

# A Theory of Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks\*

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## Abstract

We provide a theoretical model linking firm characteristics and expected returns. The key ingredient of our model is technological shocks embodied in new capital (IST shocks), which affect the profitability of new investments. Firms' exposure to IST shocks is endogenously determined by the fraction of firm value due to growth opportunities. In our structural model, several firm characteristics – Tobin's Q, past investment, earnings-price ratios, market betas, and idiosyncratic volatility of stock returns – help predict the share of growth opportunities in the firm's market value, and are therefore correlated with the firm's exposure to IST shocks and risk premia. Our calibrated model replicates: i) the predictability of returns by firm characteristics; ii) the comovement of stock returns on firms with similar characteristics; iii) the failure of the CAPM to price portfolio returns of firms sorted on characteristics; iv) the time-series predictability of market portfolio returns by aggregate investment and valuation ratios; and v) a downward sloping term structure of risk premia for dividend strips. Our model delivers testable predictions about the behavior of firm-level real variables – investment and output growth – that are supported by the data.

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# 1 Introduction

Recent empirical research identifies a number of firm characteristics that forecast stock returns. There is also strong evidence of comovement in stock returns of firms with similar characteristics. Returns on portfolios formed by sorting firms on such characteristics exhibit a strong low-dimensional factor structure, with the common factors accounting for a significant share of their time-series variation. Furthermore, cross-sectional differences in portfolio exposures to the common factors typically account for a substantial fraction of the cross-sectional differences in their average returns.<sup>1</sup> A common interpretation of such patterns is that the relevant firm characteristics are correlated with the firms' exposures to common systematic risk factors. Despite the pervasiveness of such results in the empirical literature and their importance for understanding the risk-return tradeoff in the cross-section of stock returns, the economic origins of thus constructed empirical return factors are often poorly understood. This paper provides a theoretical explanation for the success of empirical multi-factor models.<sup>2</sup>

We focus on five firm characteristics that have received considerable attention in the literature. Prior studies have documented that firms with lower Tobin's Q (or equity book-to-market ratios), lower investment rates (IK), higher earnings-to-price (EP), lower market beta (BMKT) and lower idiosyncratic volatility (IVOL) earn abnormally high risk-adjusted returns relative to the standard CAPM model. First, we show that these patterns are closely related. Specifically, the five sets of portfolios formed on these characteristics largely share a

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<sup>1</sup>Specifically, the cross-section of returns on well-diversified portfolios formed by sorting firms on a certain characteristic  $c$ ,  $R_{it}^c$ ,  $i = 1, \dots, N$ , exhibits a strong factor structure. The returns on the long-short positions in the extreme portfolios,  $R_{Nt}^c - R_{1t}^c$ , a standard empirical approximation for the common return factor, tend to capture a substantial share of the time-series variation in realized portfolio returns. Furthermore, differences in exposures of the characteristic-sorted portfolios to such factors typically account for a significant fraction of the differences in their average returns.

<sup>2</sup>The ICAPM (Merton, 1973) or APT (Ross, 1976) are typically cited as theoretical motivations behind empirical multifactor models. However, a complete explanation of the empirical return patterns should address: a) why these returns factors are priced, and b) why firm characteristics are correlated with return exposures to these risk factors.

common factor structure. After removing their exposure to the market portfolio, not only do high-IK firms comove with other high-IK firms, but they also comove with firms that have high Q, low-EP, high IVOL, and high BMKT. The first principal component extracted from the pooled cross-section of portfolio returns – after removing the market component from each portfolio return – largely captures average return differences among portfolios sorted on each of the characteristics.<sup>3</sup> These results suggest that the firm characteristics above are correlated with firms’ exposures to the same common risk factor, which generates a significant share of variation in realized portfolio returns and captures cross-sectional differences in their risk premia.

