations.

2.2 Second Stage

At this stage, a uniform divisible good auction is held. If $NOI = 0$ then $p^A = p^M$. If $NOI > 0$, participants bid to buy NOI units of bonds. In this case, each agent i may submit a left-continuous non-increasing demand schedule $x_i(p) : [0, p^M + s] \to \mathbb{R}_+ \cup 0$.

Let $X(p) = \sum_{i \in \mathcal{N}} x_i(p)$ be the total demand. The final auction price p^A is the highest price at which the entire NOI can be matched:

$$p^A = \max\{p | X(p) \geq NOI\}.$$

If $X(0) \leq NOI$, $p^A = 0$. Given p^A, the allocations $q_i(p^A)$ are determined according to the 'pro-rata at the margin' rule:

$$q_i(p^A) = x_i^+(p^A) + \frac{x_i(p^A) - x_i^+(p^A)}{X(p^A) - X^+(p^A)} \times (NOI - X^+(p^A)), \tag{2}$$

where $x_i^+(p^A) = \lim_{p \downarrow p^A} x_i(p)$ and $X^+(p) = \lim_{p \downarrow p^A} X(p)$ are the individual and total demands, respectively, above the auction clearing price.

If $NOI < 0$, participants offer to sell $|NOI|$ units of bonds. Each agent i may then submit a right-continuous non-decreasing supply schedule $x_i(p) : [100, p^M - s] \to \mathbb{R}_- \cup 0$.

As before, the total supply is $X(p) = \sum_{i \in \mathcal{N}} x_i(p)$. And the final auction price p^A is the lowest price at which the entire NOI can be matched:

$$p^A = \min\{p | X(p) \leq NOI\}.$$

If $X(100) \geq NOI$, $p^A = 100$. Given p^A, the allocations $q_i(p^A)$ are given by:

$$q_i(p^A) = x_i^-(p^A) + \frac{x_i(p^A) - x_i^-(p^A)}{X(p^A) - X^-(p^A)} \times (NOI - X^-(p^A)),$$

where $x_i^-(p^A) = \lim_{p \uparrow p^A} x_i(p)$ and $X^-(p) = \lim_{p \uparrow p^A} X(p)$ are the individual and total supplies, respectively, below the auction clearing price.

2.3 Preferences

Two types of agents participate in the auction: dealers and common participants. In our setup, all agents are risk-neutral and have identical expected valuations of the bond payoff, v. The agents' objective is to maximize their wealth, Π_i, at date 1,

where

$$\begin{aligned}\Pi_i &= \underbrace{(v-p^A)q_i}_{\text{auction-allocated bonds}} + \underbrace{(n_i-y_i)(100-p^A)}_{\text{net CDS position}} \\ &+ \underbrace{100y_i}_{\text{physical settlement}} + \underbrace{v(b_i-y_i)}_{\text{remaining bonds}}. \end{aligned} \tag{3}$$

and q_i is the number of auction-allocated bonds.

Dealers differ from common participants in that they submit quotes (π_i) in the first stage, which are made public after the auction. Thus, due to regulatory and reputational concerns, dealers may be reluctant to quote prices that are very different from v unless the auction results in a large gain. To model these concerns we assume that dealers' utility has an extra term $-\frac{\gamma}{2}(\pi_i - v)^2$, $\gamma \geq 0$.

2.4 Trading Constraints

So far we have assumed a frictionless world in which every agent can buy and sell bonds freely. This is a very strong assumption which is violated in practice. Therefore, we extend our setup to allow market imperfections. Specifically, we place importance on the following two frictions.

First, some auction participants, such as pension funds or insurance companies, may not be allowed to hold bonds of defaulted companies. To model this, we introduce Assumption 1.

Assumption 1 *Only a subset $\mathcal{N}_+ \subseteq \mathcal{N}$, $\mathcal{N}_+ \neq \emptyset$ of the set of agents can hold a positive amount of bonds after the auction.*

Second, because bonds are traded in OTC markets, short-selling a bond is generally difficult. To model this, we introduce Assumption 2.

Assumption 2 *Each agent i can sell only b_i units of bonds.*

In what follows, we solve for the auction outcomes both in the frictionless world and under Assumptions 1 and 2.

3 Analysis

We now turn to a formal analysis of the auction described in the preceding section. We solve for the auction outcomes using backward induction. We start by solving for the equilibrium outcome in the second stage of the auction, for a given IMM and NOI. We then find optimal dealer quotations π_i and optimal physical settlement requests in the first stage given the equilibrium outcomes of the second stage.

3.1 Second Stage

As previously noted, stage two consists of a uniform divisible good auction with the goal of clearing the net open interest generated in the first stage. A novel feature of our analysis is that we study auctions where participants have prior positions in derivative contracts written on the asset being auctioned. We show that equilibrium outcomes in this case can be very different from those realized in 'standard' auctions (that is, auctions in which $n_i = 0$ for all i).

We first consider the case in which all CDS positions are common knowledge. (This assumption is relaxed later.) If this is the case, each agent i takes the following as given: the NOI, a set of all CDS positions n_i, a set of physical settlement requests y_i, $i \in \mathcal{N}$, and the demand of other agents $x_{-i}(p)$. Therefore, from equation (3), each agent's demand schedule $x_i(p)$ solves the following optimization problem:

$$\max_{x_i(p)} \; (v - p(x_i(p), x_{-i}(p)))\, q_i(x_i(p), x_{-i}(p)) + (n_i - y_i)\,(100 - p(x_i(p), x_{-i}(p)))\,. \quad (4)$$

The first term in this expression represents the payoff realized by participating in the auction, while the second term accounts for the payoff from the remaining CDS positions, $n_i - y_i$, which are settled in cash on the basis of the auction results.

To develop intuition about the forthcoming theoretical results, consider the bidding incentives of the auction participants. The objective function (4) implies that, holding the payoff from the auction constant, an agent who has a short (long) remaining CDS position wishes the final price to be as high (low) as possible. However, agents with opposing CDS positions do not have the same capacity to affect the auction price. The auction design restricts participants to submit one-sided limit orders depending on the sign of the NOI. If the $NOI > 0$, only buy limit orders are allowed,

and therefore agents with short CDS positions are capable of bidding up the price. By contrast, all that an agent with a long CDS positions can do to promote her desired outcome is not to bid at all. The situation is reversed when the $NOI < 0$.

Continuing with the case of the $NOI > 0$, consider an example of one agent with a short CDS position. She has an incentive to bid the price as high as possible if the NOI is lower than the notional amount of her CDS contracts (provided she is allowed to hold defaulted bonds). This is because the cost of purchasing the bonds at a high auction price is offset by the benefit of cash-settling her CDSs at the same high price. In contrast, if the NOI is larger than the notional amount of her CDS position, she would not want to bid more than the fair value of the bond, v. This is because the cost of purchasing bonds at a price above v is not offset by the benefit of cash-settling CDSs. In what follows, we show that this intuition can be generalized to multiple agents, as a long as we consider the size of their aggregate net CDS positions relative to the NOI.

Proposition 1 *Suppose that $NOI > 0$ and Assumption 1 holds.*

1. *If*

$$\sum_{i \in \mathcal{N}_+ : n_i < 0} |n_i - y_i| \geq NOI, \tag{5}$$

and $p^M + s > v$, then in any equilibrium the final auction price $p^A \in [v, p^M + s]$. Furthermore, there always exists an equilibrium in which the final price is equal to the cap: $p^A = p^M + s$. If $p^M + s < v$ then the final price is always equal to the cap: $p^A = p^M + s$.

2. *If*

$$\sum_{i \in \mathcal{N}_+ : n_i < 0} |n_i - y_i| < NOI, \tag{6}$$

then only equilibria with $p^A \leq \min\{p^M + s, v\}$ exist.

Proof. *Part 1.* Intuitively, if condition (5) holds, there is a subset of agents for whom a joint loss incurred by acquiring a number of bonds equal to the NOI, at a price above v, is dominated by a joint gain from paying less on a larger number of short CDS contracts that remain after the physical settlement. As a result, these agents

bid aggressively and can push the auction price above v unless it is constrained by the IMM. In the latter case, $p^A = p^M + s$.

Formally, suppose that $p^M + s > v$, $p^A < v$ and condition (5) holds. We show that this cannot be true in equilibrium. Let the equilibrium allocation of bonds to agent i be q_i. Consider a change in the demand schedule of player i from x_i to x_i' that leads to the auction price $p \in [p^A, v]$. Denote the new bond allocation of agent i by q_i'. Since demand schedules are non-decreasing, $q_i' \geq q_i$. Agent i's change in profit is thus

$$\delta_i = \left[(v - p^A)q_i - p^A(n_i - y_i)\right] - \left[(v - p)q_i' - p(n_i - y_i)\right] =$$

$$= (p - p^A)(n_i - y_i + q_i) - (v - p)(q_i' - q_i) \leq (p - p^A)(n_i - y_i + q_i). \tag{7}$$

Equilibrium conditions require that $\delta_i \geq 0$ for all i. Summing over all i such that $n_i < 0$, it must be that

$$0 \leq \sum_{i:n_i<0} \delta_i \leq (v - p^A) \sum_{i:n_i<0} (n_i - y_i + q_i)\,.$$

Because all $q_i \geq 0$,

$$\sum_{i:n_i<0} (n_i - y_i + q_i) \leq \sum_{i:n_i<0} (n_i - y_i) + NOI \leq 0, \tag{8}$$

where we use (5). Thus in any equilibrium with $p^A < v$, it must be that $\delta_i = 0$ for all i with $n_i < 0$. (7) and (8) then imply that for any deviation x_i' that leads to $p \in [p^A, v]$, it must be that $q_i' = q_i$. Since this is true for any $p \in [p^A, v]$ the initial total demand $X(p)$ must be constant over $[p^A, v]$, and therefore $p^A = v$. Thus we arrive at a contradiction.

Next, consider the following set of equilibrium strategies:

$$x_i(p): \quad \begin{cases} x_i = NOI \times (n_i - y_i)/(\sum_{j:n_j<0}(n_j - y_j)) & \text{if} \quad v < p \leq p^M + s, \\ x_i = NOI & \text{if} \quad p \leq v, \end{cases}$$

for agents with net negative CDS positions after physical settlement request submission, and $x_i(p) \equiv 0$ for agents with positive CDS positions. It is not difficult to see

that it supports $p^A = p^M + s$.

Part 2. Finally, suppose that condition (5) does not hold and there exists an equilibrium with $p^A > v$. Then there also exists an i such that agent i's equilibrium second stage allocation $q_i > |n_i - y_i|$. Consider a variation of this agent's demand schedule, in which she submits zero demand at $p^A > v$ and demand equal to the NOI at $p^A = v$. Given this variation, the new auction price will be higher than or equal to v. Thus her profit increases by at least $(p^A - v)(q_i + n_i - y_i) > 0$, so $p^A > v$ cannot be an equilibrium outcome. *QED.*

The next lemma shows that when all agents are allowed to hold bonds after the auction (that is, Assumption 1 does not hold), condition (5) always holds. As a result, the final price is always at least v unless it is capped.

Lemma 1 *If $\mathcal{N}_+ = \mathcal{N}$ then condition (5) holds.*

Proof.

$$\sum_{i:n_i<0}(n_i - y_i) + NOI = \sum_{i:n_i<0}(n_i - y_i) + \sum_i y_i = \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} y_i \leq \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} n_i = 0.$$

QED.

Proceeding to the case where $NOI < 0$, we obtain the following result.

Proposition 2 *Suppose that $NOI < 0$ and there are no short-selling constraints. If $p^M - s < v$, then in any equilibrium, $p^A \in [p^M - s, v]$. If $p^M - s > v$ then $p^A = p^M - s$.*

This result is a natural counterpart of Part 1 of Proposition 1, and the proof follows the same logic. Without Assumption 2, we do not have a counterpart to Part 2 because all agents can participate in the second stage. With short-selling constraints, equilibria in which the bond is overpriced and the price is not capped can also exist. The conditions allowing for these equilibria are more stringent than those in Part 2 of Proposition 1 because the short-selling constraints are assumed to hold at the individual level. Proposition 3 characterizes these conditions.

Proposition 3 *Suppose that $NOI < 0$ and Assumption 2 is imposed.*

1. *If for all i such that $n_i > 0$,*

$$b_i \geq -NOI \times \frac{n_i - y_i}{\sum_{j:n_j>0}(n_j - y_j)} \tag{9}$$

then there exists an equilibrium in which $p^A = p^M - s$.

2. *If*

$$\sum_{i:n_i>0} b_i < -NOI, \tag{10}$$

then only equilibria with $p^A \geq \max\{p^M - s, v\}$ *exist.*

Proof. Part 2 is straightforward: under the assumption of short-sale constraints and (10), NOI units of bonds cannot be sold solely by agents with long CDS positions. Agents with non-positive CDS positions, however, will not sell bonds at a price below v. Thus we only need to prove Part 1. To do this, consider the following set of strategies (assuming that $p^M - s < v$):

$$x_i(p): \quad \begin{cases} x_i = NOI \times (n_i - y_i)/(\sum_{j:n_j<0}(n_j - y_j)) & \text{if} \quad p^M + s \leq p < v, \\ x_i = -b_i & \text{if} \quad p \geq v, \end{cases}$$

for agents with net positive CDS positions after physical request submission, and

$$x_i(p): \quad \begin{cases} x_i = 0 & \text{if} \quad p^M + s \leq p < v, \\ x_i = -b_i & \text{if} \quad p \geq v. \end{cases}$$

for agents with positive CDS positions. It is not difficult to see that this set of strategies constitutes an equilibrium and supports $p^A = p^M - s$. QED.

3.2 First Stage

To solve for a full game equilibrium, the last step is to determine physical settlement requests y_i, the NOI and the IMM, given the outcomes in the second stage of the auction. The IMM does not contain any information in our setup, which precludes uncertainty. Nevertheless, it can still play an important role because it provides a cap on the final price. We start our analysis by assuming that the second-stage auction does not have a cap. After we solve for (and develop intuition about) the optimal physical settlement requests and the NOI, we discuss the effect of the cap.

3.2.1 Second-Stage Auction Without a Cap

First, we show that in a frictionless world, only equilibria with the auction price different from v exist. Furthermore, in all of these equilibria agents obtain the same utility.

Proposition 4 *Suppose that there are no trading frictions, i.e. Assumptions 1 and 2 are not imposed. Then any equilibrium will be one of three types: (i) $p^A \in (v, 100]$ and $NOI \geq 0$, where agents with initial long CDS positions choose physical delivery and receive zero bond allocation in the auction; (ii) $p^A \in [0, v)$ and $NOI \leq 0$, where agents with initial short CDS positions choose physical delivery and do not sell bonds in the auction; and (iii) $p = v$. In each of the three cases, all agents attain the same utility.*

Proof. Suppose that $p^A \in (v, 100]$. Lemma 1, Part 2 of Proposition 1, and Proposition 2 imply that this can be the case only if $NOI \geq 0$. Clearly, only agents with negative remaining CDS positions after the first stage of the auction will be willing to buy bonds at a price above v. Agents with initial long CDS positions receive zero bond allocation. From (3) each of their utility functions will be

$$\Pi_i = n_i(100 - v) + (y_i - n_i)(p^A - v) + b_i v. \tag{11}$$

If $p^A > v$, utility (11) is maximized if y_i is as large as possible. Therefore, $y_i = n_i$ and $\Pi_i = n_i(100 - v)$ for $n_i > 0$. Thus in any such equilibrium agents with initial long CDS positions choose physical delivery, receive zero bond allocation, and attain the same utility. The NOI is

$$NOI = \sum_i y_i = \sum_{i:n_i>0} n_i + \sum_{i:n_i<0} y_i = -\sum_{i:n_i<0} (n_i - y_i) \geq 0. \tag{12}$$

In other words, the NOI is equal to the sum of outstanding CDS positions (after the first stage) held by agents with initial short CDS positions. As a result, any gain from buying at a price above v (due to the existing CDS positions) is exactly offset by the loss incurred by buying bonds at this price. From (3), the utility of agents

with initial short CDS positions is given by

$$\Pi_i = n_i(100 - v) + (y_i - n_i - q_i)(p^A - v) + b_i v. \tag{13}$$

Because every agent can always guarantee utility $\Pi_i = n_i(100-v)$ by choosing physical delivery, q_i cannot be higher than $-(n_i - y_i)$. In addition, (12) implies that q_i cannot be lower than $-(n_i - y_i)$. Therefore, $q_i = -(n_i - y_i)$ and $\Pi_i = n_i(100 - v)$ for each $i : n_i < 0$. The proof when $p^A \in [0, v)$ is similar. *QED*.

Proposition 4 shows that in a frictionless world, all mispricing equilibria are unidirectional – that is, there is no under- (over-) pricing if the *NOI* is positive (negative). Furthermore, agents can undo any loss of utility resulting from auction mispricing by optimally choosing between cash and physical settlement of their positions.

We now turn to more realistic setups that include trading frictions. Our analysis in section 3.1 shows that there can be a continuum of equilibria in the second stage, which makes solving for every equilibrium in a two-stage auction a daunting problem. Instead of characterizing all of the equilibria, we show that in the presence of trading frictions, as outlined in Section 2.4, there exists a subset of equilibria of the two-stage game that results in bond mispricing. This result answers, in the affirmative, our main question as to whether mispricing is possible in the auction. Proposition 5 characterizes sufficient conditions for underpricing to occur.

Proposition 5 *Suppose that Assumption 1 holds,*

$$(i) \quad \sum_{i:n_i>0} n_i + \sum_{i \in \mathcal{N}_+ : n_i<0} n_i > 0, \tag{14}$$

and for any $n_i > 0$,

$$(ii) \quad n_i > \frac{\sum_{j:n_j>0} n_j + \sum_{j \in \mathcal{N}_+ : n_j<0} n_j}{K+1}, \tag{15}$$

where K is a total number of agents with initial long CDS positions. Then there exist a multitude of subgame perfect underpricing equilibria for the two-stage auction, in which (i) $NOI > 0$,

$$(ii) \quad \frac{\partial p^A(NOI)}{\partial NOI} < 0, \quad \textit{and (iii)} \quad 0 \le v - p^A(NOI) \le NOI \times \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|. \tag{16}$$

In particular, there exists a subset of full-game equilibria where for any NOI that can be realized in the first stage, the second stage leads to a final price p^A which is a linear function of the NOI:

$$p^A = v - \delta \times NOI \geq 0, \quad \delta > 0. \tag{17}$$

Proof. See Appendix.

We give a formal proof by construction in the Appendix and merely describe the intuition here. In the proof, we show that if optimal physical settlement requests satisfy condition (6) instead then there exist second-stage equilibria with $p^A \leq v$, where agents play the following strategies:

$$x_i(p) = \max\{c(v-p)^\lambda - n_i + y_i, 0\}, \tag{18}$$

c and λ are specified in the Appendix. A similar set of strategies is used in Back and Zender (1993) to construct equilibria in a standard auction without CDS positions. There could also be other classes of equilibrium second-stage strategies. We use strategies (18) mainly because they lead to a closed form solution. The main challenge in the rest of the proof is to solve jointly for equilibrium physical settlement requests and the second-stage equilibrium price.