We connect this common return factor to investment-specific technology (IST) shocks using a structural model, based on Kogan and Papanikolaou (2011). Our model features two aggregate sources for risk, disembodied technology shocks and technological improvements that are embodied in new capital goods (investment-specific shocks). Firms are endowed with a stochastic sequence of investment opportunities, which they implement by purchasing and installing new capital. In our model, a positive IST shock – a reduction in the relative price of capital goods – benefits firms with more growth opportunities relative to firms with limited opportunities to invest. Hence, differences in the ratio of growth opportunities to firm value  $PVGO/V$  lead to return comovement, and if IST shocks are priced by the market, to differences in average stock returns.

We formally illustrate the endogenous connection between the above firm characteristics, their growth opportunities, and their risk exposure to IST shocks.<sup>4</sup> To do so, we extend the

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<sup>3</sup>This result is not driven by the same stocks being ranked similarly using each of the above characteristics – correlations among portfolio assignments using various characteristics are low.

<sup>4</sup>Prior research already alludes to such connections. Firms with more growth opportunities are likely to invest more. Furthermore, such firms are likely to have higher valuation ratios (Tobin’s Q, price-earnings ratios) since their market value reflects the NPV of future investment projects. In addition, by the logic of the real options theory, growth opportunities are likely to have higher exposure to market conditions, and hence higher market betas. Finally, the literature also connects growth opportunities to the firms’ idiosyncratic risk, appealing to the common assumption that there is more uncertainty about firms’ growth opportunities than their assets in place (see, e.g., Myers and Majluf, 1984; Bartram, Brown, and Stulz, 2011). In addition,

model of Kogan and Papanikolaou (2011) by allowing the arrival rate of firms' investment opportunities to be unobservable. Market participants learn about firms' future growth opportunities from public signals and firms' investment decisions. This learning channel has two important effects. First, it formalizes the idea that revelation of information about firms' future growth opportunities contributes to their return variation. Second, due to learning, firms' past investment rates are informative about their future investment opportunities and their future expected stock returns.

Our calibrated model replicates key features of asset prices. First, our model generates empirically plausible average return spreads between firms with high and low Tobin's  $Q$ , investment rates, earnings-to-price ratios, market betas, and idiosyncratic volatility. Second, it replicates the existence of the common return factor among the portfolios sorted on these characteristics and the resulting failure of the CAPM to price the portfolio returns. Third, the same mechanism based on asset composition that leads to cross-sectional dispersion in risk premia also leads to time-variation in the aggregate equity premium. As a result, variables that are correlated with the aggregate fraction of growth opportunities to firm value – aggregate investment rate and valuation ratios – forecast excess returns on the market portfolio.<sup>5</sup> A key parameter in our calibration is the price of IST shocks, which we assume to be negative. We show that a negative price of risk for IST shocks implies that risk premia on stock market dividend strips are declining with maturity. In particular, a positive IST shock leads to a decline in short-term dividends as investment outlays rise, and to an increase in long-term dividends due to a higher rate of capital accumulation. This differential IST-shock exposure among different tenors of aggregate dividends implies that the term structure of equity risk premia is downward sloping.

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Comin and Philippon (2006) relate the rise in idiosyncratic volatility to the increasing importance of research and development, which has a natural relation to the firms' growth opportunities. Hence, *ceteris paribus*, firms with more growth opportunities are likely to have higher idiosyncratic volatility.

<sup>5</sup>See Cochrane (2011) for a summary of recent evidence for return predictability.

Our model’s implications for asset prices are supported by the data. Using proxies for the IST shock, we find that differences in IST exposures among the test portfolios account for a significant portion of their average return spreads. In addition, we explore to what extent IST shock exposures can reduce the predictive power of firm characteristics in cross-sectional regressions. In our model, return covariances and firm characteristics are jointly determined by firms’ growth opportunities and assets in place. Hence, firm characteristics forecast returns because they forecast future return exposure to IST shocks. In empirical tests, firm characteristics often dominate conventional empirical risk measures, or at least add nontrivial explanatory power. We argue that this finding is partly driven by the difficulty of accurately estimating risk exposures using stock return data alone. In particular, both in the data and in the model, estimated risk exposures using stock return data alone are too noisy to drive out characteristics in Fama-McBeth regressions. However, using predicted risk exposures, constructed as linear functions of characteristics and stock return betas, significantly reduces the incremental power of characteristics to forecast average returns.