A closer inspection of (3) reveals that if the final auction price is lower than v and is not affected by agents' physical requests (i.e., participants always choose to play same-price equilibria as long as the NOI is high enough to ensure second-stage underpricing), agents with long (short) CDS positions only have an incentive to choose full cash (physical) settlement in the first stage. This first-stage play implies that the NOI must be negative. As a result, second-stage underpricing equilibria in which $\partial p^A / \partial NOI = 0$ cannot be equilibria of the full game. However, if the strategies played in the second stage are such that the final auction price is a negative function of the NOI, then the incentives of agents with long CDS positions become non-trivial. Submission by such agents of a partial physical settlement request could lead to a larger NOI and in turn to a lower final auction price, increasing the payoff they receive from their partial cash settlement. The larger the initial positions of agents with long CDS positions, the stronger the incentives to lower the price via partial physical settlement. Condition (15) guarantees that the long positions of agents are

sufficiently large to ensure that they choose physical settlement of enough positions to render the resulting NOI positive.

The subset of equilibria characterized in Proposition 5 is the simplest and serves as an example of underpricing. There may be other equilibria resulting in underpricing that we have not found. While Lemma 1 implies that condition (14) is necessary for an underpricing equilibrium to exist, condition (15) can be relaxed at the expense of a more complicated proof.

Finally, notice that if there are short-sale constraints (that is, Assumption 2 is imposed), the logic of Proposition 4 may also break down. In this case, agents with initial long CDS positions are able to choose only b_i units of bonds for physical settlement. If for at least one such agent $n_i > b_i$, and sufficiently many agents with remaining short CDS positions participate, the agents as a group could become strictly better off by pushing the price above v. Proposition 6 characterizes the effect of short-sale constraints on auction outcomes.

Proposition 6 *Suppose that only Assumption 2 is imposed and there exists an i such that*

$$n_i > b_i > 0, \tag{19}$$

and

$$\sum_{j:n_j<0} |n_j| > \sum_{j:n_j>0} \max\{b_j, 0\}. \tag{20}$$

Then for the two-stage auction there exists a subgame perfect overpricing equilibrium in which $NOI = \sum_{j:n_j>0} \max\{b_j, 0\} > 0$, $p^A = 100$, and agents with initial short CDS positions attain strictly greater utility than when $p^A = v$.

Proof. The proof is by construction. As in Proposition 4, if $p^A = 100$, agents who are initially long CDS contracts will choose physical delivery, and only agents with negative remaining CDS positions after the first stage will be willing to buy bonds in the auction. Proposition 1 Part 1 shows that for any $NOI > 0$, if condition (5) holds (which turns out to be the case in the constructed equilibrium), then $p^A = 100$ is an equilibrium of the second stage if agents play the following strategies:

$$x_i(p): \quad \begin{cases} x_i = NOI \times (n_i - y_i)/(\sum_{j:n_j<0}(n_j - y_j)) & \text{if} \quad v < p \leq 100, \\ x_i = NOI & \text{if} \quad p \leq v. \end{cases}$$

for agents with net negative CDS positions after physical request submission, and $x_i(p) \equiv 0$ for other agents. The profit earned by agent i with $n_i < 0$ is therefore

$$\Pi_i = \left(y_i - NOI \frac{n_i - y_i}{\sum_{j:n_j<0}(n_j - y_j)} \right) (100 - v) + b_i v. \tag{21}$$

Taking the F.O.C. at $y_i = 0$, one can verify that it is optimal for agents with initial short CDS positions to choose cash settlement. Thus $NOI = \sum_{j:n_j>0} \max\{b_j, 0\}$ and the profit accruing to any agent i with an initial short CDS position $n_i < 0$ is

$$\Pi_i = -(100 - v)n_i \times \frac{\sum_{j:n_j>0} \max\{b_j, 0\}}{\sum_{j:n_j<0} n_j} + b_i v > (100 - v)n_i + b_i v,$$

where the expression on the right hand side is the agent's utility if p^A is equal to v. QED.

Propositions 5 and 6 show that there can be either underpricing or overpricing equilibria in the two-stage game with $NOI > 0$, if there are trading frictions. A similar set of results can be obtained for $NOI < 0$.

3.2.2 Second-Stage Auction with a Cap

We now discuss the implications of the second-stage price cap, which imposes an upper bound of $p^M + s$ on the final price. In the presence of the cap, mispricing in the auction depends on the bidding behavior of dealers in the first stage. The next proposition shows that the IMM is equal to v when there are no trading frictions.

Proposition 7 *Suppose that there are no trading frictions (Assumptions 1 and 2 are not imposed) and $\gamma > 0$. Then $IMM = v$. Therefore, of the overpricing equilibria described in Proposition 4 there can exist only equilibria with $|p^A - v| \leq s$.*

Proof. Proposition 4 shows that in all possible equilibria, common participants attain the same utility. Because dealers have regulatory and reputational concerns, captured by the extra term $-\gamma(\pi_i - v)^2$, their optimal quotes, π_i, are equal to v. Thus, $IMM = v$. QED.

This result further restricts the set of possible full-game equilibria. When there are no frictions, the final auction price cannot differ from the fair value of the bond by more than the size of the spread, s.

In the presence of frictions, there can be either underpricing or overpricing equilibria in the auction without a cap (Propositions 5 and 6). The cap cannot eliminate underpricing equilibria.[4] Additionally, if the cap is set too low it rules out equilibria with $p^A = v$.

The cap can, however, be effective at eliminating overpricing equilibria. As an illustration, consider a simple example in which all dealers have zero CDS positions.[5] Proposition 6 shows that when there are short-sale constraints, the final auction price can be as high as 100 in the absence of a cap. Following the same logic as in Proposition 6, one can show that if the cap is greater than v, then there exists an equilibrium with the final price equal to the cap. Since in any such equilibrium dealers do not realize any profit but have regulatory and reputational concerns, their optimal quotes are equal to v. Thus, $IMM = v$ and $p^A = v + s$.

4 Empirical Evidence

Our theoretical analysis shows that CDS auctions may result in both overpricing and underpricing of the underlying bonds. In this section, we seek to provide empirical evidence indicating which outcomes occur in practice. Unfortunately, the true value of deliverable bonds is never observed. Because of this, we use available bond prices from the day before the auction to construct a proxy for the bond value v. Admittedly this measure is not a perfect substitute for the true value of the bond, and so we also consider a number of alternatives to show the robustness of our results. We first describe our data before presenting the empirical analysis.

[4]For example, consider an extreme case in which all dealers have large positive CDS positions and the conditions of Proposition 5 hold. Following the logic of Proposition 5, one can show that there exists a subgame perfect equilibrium in which $p^A = 0$ and $IMM = v$.

[5]While this is a simplification it is arguably also realistic, as dealers try to maintain zero CDS positions in their capacity as market makers.

4.1 Data

Our data come from two primary sources. The details of the auction settlement process are publicly available from the Creditfixings website (www.creditfixings.com). As of December 2010, there have been 86 CDS and Loan CDS auctions, settling contracts on both US and international legal entities. To study the relationship between auction outcomes and the underlying bond values, we merge these data with bond price data from the TRACE database. TRACE reports corporate bond trades for US companies only. Thus, our merged dataset contains 23 auctions.

Table 2 summarizes the results of the auctions for these firms. It reports the settlement date, the type of credit event and the auction outcomes. Most of the auctions took place in 2009 and were triggered by the Chapter 11 event. In only two of the 23 auctions (Six Flags and General Motors) was the net open interest to buy ($NOI < 0$). The full universe of CDS auctions contains 61 auctions in which the net open interest was to sell, 19 auctions where the net open interest was to buy, and 6 auctions with zero net open interest.

Table 3 provides summary statistics of the deliverable bonds for each auction for which we have bond data.[6] Deliverable bonds are specified in the auction protocols, available from the Creditfixings website. The table also reports the ratio of net open interest to the notional amount of deliverable bonds (NOI/NAB). This shows how many units of bonds changed hands during an auction, as a percentage of the total amount of bonds. There is strong heterogeneity in NOI/NAB across different auctions, with absolute values ranging from 0.38% to 56.81%. In practice, NOI has never exceeded NAB.

We construct daily bond prices by weighing the price for each trade against the trade size reported in TRACE, as in Bessembinder, Kahle, Maxwell, and Xuet (2009). These authors advocate eliminating all trades under $100,000 as they are likely to be non-institutional. The larger trades have lower execution costs; hence they should reflect the underlying bond value with greater precision. For each company, we build

[6] A clarification regarding the auctions of Abitibi and Bowater is in order. AbitibiBowater is a corporation, formed by Abitibi and Bowater for the sole purpose of effecting their combination. Upon completion of the combination, Abitibi and Bowater became subsidiaries of AbitibiBowater and the businesses that were formerly conducted by Abitibi and Bowater became the single business of AbitibiBowater. The CDS contracts were linked to the entities separately, and, as a result, there were two separate auctions.

a time-series of bond prices in the auction event window of -30 to +30 trading days. Because all credit events occur no more than one calendar month before the CDS auction, our choice of the event window ensures that our sample contains all relevant data for the post-credit-event prices. The last column of Table 3 reports a weighted average bond price on the day before the auction, p_{-1}. We use this as our proxy for the bond value v.

4.2 The Impact of the First Stage

The theoretical results of Section 3 imply that the first and the second stages of the auction are not independent. The first stage yields the mid-point price, p^M, which determines a cap on the final settlement price. Our model shows that when the final price, p^A, is capped, it can be either above or below the true value of the bond, v, depending on the initial CDS and bond positions of different agents.

Our analysis suggests a way of differentiating between the two cases. To be more specific, consider outcomes in which $NOI > 0$ (outcomes in which $NOI < 0$ follow similar logic). According to Proposition 1 Part 1, the price can be higher than v if, after the first stage, the aggregate short net CDS position of agents participating in the second stage is larger than the net open interest. In this case, protection sellers have an incentive to bid above the true value of the bond to minimize the amount paid to their CDS counterparties. Notice that while bidding at a price above v, they would like to minimize the amount of bonds acquired at the auction for a given final auction price. Thus, if the price is above v they will never bid to buy more than NOI units of bonds.

The case in which p^A is capped and lies below the true value of the bond is brought about when dealers set p^M so that $p^M + s$ is below v. This prevents the agents from playing second-stage equilibrium strategies with the final price above the cap. In this case, submitting a large demand at the cap price leads to greater profit. Thus, in the presence of competition and sharing rules, agents have an incentive to buy as many bonds as possible and would bid for substantially more than NOI units.

The final price is capped in 19 of the 86 credit-event auctions.[7] Figure 2 shows the entities and the individual bids at the cap price. The individual bids are represented

[7]Of these 19 auctions, only one (Ecuador) has a negative NOI. So the above discussion for the case of positive NOI should be adjusted appropriately for Ecuador.

by different colors, and bid sizes are scaled by NOI to streamline their interpretation. For example, there are seven bids at the cap price in the case of General Growth Properties. Six of these are equal to NOI and the seventh one is approximately one-fourth of NOI.

We can see that in all but two auctions (Kaupthing Bank and Glitnir), the bids at the price cap do not exceed NOI. The results suggest that in these cases the final auction price is above the true bond value. Of the 19 auctions with a capped price, we have bond data for only five companies: Smurfit-Stone, Rouse, Charter Communications, Capmark and Bowater. Comparing the final auction price from Table 2 with the bond price from Table 3, we can see that that the bond price (our proxy for the true bond value) is below the final auction price for these five companies, as expected.

We can compare the bond and auction prices for the rest of the companies for which TRACE data are available. Figure 3 shows the ratio of final auction prices to bond prices, p^A/p_{-1}. We see that in all but seven auctions, the final auction price, p^A, is below the bond price, p_{-1}. It seems likely that underpricing equilibria were played out in these auctions. The exceptions include the aforementioned five companies with capped auction prices, as well as the General Motors and Six Flags auctions, where the price was not capped but $NOI < 0$. In these last two cases, the auction prices are expected to exhibit a reverse pattern.

4.3 Price Impact at the Second Stage

In the preceding section, our evidence showed that in the absence of a cap, the auction yields a price below the bond value. According to Proposition 1, if $NOI > 0$ such an outcome can occur only if the aggregate net short CDS position of the agents who participate in the second stage, $\sum_{i\in\mathcal{N}_+:n_i<0} |n_i - y_i|$, is smaller than or equal to the net open interest. But as we do not have data on individual bids and positions we cannot test this proposition directly. Instead, we provide empirical evidence that complements our theoretical analysis. Specifically, we study the effect of the NOI on the degree of price discrepancy resulting from the auction. We scale the net open interest by the notional amount of deliverable bonds, giving the quantity NOI/NAB, to allow for a meaningful cross-sectional examination.

Tables 2 and 3 reveal that NOI/NAB is greatest in the auctions with the largest discrepancy in prices. At the same time, NOI/NAB is lowest in the auctions where the final price is capped, which is again consistent with Propositions 1 and 5. We quantify this relationship using a simple cross-sectional regression of p_{-1}/p^A on NOI/NAB :

$$p^A/p^{-1} = \alpha + \beta \times NOI/NAB + \varepsilon. \tag{22}$$

Figure 4 shows the results. The normalized NOI explains 55% of the variation in the ratio of the lag of the market price of bonds to the final price. The β is significantly negative. For every one-percentage-point increase in the normalized NOI, the underpricing increases by 1.2%.

This evidence is consistent with Proposition 5, which shows that there exist second-stage equilibria in which the final price, p^A, depends linearly on the NOI (equation (17)). Since the only theoretical restriction on the slope (δ) is its sign, the linear relationship (17) can be written as

$$p^A/v = 1 + \beta \times NOI/NAB, \quad \beta < 0.$$

If agents play equilibrium strategies with the same β across auctions, the estimated cross-sectional regression β will also be an estimate of the within-auction relationship. While the assumption of the same linear dependence across auctions is admittedly strong, it can be accommodated by the following argument. If all agents in an auction take historical information about previous types of equilibria into account when forming their perceptions, then β is unlikely to vary much across auctions. Finally, the estimated α is insignificantly different from one, which is again consistent with the theory.

4.4 Robustness Checks

4.4.1 Fair Value Proxy

Our conclusions so far rest on the assumption that p_{-1} is a good proxy for the actual fair value v. One could argue that auctions exist precisely because it is difficult to establish a bond's fair value by observing bond markets. Moreover, even if p_{-1} were to reflect the bond value accurately, it would still be the value on the day before the

auction. It is conceivable that the auction process establishes a v that differs from p_{-1} simply due to the arrival of new information between time -1 and 0, and/or as a result of the centralized clearing mechanism of the auction.

We expand the auction event window to check the robustness of our results to these caveats. In our sample, the shortest time between a credit event and an auction is 8 days. This prompts us to select an event window of -8 to +12 days. The choice of boundary is dictated by liquidity considerations: liquidity generally declines after the auction. Figure 5 (a) displays daily bond prices normalized by the auction final price, p_t/p^A, equally weighted across the 22 auctions for which we have reliable bond data.[8] We see that the price generally declines, reaches its minimum on the auction day, then reverts to its initial level. The figure shows that no matter which day we look at, the auction final price is, on average, at least 10% lower.

We have a sample of 22 auctions with reliable data. This small sample size may raise concerns that our results are sensitive to outliers. In what follows, we discuss the effects of two types of such outliers.

First, the Tribune auction stands out because of the large magnitude of the underpricing it generated. This can be seen in Figure 4: the point in the lower right corner of the plot. The normalized NOI was also the largest, so the magnitude of the underpricing on its own is consistent with our theory. Nonetheless, to be sure that the pattern of average prices is not driven by this one company, we remove Tribune from our sample and recompute the pattern. Figure 5 (b) shows the results. We see that the magnitude of the average smallest underpricing declines to 5%, but all qualitative features remain intact.

Second, there are six auctions that resulted in overpricing. Four auctions (Smurfit-Stone, Rouse, Capmark and Bowater) had positive NOI and final price equal to the cap (see Section 4.2 for details). The two remaining auctions (GM and Six Flags) had negative NOI. Therefore, the presence of these names in our average may only bias our results against finding underpricing.

The documented V shape of the discrepancy alleviates the concern that the correct value v differs from p_{-1} simply because the latter does not reflect the bond value

[8] We exclude the auction for Charter, which has only 10 trades in the [-10,0] window during which our proxy for v is constructed. Of these 10 trades, only 6 are in sizes greater than \$1M. The second-worst company in terms of data reliability, Chemtura, has 35 trades and all of them are above \$1M.

correctly. If this were the case, one would expect bond prices to remain in the region of the auction price after the auction, whereas in practice they increase.

4.4.2 The Cheapest-to-Deliver Option

Another potential concern about using weighted daily bond prices as a proxy for the underlying value of auctioned bonds is that agents will likely use only cheapest-to-deliver bonds for physical delivery. As a result, our methodology may overestimate the fair value. This argument is not applicable when the credit event is Chapter 11, and all the deliverable bonds are issued by the holding company and cross-guaranteed by all subsidiaries. In Chapter 11, bonds with no legal subordination are treated as identical, see for example Guha (2002).[9] The reasons for this are that all the bonds stop paying coupons and mature (cease to exist) at the same time, with identical terminal payouts to all bondholders. Hence there is no concern that some bonds are cheaper to deliver due to the difference in their fundamental value.

As an example, Figure 6 shows weighted daily prices of each individual WaMu bond issue, identified by its CUSIP. We see that there are large difference between the prices of different bonds in the period leading to the credit event (trading day -19). After this day the prices of all bonds are very similar. The prices cannot be literally identical because trades may occur at different times of the day, and because trades may be either buyer- or seller-initiated which means prices will be closer to bid or ask prices, respectively.

In our sample, 13 out of 23 credit events are triggered by Chapter 11 bankruptcy and have one issuer. These companies should not have bonds that diverge in value. Nonetheless, we manually confirm that this is indeed the case. There are three companies that filed for Chapter 11 and have multiple subsidiaries issuing bonds, but for which TRACE contains trade data for only one subsidiary in the event window (CIT, Lyondell, and Quebecor). We treat these three names the same way as the 13 firms without subsidiaries.

There are four companies that filed for Chapter 11 and have multiple subsidiaries, and where we have data for the bonds of these subsidiaries (Bowater, Charter, Nortel and Smurfit-Stone). In all of these cases the bonds of the different subsidiaries are

[9] CDS contracts on bonds with different seniorities are settled in different auctions. Examples of this in our data are the Dura/Dura Sub auctions.

legally pari-passu with each other, but some of them may be structurally subordinated to others and, therefore, could be cheaper. For this reason, we select the cheapest bonds in the case of these four companies (however, the differences are not large in practice). There are three companies with a credit event other than Chapter 11 (Abitibi, Capmark and Rouse) in which we also select the cheapest bonds.

Finally, to account for other potential deliverables selection issues that could work against our findings, we treat the aforementioned differences in bond prices (due to bid-ask spread and timing differences) as real differences, and select the lowest-priced bonds. Specifically, we take representative daily prices of a company's deliverable bonds to be equal to the weighted daily prices of their bond issues with the lowest pre-auction price, provided that these bond issues are relatively actively traded.[10] The results are displayed in Figure 5. It can be seen that even with these conservative bond selection criteria, the average underpricing on the day of the auction is still 10%, and follows a V pattern as before.[11]

5 Extensions

Section 4 documents our finding that when NOI/NAB is large, the auction generally results in a price considerably below fair value. We now suggest several modifications to the auction design that can reduce mispricing, and discuss some of the assumptions of the model.