We explore the testable implications of our core mechanism for real economic variables – firm investment decisions and output growth. In particular, firms with more growth opportunities increase their investment by a greater amount following a positive IST shock. Such firms also experience higher subsequent output growth relative to firms with few growth opportunities as a result of their faster capital accumulation. We find support for both of these predictions in the data. Following a positive IST shock, firms with high Tobin’s Q, high investment rates, low earnings-to-price, high market beta, and high idiosyncratic volatility increase their investment by more and experience higher future output growth relative to their peers. The magnitude of these effects in the data is comparable to the patterns produced by our calibrated model.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the theoretical model. Section 4 describes the empirical procedures and

calibration. Section 5 compares the patterns in historical data to simulated model output. In Section 6, we test the model’s implications for investment and output. In Section 7 we derive additional predictions of our model for return predictability and the term structure of risk premia. We briefly describe robustness tests in Section 8. Section 9 concludes.

## 2 Related Research

The empirical literature linking average returns and firm characteristics is extensive. Related to our study, Basu (1977) and Haugen and Baker (1996) document the relation between profitability and average returns; Fama and French (1992) and Lakonishok, Shleifer, and Vishny (1994) study market-to-book and earnings-to-price ratios; Titman, Wei, and Xie (2004) and Anderson and Garcia-Feijo (2006) relate investment to average returns; Ang, Hodrick, Xing, and Zhang (2006, 2009) document the negative relation between idiosyncratic volatility and average returns; and Black, Jensen, and Scholes (1972), Frazzini and Pedersen (2010) and Baker, Bradley, and Wurgler (2011), among others, document that the security market line is downward sloping. In this paper we show that all of the above empirical patterns are related to each other, and propose differences in the firms’ exposures to IST shocks as a common source of return comovement and cross-sectional differences in expected returns.

A number of models with production relate average returns to investment rates or valuation ratios.<sup>6</sup> Our model shares some of the features of these models, namely that variation in the firms’ mix of assets in place and growth opportunities leads to heterogeneous and time-varying risk exposures. However, most of the existing models feature a single aggregate shock, implying that firms’ risk premia are highly correlated with their conditional market

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<sup>6</sup>Examples include Berk, Green, and Naik (1999); Gomes, Kogan, and Zhang (2003); Carlson, Fisher, and Giammarino (2004); Zhang (2005); Bazdrech, Belo, and Lin (2009); Ai, Croce, and Li (2011); Ai and Kiku (2011); Kogan and Papanikolaou (2011). See Kogan and Papanikolaou (2012) for a recent survey of the related literature.

betas.<sup>7</sup> As a result, return factors constructed by sorting firms on various characteristics are conditionally perfectly correlated with the market portfolio. Hence, these models fail to capture the patterns of return comovement in the cross-section, and the resulting failure of the conditional CAPM (e.g., Lewellen and Nagel, 2006).

The difficulty of standard models in reproducing the negative relation between market betas and future returns has led to several recent explanations based on market frictions (e.g., Frazzini and Pedersen, 2010; Baker et al., 2011; Hong and Sraer, 2012). However, explanations based on deviations of market values from fundamentals need additional assumptions to generate comovement of firms with similar characteristics. Furthermore, we provide evidence that this comovement in stock returns is related to comovement in real economic variables – firm investment rates and output growth – consistent with the mechanism operating through the real channel.