5.1 Allocation Rule at the Second Stage

As usual, we focus on the case of $NOI > 0$. Proposition 1 shows that if condition (6) holds, the CDS auction is similar to a 'standard' auction, so the price can be below v. Kremer and Nyborg (2004) show that in a setting without CDS positions, a simple change of the allocation rule from pro-rata on the margin (2) to 'pro-rata' destroys all underpricing equilibria, so that only $p^A = v$ remains. Under the pro-rata rule, the

[10] The requirement is that the trading volume over the five trading days before the auction constitutes at least 5% of total trading volume for the company.

[11] Gupta and Sundaram (2011) address the cheapest-to-deliver issue using an alternative procedure based on econometric modelling of issue-specific pricing biases, and arrive at similar conclusions.

equilibrium allocations q_i are given by

$$q_i(p^A) = \frac{x_i(p^A)}{X(p^A)} \times NOI.$$

That is, the total rather than marginal demand at p^A is rationed among agents. The next proposition extends the result of Kremer and Nyborg (2004) to our setting. We demonstrate that if $p^M + s \geq v$, then the second-stage equilibrium price p^A cannot be less than v. This is true even if the agents are allowed to hold non-zero quantities of CDS contracts.

Proposition 8 *Suppose that the auction sharing rule is pro-rata. In this case, if* $NOI > 0$ *then* $p^A \geq \min\{p^M + s, v\}$. *If* $NOI < 0$ *then* $p^A \leq \max\{p^M - s, v\}$.

Proof. See Appendix.

To develop intuition for this result, consider the case of positive NOI. According to Proposition 1 Part 2, if condition (6) holds, the pro-rata on the margin allocation rule may inhibit competition and lead to underpricing equilibria. The presence of agents who are short CDS contracts does not help in this case. The pro-rata allocation rule (i) does not guarantee the agents their inframarginal demand above the clearing price, and (ii) closely ties the proportion of allocated bonds to the ratio of individual to total demand at the clearing price. Therefore, a switch to such a rule would increase competition for bonds among agents. As a result, even agents with long positions would bid aggressively. If $p^A < v$, demanding the NOI at a price only slightly higher than p^A allows an agent to capture at least half of the surplus. As a result, only fair-price equilibria survive.

5.2 The Price Cap

Our theoretical analysis in Section 4.2 shows that the presence of a price cap can result in auction outcomes with either lower or higher mispricing. The cap is likely to help when $|NOI|$ is small and the temptation to manipulate the auction results is highest. At the same time, the cap allows dealers to limit the final price to below v in the second stage.

These results suggest that making the cap conditional on the outcome of the first stage of a CDS auction can lead to better outcomes. In our base model without uncertainty, the optimal conditional cap is trivial. Again, we consider the case of $NOI > 0$. If $p^M < v$, setting $s^* = v - p^M$ ensures that the set of second-stage equilibria includes v. If $p^M \geq v$, it is best to set $s^* = 0$. While the conditional cap cannot eliminate the worst underpricing equilibria, it can ensure that agents who want to bid aggressively will be able to do so.

In practice, v and n_i are unobservable. Thus, making the cap conditional on NOI and on the ratio p^M/p_{-1} could lead to the final auction price being closer to the fair bond value. For example, if $p^M/p_{-1} \leq \alpha$, NOI is large and $\alpha < 1$ is reasonably small, the auctioneer can set a higher cap; if $p^M/p_{-1} > \alpha$ and NOI is small, a lower cap can be set.

5.3 Risk-averse agents

So far we have considered only risk-neutral agents. This allowed us to abstract from risk considerations. If agents are risk-averse, the reference entity's risk is generally priced. Even though a CDS is in zero net supply, its settlement leads to a reallocation of risk among the participants in the auction; hence it can lead to a different equilibrium bond price. In particular, when NOI/NAB is large and positive, and there are only a few risk-averse agents willing to hold defaulted bonds, the auction results in highly-concentrated ownership of the company's risk and can thus lead to a lower equilibrium bond price.

Notice, however, that risk-aversion does not automatically imply a lower auction price. For example, if marginal buyers of bonds in the auction are agents who previously had large negative CDS positions (as in Proposition 5), their risk exposure after the auction may actually decrease. As a result they could require a lower risk premium.

Due to the fact that we do not have data on individual agents' bids and positions, we cannot determine whether the observed price discrepancy is due to mispricing equilibria or risk-aversion. It is likely that both factors work together in the same direction. Data on individual agents' bids and positions could help to quantify the effect of the two factors on the observed relationship between the auction price and

the size of net open interest.

5.4 Private information

Up to this point we have restricted our attention to the simplest case in which agents' CDS positions are common knowledge. This may seem like a very strong assumption given that CDS contracts are traded in the OTC market. Notice, however, that in the type of equilibria constructed in Propositions 5 (linear case) and 6, conditions (14), (15) and (19), (20) completely define the two equilibria. Therefore, Propositions 5 and 6 continue to hold with private CDS positions as long as (14), (15) and (19), (20) are public knowledge.[12] One can argue that this is likely to be the case. For example, (20) assumes that total short CDS positions are larger than total bond holdings of agents with long CDS positions. The aggregate net CDS positions are known to market participants.[13] Therefore, whether condition (20) holds can be easily verified in every auction. Similarly, (19) assumes that there is an agent whose long position in CDSs is larger than her bond holdings. Given the much larger size of CDS contracts compared to the value of bonds outstanding, (19) holds as long aggregate long CDS positions are larger than the value of the outstanding bonds. The latter is true for most (if not all) of the auctions.

We also assume that agents value bonds identically, and that this value is common knowledge. This assumption provides a stark benchmark: we are able to show that the auction results in mispricing even in such a basic case. We conjecture that it would be even harder for the current auction mechanism to arrive at the fair value when agents have private or heterogeneous valuations.

6 Conclusion

We present a theoretical and empirical analysis of the settlement of CDS contracts when a credit event takes place. A two-stage, auction-based procedure aims to establish a reference bond price for cash settlement and to provide market participants

[12]The formal proofs follow closely the original proofs for the full information case and are available upon request.

[13]For example, they are available from Markit reports.

with the option to replicate a physical settlement outcome. The first stage determines the net open interest (NOI) in the physical settlement and the auction price cap (minimum or maximum price, depending on whether the NOI is to sell or to buy). The second stage is a uniform divisible good auction with a marginal pro-rata allocation rule that establishes the final price by clearing the NOI.

In our theoretical analysis, we show that the auction may result in either overpricing or underpricing of the underlying bonds. Our empirical analysis establishes that the former case is more prevalent in practice. Bonds are underpriced by 10% on average, and the amount of underpricing increases with the NOI (normalized by the notional amount of deliverable bonds). We propose introducing a pro-rata allocation rule and a conditional price cap to mitigate this mispricing.

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Appendix

Proof of Proposition 5

The proof is by construction. We construct a subgame perfect two-stage equilibrium in which the final auction price is a decreasing function of the NOI. In a similar fashion to Kremer and Nyborg (2004), it can be shown that one's attention can be restricted w.l.o.g. to equilibria in differentiable strategies. For simplicity, we provide the proof for the case in which agents have large long CDS positions. Specifically, we assume that for all $i : n_i > 0$:

$$n_i \geq NOI. \tag{A1}$$

Under this additional assumption, we can solve for the equilibrium in closed form. The general case follows similar logic, except that the number of the agents who submit nonzero demand for bonds at the second stage depends on the configuration of CDS positions. When A1 holds, only agents with non-positive CDS positions receive nonzero allocations in the equilibrium.

The proof consists of several steps. In step 1, we derive the F.O.C. for the optimal strategies at the second stage, given the remaining CDS positions of the agents after the first stage. In step 2, we derive the F.O.C. for the optimal physical settlement requests. In step 3, we show that the second-stage equilibrium with price p^A can be supported if agents play the following second-stage strategies:

$$x_i(p) = \max\{c(v-p)^\lambda - n_i + y_i, 0\}$$

(c and λ are specified later). In step 4, we solve for optimal physical requests of agents, given the above second-stage strategies. Finally, we solve for the NOI.

Step 1. Recall that at the second stage, player i solves problem (4):

$$\max_{x_i(p)} \; (v - p(x_i(p), x_{-i}(p)))\, q_i(x_i(p), x_{-i}(p)) + (n_i - y_i) \times (100 - p(x_i(p), x_{-i}(p)))\,.$$

In any equilibrium of the second stage, the sum of the demand of agent i, $x_i(p^A)$, and the residual demand of the other players, $x_{-i}(p^A)$, must equal the NOI. Therefore, solving for the optimal $x_i(p)$ is equivalent to solving for the optimal price, p^A, given the residual demand of the other players. Thus, the F.O.C. for agent i at the equilibrium

price, p^A, can be written as

$$(v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i = 0 \qquad \text{if} \qquad x_i(p^A) > 0, \tag{A2}$$

$$(v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i \geq 0 \qquad \text{if} \qquad x_i(p^A) = 0. \tag{A3}$$

Step 2. Recall that agent i's profit is given by equation (3):

$$\begin{aligned}\Pi_i \;=\; & \underbrace{(v - p^A)q_i}_{\text{auction-allocated bonds}} + \underbrace{(n_i - y_i) \times (100 - p^A)}_{\text{remaining CDS}} \\ & + \underbrace{100y_i}_{\text{physical settlement}} + \underbrace{v(b_i - y_i)}_{\text{remaining bonds}}.\end{aligned}$$

Using the fact that $\partial NOI/\partial y_i = 1$, we have that the F.O.C. for the optimal settlement amount, y_i, for agent i, satisfies

$$\frac{\partial \Pi_i}{\partial y_i} = 0 \quad \text{if} \quad y_i \neq 0 \quad \text{and} \quad y_i \neq n_i, \tag{A4}$$

$$\frac{\partial \Pi_i}{\partial y_i} \leq 0 \quad \text{if} \quad y_i = 0 \quad \text{and} \quad n_i > 0, \quad \text{or} \quad y_i = n_i \quad \text{if} \quad n_i < 0, \tag{A5}$$

$$\frac{\partial \Pi_i}{\partial y_i} \geq 0 \quad \text{if} \quad y_i = 0 \quad \text{and} \quad n_i < 0, \quad \text{or} \quad y_i = n_i \quad \text{if} \quad n_i > 0, \tag{A6}$$

where

$$\frac{\partial \Pi_i}{\partial y_i} = -\frac{\partial p^A(NOI)}{\partial NOI}(n_i - y_i + q_i) - (v - p^A(NOI))\left(1 - \frac{\partial q_i}{\partial y_i}\right). \tag{A7}$$

Step 3. Let M be the number of agents with nonpositive CDS positions who are allowed to hold bonds, and let $\lambda = 1/(M-1)$. Then consider the following set of strategies at the second stage:

$$x_i(p) = \max\left\{\frac{NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0}(n_j - y_j)}{M}\frac{(v-p)^\lambda}{(v - p^A(NOI))^\lambda} - n_i + y_i, 0\right\}. \tag{A8}$$

Demand schedules (A8) imply that agents with non-positive CDS positions who

are allowed to hold bonds receive, at $p = p^A$, the following bond allocations:

$$q_i = \frac{NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j)}{M} - (n_i - y_i). \tag{A9}$$

Equation (A3) implies that agents with initial long CDS positions receive zero equilibrium bond allocations at the second stage, as long as

$$n_i - y_i \geq \frac{NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j)}{M - 1}. \tag{A10}$$

If this is the case, equation (A2) implies that strategies (A8) form an equilibrium at the second stage, with the equilibrium price equal to p^A.[14]

Step 4. Consider now the optimal physical settlement requests of agents with initial short CDS positions. We need consider only those agents who are allowed to hold bonds after the auction. As part of the equilibrium constructed in step 3, these agents receive q_i units of bonds, as given in (A9). So we can write condition (A7) as

$$\frac{\partial \Pi_i}{\partial y_i} = -\frac{\partial p^A(NOI)}{\partial NOI} \frac{NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j)}{M} - \frac{v - p^A(NOI)}{M}. \tag{A11}$$

For simplicity, we solve for the interior solution so that $\frac{\partial \Pi_i}{\partial y_i} = 0$. Direct computations show that in such an equilibrium it must be the case that

$$NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j) = \left(v - p^A(NOI)\right) / \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|. \tag{A12}$$

Now consider the optimal physical settlement requests of agents with initial long CDS positions. If these agents receive a zero equilibrium bond allocation, conditions

[14] Technically, one extra condition is needed to ensure the existence of the constructed equilibrium. Inequality (A10) must continue to hold for every possible deviation $\hat{y}_j > y_j$ by each participant $j \in \mathcal{N}$. If for some such $\hat{y}_j$ this condition breaks down for agent i with a long CDS position, then this agent will participate in the second stage of the auction, which could increase the profit earned by agent j. (Of course, agent i could increase y_i itself, in which case $j = i$). This extra condition does not hold for $\hat{y}_i > y_i$ when $M = 2$, which leads to existence of $p^A = 0$ underpricing equilibria only. When $M > 2$ there exist underpricing equilibria with $p^A > 0$, in which out-of-equilibrium submission of physical settlement requests does not lead agents with long CDS positions to participate in the second stage of the auction. The details are available upon request.

(A4) and (A7) imply that their optimal physical requests satisfy

$$y_i = \max\left\{ n_i - \left(v - p^A(NOI)\right) / \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|, 0 \right\}. \tag{A13}$$

Using equilibrium condition (A12) together with condition (A10), we can see that agents with initial long CDS positions will receive a zero equilibrium bond allocation at the second stage if

$$n_i \geq \frac{\left(v - p^A(NOI)\right) / |\frac{\partial p^A(NOI)}{\partial NOI}|}{M-1}. \tag{A14}$$

Assumption (A1), along with condition (16), guarantee an interior solution for the optimal physical requests of agents with initial long CDS positions.

Step 5. Finally, the optimal physical requests of the agents must sum to the NOI:

$$\sum_{i:n_i>0} \left(n_i - \frac{v - p^A(NOI)}{\left| \frac{\partial p^A(NOI)}{\partial NOI} \right|} \right) + \sum_{i \in \mathcal{N}_+ : n_i < 0} y_i = NOI. \tag{A15}$$

Using (A12), we can write (A15) as

$$\sum_{i:n_i>0} n_i + \sum_{i \in \mathcal{N}_+ : n_i < 0} n_i - \frac{v - p^A(NOI)}{\left| \frac{\partial p^A(NOI)}{\partial NOI} \right|} (K+1) = 0, \tag{A16}$$

where K is the number of agents with initial long CDS positions. Consider the case where $p^A(NOI) = v - \delta \times NOI$. Under this specification,

$$\frac{v - p^A(NOI)}{\left| \frac{\partial p^A(NOI)}{\partial NOI} \right|} = NOI.$$

Condition (A15) gives a simple formula for the NOI:

$$NOI = \frac{\sum_{i:n_i>0} n_i + \sum_{i \in \mathcal{N}_+ : n_i < 0} n_i}{K+1} > 0. \tag{A17}$$

QED.

Proof of Proposition 8

As usual we focus on the case where $NOI > 0$. Note that the pro-rata allocation rule satisfies the *majority property* (Kremer and Nyborg, 2004): an agent whose demand at the clearing price is above 50% of the total demand is guaranteed to be allocated at least $(50\% + \eta) \times NOI$, where $\eta > 0$.

First, suppose that $v \leq p^M + s$. The proof that p^A cannot be above v is the same as in Proposition 1. We now prove that p^A cannot be below v. Suppose instead that $p^A < v$. The part of agent i's utility that depends on her equilibrium allocation and the final price is:

$$(v - p^A) \times q_i - p^A \times (n_i - y_i).$$

Suppose first that there is at least one agent for which $q_i < 0.5$. Suppose that this agent changes her demand schedule to:

$$x_i'(p) = \begin{cases} NOI, & p \leq p^A + \varepsilon \\ 0, & \text{otherwise,} \end{cases} \tag{A18}$$

where $0 < \varepsilon < v - p^A$. After this deviation, the new clearing price is $p^A + \varepsilon$. Since $X_{-i}(p^A + \varepsilon) < NOI$ (otherwise $p^A + \varepsilon$ would have been the clearing price), agent i demands more than 50% at $p^A + \varepsilon$, and under the pro-rata allocation rule receives $q_i' > 0.5 \times NOI$. The lower bound on the relevant part of agent i's utility is now:

$$(v - p^A - \varepsilon) \times 0.5 \times NOI - (p^A + \varepsilon) \times (n_i - y_i).$$

We can write the difference between agent i's utility under deviation and her utility under the assumed equilibrium as follows:

$$(0.5 \times NOI - q_i) \times (v - p^A) - \varepsilon(n_i - y_i + 0.5 \times NOI). \tag{A19}$$

For small enough ε and under the assumption that $p^A < v$, (A19) is greater than zero and hence equilibria with $p^A < v$ cannot exist.

If there are no agents with $q_i < 0.5 \times NOI$ we are in an auction with two bidders only. In this case, each of them gets exactly $0.5 \times NOI$. At price $p^A + \varepsilon$ $(0 < \varepsilon < p^M + s - v)$, there is at least one player (player i), for which $x_i(p^A + \varepsilon) < 0.5 \times NOI$.

Then, if the opposite agent uses demand schedule (A18), the new clearing price will be $p^A + \varepsilon$ and this agent will receive at least $(0.5 + \eta) \times NOI$. For small enough ε the difference between agent i's utility under the deviation and her utility under the assumed equilibrium is:

$$\eta \times (v - p^A) - \varepsilon(n_i - y_i + (0.5 + \eta) \times NOI) > 0. \tag{A20}$$

Therefore, equilibria with $p^A < v$ cannot exist. We conclude that if $v \leq p^M + s$, then $p^A = v$ is the only clearing price in any equilibrium under the pro-rata allocation rule.

Finally, suppose that $p^M + s < v$. The proof for this case is the same, except that there is no feasible deviation to a higher price if $p^A = p^M + s$. Hence, $p^A = p^M + s < v$ is the only clearing price in any equilibrium under the pro-rata allocation rule. QED.

Tables and Figures

Table 1: *Nortel Limited Market Quotes*

Dealer	Bid	Offer
Banc of America Securities LLC	9.5	11.5
Barclays Bank PLC	4.0	6.0
BNP Paribas	7.0	9.0
Citigroup Global Markets Inc.	10.5	12.5
Credit Suisse International	6.5	8.5
Deutsche Bank AG	6.0	8.0
Goldman Sachs & Co.	6.0	8.0
J.P. Morgan Securities Inc.	7.0	9.0
Morgan Stanley & Co. Incorporated	5.0	7.0
The Royal Bank of Scotland PLC	6.5	8.5
UBS Securities LLC	7.0	9.0

Table 1 shows the two-way quotes submitted by dealers at the first stage of the Nortel Ltd. auction.