Our paper adds to the growing literature in macroeconomics and finance on the role of investment-specific technology shocks. Investment-specific shocks capture the idea that technical change is embodied in new equipment.<sup>8</sup> Starting with Solow (1960), a number of economists have proposed embodied technical change as an alternative to the disembodied

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<sup>7</sup>There are a number of exceptions: Berk et al. (1999) assume that firms' value is affected by productivity and discount rate shocks; Ai et al. (2011) and Ai and Kiku (2011) study models with both short-run and long-run productivity shocks. However, these papers do not focus on return comovement as their primary object of interest. Our model shares some of the key conceptual elements with the framework of Berk et al. (1999), but emphasizes a different source of aggregate risk, embodied technical change. The closest paper to our work is Kogan and Papanikolaou (2011). We extend the analysis in Kogan and Papanikolaou (2011) to allow for learning about firms' growth opportunities, and provide evidence that differential exposure to IST shocks accounts for a number of other stylized empirical patterns in addition to the value premium.

<sup>8</sup>The magnitude of investment-specific technical progress can be inferred from the decline in the quality-adjusted price of investment goods. A classic example is computers. In 2011, a typical computer server costs \$5,000. In 1960, a state of the art computer server (e.g., the Burroughs 205), cost \$5.1 million in 2011 dollars. Furthermore, adjusting for quality is important: a modern computer server would cost \$160.8 million in 1960, using the quality-adjusted NIPA deflator for computers and software. Greenwood (1999) offers numerous additional examples of investment-specific technological change since the industrial revolution: Watt's steam engine, Crompton's spinning mule, and the dynamo. These innovations were embodied in new vintages of capital goods, hence they required substantial new investments before they could affect the production of consumption goods.

technology shocks assumed by most macroeconomic models.<sup>9</sup> Cummins and Violante (2002) document significant instances of investment-specific technical change in numerous industries. In macroeconomics, a number of studies have shown that IST shocks can account for a large fraction of output and employment variability, especially in the long run (e.g., Greenwood, Hercowitz, and Krusell, 1997, 2000; Christiano and Fisher, 2003; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010). Given that stock prices are particularly sensitive to low-frequency movements in fundamental variables (see, e.g. Bansal and Yaron, 2004), IST shocks are likely to be an important driver of asset prices. Furthermore, since IST advances improve real investment opportunities in the economy, they naturally have a differential impact on growth opportunities of firms and their assets in place. Papanikolaou (2011) demonstrates that in a general equilibrium model, IST shocks are positively correlated with the stochastic discount factor under plausible preference specifications, implying a negative price of risk for IST shocks.

### 3 Model

We relate observable firm characteristics, such as a firm’s beta with the market portfolio, idiosyncratic volatility, investment rate and profitability, to stock return exposures to a systematic sources of risk – investment-specific technical change – using a structural model. Our model has two aggregate shocks: a disembodied productivity shock and an investment-specific shock (IST). Assets in place and growth opportunities have the same loading on the disembodied shock, but different loadings on the IST shock. This differential sensitivity to IST shocks leads to return comovement among firms with similar ratios of growth opportunities to firm value. Furthermore, given that investment shocks are priced, this heterogeneity in

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<sup>9</sup>Solow (1960, p.91) expresses scepticism about disembodied technology shocks: “...This conflicts with the casual observation that many, if not most, innovations need to be embodied in new kinds of durable equipment before they can be made effective. Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by the replacement of old-fashioned equipment by the latest models...”



risk translates into cross-sectional differences in risk premia across firms based on the fraction of firm value derived from growth opportunities.

Thus, our model links firm characteristics to the share of growth opportunities in firm value. A key part of the mechanism is that firms' growth opportunities are difficult to observe. Hence, we extend the structural model of Kogan and Papanikolaou (2011) to incorporate learning about firms' growth opportunities. To make the exposition largely self-contained, we describe all the elements of the model below, but we refer the readers to Kogan and Papanikolaou (2011) for proofs of some of the technical results.