Table 2: *Auction Summaries*

Name	Date	Credit Event	Inside Market Quote	Net Open Interest	Final Price
Dura	28 Nov 2006	Chapter 11	24.875	20.000	24.125
Dura Subordinated	28 Nov 2006	Chapter 11	4.250	77.000	3.500
Quebecor	19 Feb 2008	Chapter 11	42.125	66.000	41.250
Lehman Brothers	10 Oct 2008	Chapter 11	9.750	4920.000	8.625
Washington Mutual	23 Oct 2008	Chapter 11	63.625	988.000	57.000
Tribune	6 Jan 2009	Chapter 11	3.500	765.000	1.500
Lyondell	3 Feb 2009	Chapter 11	23.250	143.238	15.500
Nortel Corp.	10 Feb 2009	Chapter 11	12.125	290.470	12.000
Smurfit-Stone	19 Feb 2009	Chapter 11	7.875	128.675	8.875
Chemtura	14 Apr 2009	Chapter 11	20.875	98.738	15.000
Great Lakes	14 Apr 2009	Ch 11 of Chemtura	22.875	130.672	18.250
Rouse	15 Apr 2009	Failure to pay	28.250	8.585	29.250
Abitibi	17 Apr 2009	Failure to pay	3.750	234.247	3.250
Charter Communications	21 Apr 2009	Chapter 11	1.375	49.2	2.375
Capmark	22 Apr 2009	Failure to pay	22.375	115.050	23.375
Idearc	23 Apr 2009	Chapter 11	1.375	889.557	1.750
Bowater	12 May 2009	Chapter 11	14.000	117.583	15.000
R.H.Donnelly Corp.	11 Jun 2009	Chapter 11	4.875	143.900	4.875
General Motors	12 Jun 2009	Chapter 11	11.000	-529.098	12.500
Visteon	23 Jun 2009	Chapter 11	4.750	179.677	3.000
Six Flags	9 Jul 2009	Chapter 11	13.000	-62.000	14.000
Lear	21 Jul 2009	Chapter 11	40.125	172.528	38.500
CIT	1 Nov 2009	Chapter 11	70.250	728.980	68.125

Table 2 summarizes the auction results for 23 US firms for which TRACE data are available. It reports the settlement date, type of credit event, inside market quote (per 100 of par), net open interest (in millions of USD), and final auction settlement price (per 100 of par).

Table 3: *Tradable Deliverable Bond Summary Statistics*

Name	Number of deliverable bonds	Notional amount of bonds outstanding (NAB)	NOI/NAB (%)	Average price on the day before the auction
Dura	1	350,000	5.71	25.16
Dura Subordinated	1	458,500	16.79	5.34
Quebecor	2	600,000	11.00	42.00
Lehman Brothers	157	42,873,290	11.47	12.98
Washington Mutual	9	4,750,000	20.80	64.79
Tribune	6	1,346,515	56.81	4.31
Lyondell	3	475,000	30.15	26.57
Nortel Corp.	5	3,149,800	9.22	14.19
Smurfit-Stone	5	2,275,000	5.65	7.77
Chemtura	3	1,050,000	9.40	26.5
Great Lakes	1	400,000	32.65	26.71
Rouse	4	1,350,000	0.63	29.00
Abitibi	10	3,000,000	7.81	4.61
Charter Communications	17	12,769,495	0.38	2.00
Capmark	2	1,700,000	6.79	22.75
Idearc	1	2,849,875	31.21	2.15
Bowater	6	1,875,000	6.27	14.12
R.H.Donnelly Corp.	7	3,770,255	3.81	5.12
General Motors	16	18,180,552	-2.91	11.17
Visteon	2	1,150,000	15.62	74.87
Six Flags	4	1,495,000	-4.14	13.26
Lear	3	1,298,750	13.28	39.27
CIT	281	22,584,893	3.29	69.35

Table 3 provides summary statistics of deliverable bonds for 23 US firms for which TRACE data are available. Column three reports the ratio of Table 2's net open interest (NOI) to the notional amount outstanding of deliverable bonds. The last column shows a weighted average bond price on the day before the auction, constructed as described in Section 4.1.

Figure 1: *IMM Determination: The Case of Nortel*

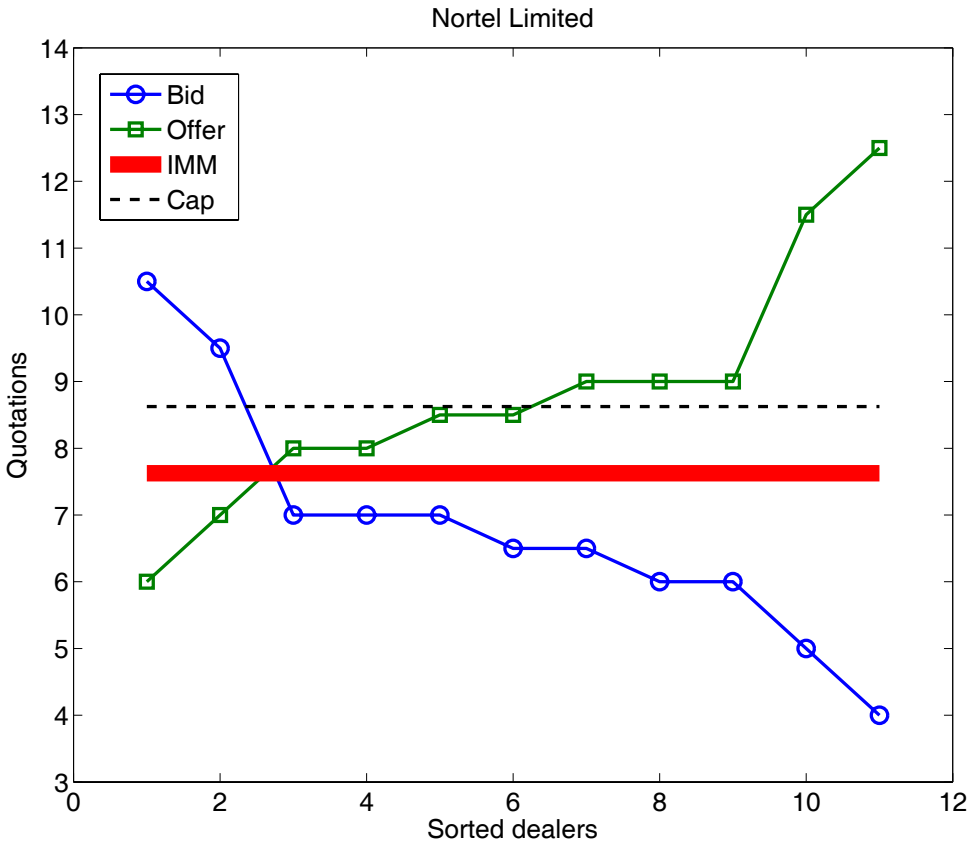

Figure 1 displays all bids (sorted in descending order) and all offers (sorted in ascending order). Tradeable quotes (bid greater than offer) are discarded for the purposes of computing IMM. Dealers quoting tradeable markets must pay a penalty (adjustment amount) to ISDA. The cap price is higher than the IMM by 1% of par and is used in determining the final price. (If the open interest is to buy, the cap price is set below the IMM.)

Figure 2: *Bids at the Cap Price*

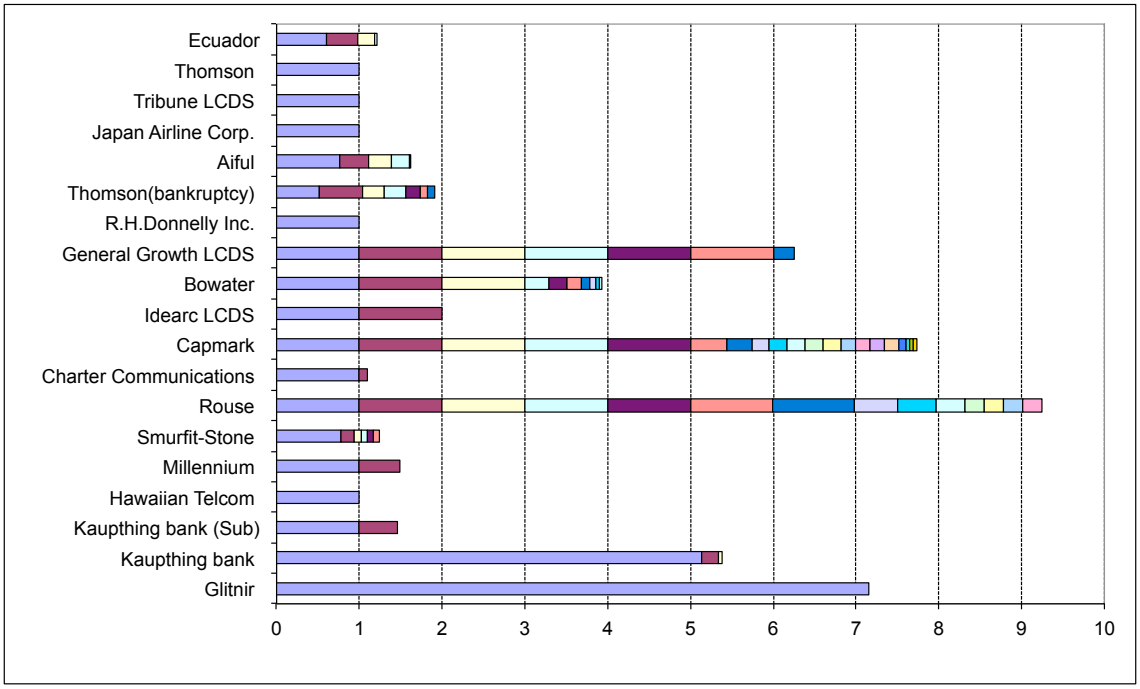

Figure 2 shows individual bids scaled by the NOI at the cap price (in auctions where the price is capped). Each bid within an auction is represented by a different color.