### 3.1 Setup

There are two sectors of production, a sector producing consumption goods and a sector producing investment goods. Each sector features a continuum of measure one of infinitely lived competitive firms financed only by equity. During most of our analysis we focus on the sector producing consumption goods. We use the investment-goods sector to construct a factor mimicking portfolio for IST shocks.

#### Assets in Place

Each consumption firm owns a finite number of individual projects. Firms create projects over time through investment, and projects expire randomly.<sup>10</sup> Let  $\mathcal{F}$  denote the set of firms and  $\mathcal{J}_{ft}$  the set of projects owned by firm  $f$  at time  $t$ .

Project  $j$  produces a flow of output equal to

$$y_{fjt} = u_{jt} x_t K_j^\alpha, \tag{1}$$

where  $K_j$  is physical capital chosen irreversibly at the project  $j$ 's inception date,  $u_{jt}$  is the

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<sup>10</sup>Firms with no current projects may be seen as firms that temporarily left the sector. Likewise, idle firms that begin operating a new project can be viewed as new entrants. Thus, our model implicitly captures entry and exit by firms.

project-specific component of productivity, and  $x_t$  is the disembodied productivity shock affecting output of all existing projects. There are decreasing returns to scale at the project level,  $\alpha \in (0, 1)$ . Firm's projects expire independently at rate  $\delta$ .

The project-specific component of productivity  $u$  follows a mean-reverting, stationary process, while the process for the disembodied shock  $x$  follows a Geometric Brownian motion,

$$du_{jt} = \theta_u(1 - u_{jt}) dt + \sigma_u \sqrt{u_{jt}} dB_{jt}, \quad (2)$$

$$dx_t = \mu_x x_t dt + \sigma_x x_t dB_{xt}, \quad (3)$$

where  $dB_{jt}$  and  $dB_{xt}$  are independent standard Brownian motions.

## Investment

Consumption firms acquire new projects exogenously according to a Poisson process with a firm-specific arrival rate  $\lambda_{ft}$ . At the time of investment, the project-specific component of productivity is at its long-run average value,  $u_{jt} = 1$ .

The firm-specific arrival rate of new projects has two components:

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{f,t}. \quad (4)$$

The first component of firm arrival rate  $\lambda_f$  is constant over time. In the long run,  $\lambda_f$  determines the size of the firm. The second component of firm arrival rate  $\tilde{\lambda}_{f,t}$  captures the current growth state of the firm. We assume that  $\tilde{\lambda}_{f,t}$  follows a two-state, continuous-time Markov process with transition probability matrix between time  $t$  and  $t + dt$  given by

$$P = \begin{pmatrix} 1 - \mu_L dt & \mu_L dt \\ \mu_H dt & 1 - \mu_H dt \end{pmatrix}. \quad (5)$$

Thus, at any point in time, a firm can be either in the high-growth ( $\lambda_f \cdot \lambda_H$ ) or in the low-growth state ( $\lambda_f \cdot \lambda_L$ ), and  $\mu_H$  and  $\mu_L$  denote the transition rates between the two states.

Without loss of generality, we impose the normalization  $E[\tilde{\lambda}_{f,t}] = 1$ .<sup>11</sup> Hence,  $\lambda_f$  denotes the average project arrival rate of firm  $f$ .

When presented with a new project at time  $t$ , a firm must make a take-it-or-leave-it decision. If the firm decides to invest in a project, it chooses the associated amount of capital  $K_j$  and pays the unit investment cost  $p_t^I = z_t^{-1}x_t$ . The price of investment goods relative to the average productivity of capital depends on the stochastic process  $z_t$ , which follows a Geometric Brownian motion

$$dz_t = \mu_z z_t dt + \sigma_z z_t dB_{zt}, \quad (7)$$

where  $dB_{zt} \cdot dB_{xt} = 0$ . The  $z$  shock is the embodied, investment-specific shock in our model, representing the component of the price of capital that is unrelated to its current level of average productivity  $x$ . A positive change in  $z$  reduces the cost of new capital goods and thus leads to an improvement in investment opportunities.

## 3.2 Learning

In contrast to Kogan and Papanikolaou (2011), we assume that the firm-level arrival rate  $\lambda_{ft}$  is not perfectly observable. Market participants observe a long history of the economy, hence they know its long-run mean  $\lambda_f$ . However, they do not observe whether the firm is currently in the high-growth or low-growth phase. Thus,  $\tilde{\lambda}_{ft}$  is an unobservable, latent process.

The market learns about the firm's growth opportunities through two channels. First, market participants observe a noisy public signal  $e_{ft}$  of  $\lambda_{ft}$ ,

$$de_{ft} = \lambda_{ft} dt + \sigma_e dZ_{ft}^e.$$

Second, the market updates its beliefs about  $\lambda_{ft}$  by observing the arrivals of new projects.

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<sup>11</sup>This normalization leads to the parameter restriction

$$1 = \lambda_L + \frac{\mu_H}{\mu_H + \mu_L}(\lambda_H - \lambda_L). \quad (6)$$

We derive the evolution of the probability  $p_{ft}$  that the firm is in the high growth state  $\lambda_{ft} = \lambda_f \lambda_H$  using standard results on filtering for point processes (see, e.g. Lipster and Shiryaev, 2001),

$$dp_{ft} = \left( (1 - p_{ft})\mu_H - p_{ft}\mu_L \right) dt + p_{ft} (\lambda_f \lambda_H - \bar{\lambda}_{ft}) \left( dM_{ft} + h_e d\tilde{Z}_{ft}^e \right), \quad (8)$$

where  $h_e = \sigma_e^{-1}$  is the precision of the public signal and  $\bar{\lambda}_{ft} = p_{ft}\lambda_f \lambda_H + (1 - p_{ft})\lambda_f \lambda_L$  is the market's unbiased estimate of the arrival rate of the firm's investment opportunities. The stochastic processes  $\tilde{Z}^e$  and  $M$  are martingales, given by

$$d\tilde{Z}_{ft}^e = h_e (de_{ft} - \bar{\lambda}_{ft} dt), \quad (9)$$

$$dM_{ft} = \bar{\lambda}_{ft}^{-1} (dN_{ft} - \bar{\lambda}_{ft} dt), \quad (10)$$

where  $N_{ft}$  denotes the cumulative number of projects undertaken by the firm. Hence, the market learns about  $\lambda_{ft}$  using the demeaned public signal  $\tilde{Z}_{ft}^e$ . In addition, the market adjusts its beliefs about  $\lambda_{ft}$  upwards whenever the firm invests ( $dN_{ft} = 1$ ).

### 3.3 Valuation

We denote the stochastic discount factor as  $\pi_t$ . For simplicity, we assume that the two aggregate shocks  $x_t$  and  $z_t$  have constant prices of risk,  $\gamma_x$  and  $\gamma_z$  respectively. The risk-free interest rate  $r_f$  is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \gamma_x dB_{xt} - \gamma_z dB_{zt}. \quad (11)$$

The factor structure of the stochastic discount factor is motivated by the general equilibrium model with IST shocks in Papanikolaou (2011). IST shocks endogenously affect the representative household's consumption stream, and hence they are priced in equilibrium.