Figure 3: *Bond and Auction Prices*

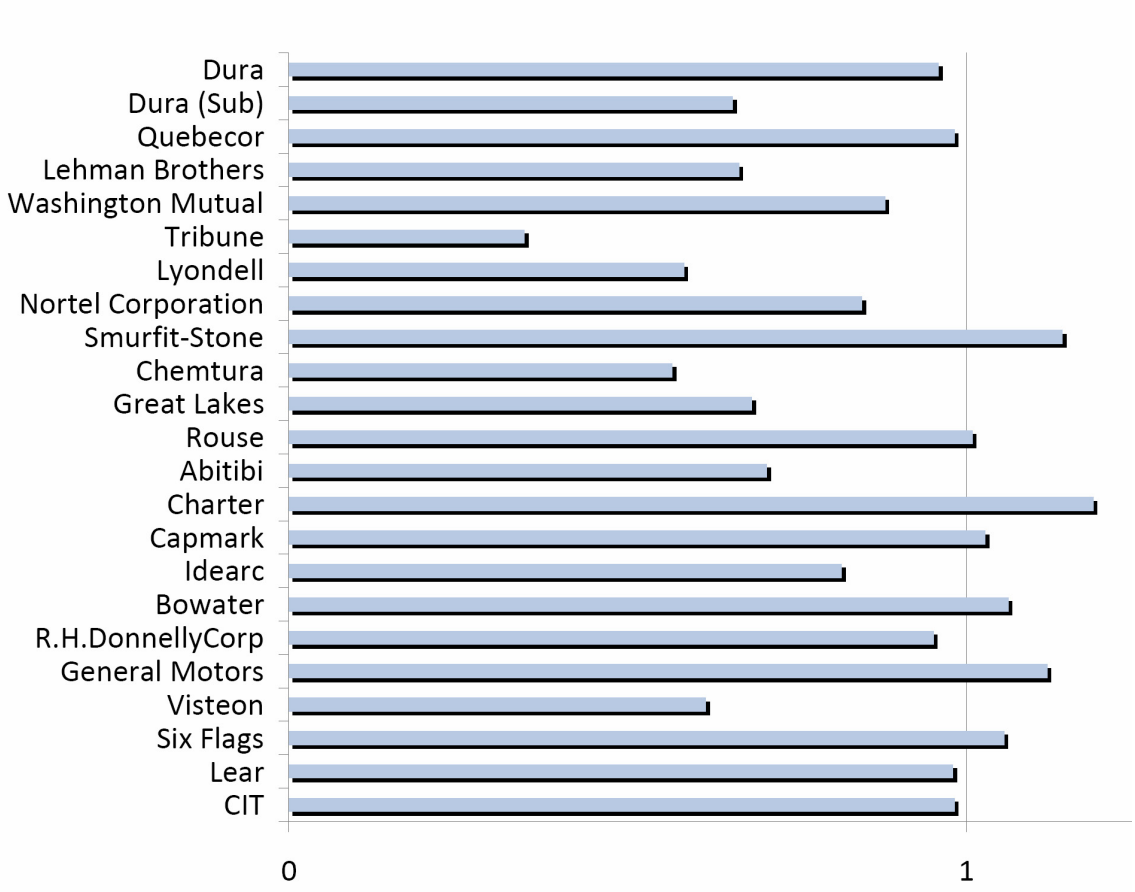

Figure 3 shows the final auction price, scaled by the weighted-average market price of the bonds a day before the auction.

Figure 4: *Price Discount*

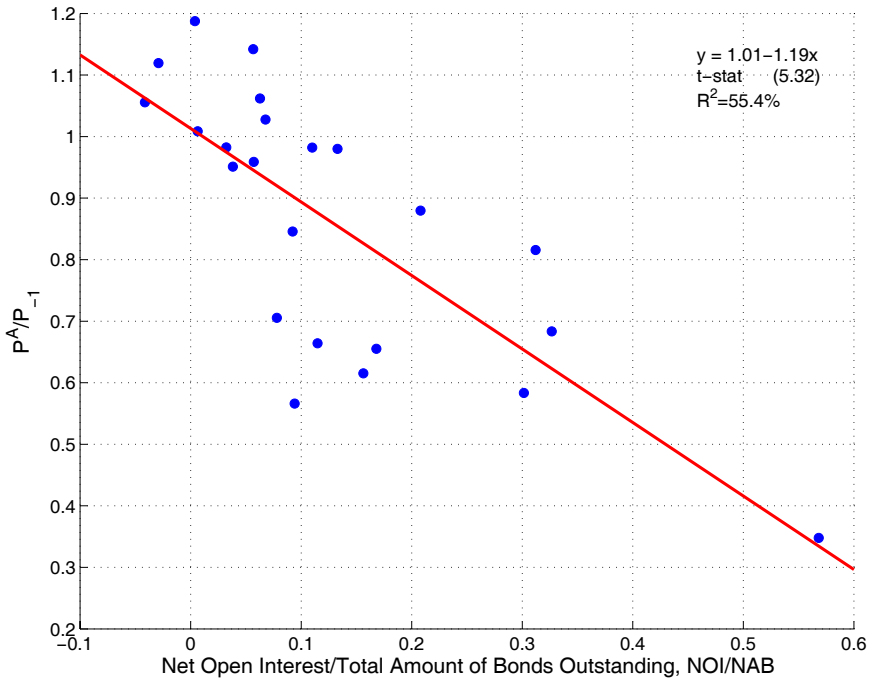

Figure 4 shows the result of an OLS regression where the dependent variable is the ratio of the final auction price to the weighted-average market price of bonds a day before the auction, and the explanatory variable is the scaled NOI:

$$y_i = \alpha + \beta \times NOI_i/NAB_i + \varepsilon_i.$$

Figure 5: *Price Impact*

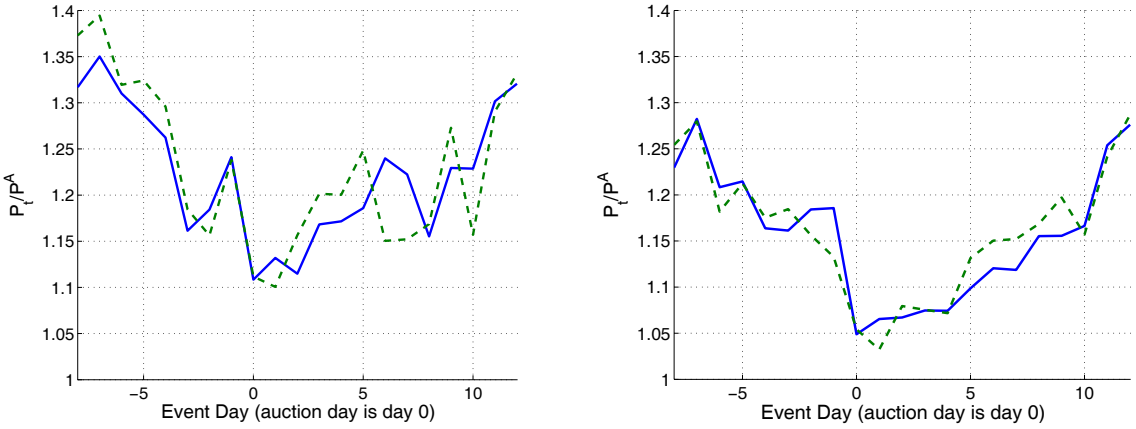

Figure 5 Panel (a) displays daily bond prices, normalized by the auction final price, p_t/p^A, and equally weighted across the 22 auctions reported in Table 2 (the Charter auction is excluded due to a lack of reliable bond data). Panel (b) shows the same prices but excluding the Tribune auction, which has the largest degree of underpricing. The blue line shows the prices based all available bond issues. The green line shows prices based only on bond issues with the lowest price.

Figure 6: *Washington Mutual Bond Prices*

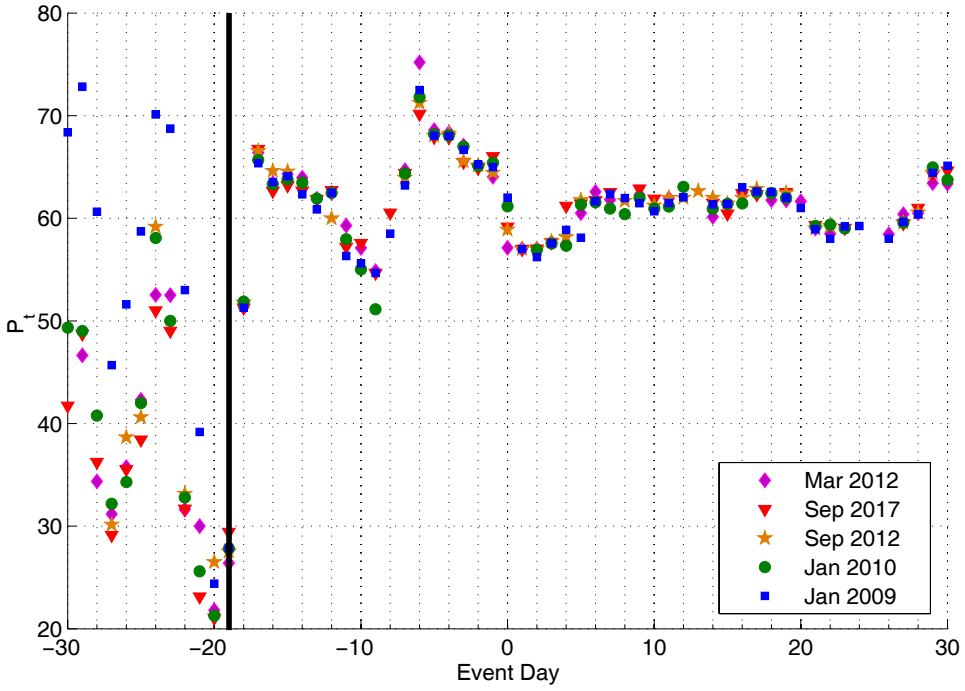

Figure 6 shows daily prices of Washington Mutual outstanding bond issues around the day of bankruptcy (indicated by a vertical black line). The legend shows the maturity date of each issue. The daily price at a given date is a volume-weighted average for all trades at this date. Further details on the construction of this graph are given in Section 4.1.