Firms' investment decisions are based on a tradeoff between the market value of a new

project and the cost of physical capital. Given (11), the time- $t$  market value of an existing project  $j$  is equal to the present value of its cashflows

$$p(u_{jt}, x_t, K_j) = \mathbb{E}_t \left[ \int_t^\infty e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} \left( u_{js} x_s K_j^\alpha \right) ds \right] = A(u_{jt}) x_t K_j^\alpha,$$

$$A(u) = \frac{1}{r_f + \gamma_x \sigma_x + \delta - \mu_X} + \frac{1}{r_f + \gamma_x \sigma_x + \delta - \mu_X + \theta_u} (u - 1). \quad (12)$$

The optimal investment decision follows the NPV rule: firm  $f$  chooses the amount of capital  $K_j$  to invest in project  $j$  to maximize its net present value

$$NPV_{jt} = \max_{K_j} p(1, x_t, K_j) - p_t^I K_j. \quad (13)$$

Because the marginal productivity of capital in (1) is infinite at the zero capital level, it is always optimal to invest a positive and finite amount. The optimal capital investment in the new project is given by

$$K^*(z_t) = \alpha^{\frac{1}{1-\alpha}} \left( \frac{p(1, x_t, K_j)}{p_t^I} \right)^{\frac{1}{1-\alpha}} = z_t^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}}. \quad (14)$$

Equation (14) illustrates the relation between the optimal level of investment  $K^*$  and the ratio of the market value of a new project  $p(1, x, K)$  to the cost of capital  $p^I$ . This ratio bears similarities to the marginal  $Q$  in the  $Q$ -theory of investment. However, in contrast to most  $Q$ -theory models, optimal investment depends on the market valuation of a new project, which in general is not directly linked to the market valuation of the entire firm. Furthermore, the relation in (14) holds conditional on the firm having the opportunity to invest. That is yet another reason why the firm's marginal (or average)  $Q$  is not a sufficient statistic for the optimal investment in our model, since investment depends on the firm's current investment opportunities  $\lambda_{ft}$ .

The market value of a firm is the sum of the value of its existing projects and the value of its future growth opportunities. Following the standard convention, we call the first

component of firm value *the value of assets in place*,  $VAP_{ft}$ , and the second component *the present value of growth opportunities*,  $PVGO_{ft}$ . The value of a firm's assets in place is the value of its existing projects

$$VAP_{ft} = \sum_{j \in \mathcal{J}_{ft}} p(u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{jt}) K_j^\alpha. \quad (15)$$

The value of assets in place is independent of the IST shock  $z$  and loads only on the disembodied shock  $x$ .

The present value of growth opportunities equals the expected discounted NPV of future investments

$$PVGO_{ft} = \text{E}_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} (\lambda_{fs} NPV_t) ds \right] = z_t^{\frac{\alpha}{1-\alpha}} x_t (G_L + p_{ft} (G_H - G_L)), \quad (16)$$

where

$$NPV_t = x_t z_t^{\frac{\alpha}{1-\alpha}} (\alpha^{-1} - 1) \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}}, \quad (17)$$

$$G_H = \lambda_f (\alpha^{-1} - 1) \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}} \left( \rho^{-1} + \frac{\mu_L}{\mu_L + \mu_H} (\lambda_H - \lambda_L) (\rho + \mu_H + \mu_L)^{-1} \right),$$

$$G_L = \lambda_f (\alpha^{-1} - 1) \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}} \left( \rho^{-1} - \frac{\mu_H}{\mu_L + \mu_H} (\lambda_H - \lambda_L) (\rho + \mu_H + \mu_L)^{-1} \right),$$

$$\rho = r - \mu_x - \frac{\alpha}{1-\alpha} \left( \mu_z + \frac{1}{2} \sigma_z^2 \right) - \frac{1}{2} \frac{\alpha^2}{(1-\alpha)^2} \sigma_z^2. \quad (18)$$

The present value of growth opportunities depends positively on aggregate productivity  $x$  and the IST shock  $z$ , because the latter affects the profitability of new projects.

Adding the two pieces, the total value of the firm is equal to

$$V_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{jt}) K_j^\alpha + z_t^{\frac{\alpha}{1-\alpha}} x_t (G_L + p_{ft} (G_H - G_L)). \quad (19)$$

Examining equation (19), we can see that the firm's stock return beta with the disembodied

productivity shock  $x$  and the IST shock  $z$  is equal to

$$\beta_{ft}^x = 1, \quad (20)$$

$$\beta_{ft}^z = \frac{\alpha}{1 - \alpha} \frac{PVGO_{ft}}{V_{ft}}. \quad (21)$$

This differential sensitivity to IST shocks has implications for stock return comovement and risk premia. In particular, equations (20-21) imply that stock returns have a conditional two-factor structure. The disembodied shock  $x$  affects all firms symmetrically, whereas firms' sensitivity to the IST shock is a function of the ratio of growth opportunities to firm value. Moreover, the firm's asset mix between growth opportunities and assets in place determines its risk premium

$$\frac{1}{dt} E_t[R_{ft}] - r_f = \gamma_x \sigma_x + \frac{\alpha}{1 - \alpha} \gamma_z \sigma_z \frac{PVGO_{ft}}{V_{ft}}. \quad (22)$$

Whether firm's expected returns are increasing or decreasing in the share of growth opportunities in firm value depends on the risk premium attached to the IST shock,  $\gamma_z$ .<sup>12</sup>

The ratio of the firm's growth opportunities to its total market value,  $PVGO/V$ , evolves endogenously as a function of the firm-specific project arrival rate  $\lambda_{ft}$ , the history of project arrival and expiration, and the project-specific level of productivity  $u$ . In the short run, firms with a large expected number of new projects  $\lambda_{ft}$  relative to the number of active projects are likely to be firms with high growth opportunities. In addition, firms with productive existing projects (high  $u$ ) are more likely to be firms where the value of assets in place accounts for a larger share of firm value.

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<sup>12</sup>Most equilibrium models imply a positive price of risk for disembodied technology shocks, so  $\gamma_x > 0$ . The price of risk of the IST shock  $\gamma_z$  depends on preferences. Papanikolaou (2011) shows that under plausible preference parameters, states with low cost of new capital (high  $z$ ) are high marginal valuation states, which is analogous to a negative value of  $\gamma_z$ . In Papanikolaou (2011), households attach higher marginal valuations to states with a positive IST shock because in those states households substitute resources away from consumption and into investment. We infer the price of risk of IST shocks from the cross-section of stock returns. In particular, growth firms, which derive a relatively large fraction of their value from growth opportunities, have relatively high exposure to IST shocks and relatively low expected excess returns. This implies that the market price of IST shocks is negative.

To the extent that the ratio  $PVGO/V$  is correlated with observable firm characteristics, our model implies that portfolios of firms sorted on these characteristics exhibit dispersion in risk premia. Furthermore, long-short portfolios formed on various characteristics are conditionally spanned by the IST shock  $z$ . Consequently, each of these long-short portfolios, together with the market portfolio, spans the two systematic sources of risk in the model,  $x$  and  $z$ .

### 3.4 Investment Sector

There is a continuum of firms producing new capital goods. The investment firms produce the demanded quantity of capital goods at the current unit price  $p_t^I$ , and have a constant profit margin  $\phi$ . Given (11), the price of the investment firm is given by

$$V_{I,t} = x_t z_t^{\frac{\alpha}{1-\alpha}} \frac{\phi}{\rho} \left( \int_{\mathcal{F}} \lambda_f df \right) \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}}. \quad (23)$$

A positive IST shock  $z$  benefits the investment-good producers. Even though the price of their output declines, the elasticity of investment demand with respect to price is greater than one, so their profits increase. Hence, we can use the relative stock returns of the investment and consumption good producers to create a factor-mimicking portfolio for the IST shock.

### 3.5 Growth Opportunities and Firm Characteristics

Our model maps the ratio of growth opportunities to firm value into observable firm characteristics. Any particular characteristic is an imperfect proxy for growth opportunities, and the sign of its relation with  $PVGO/V$  may be ambiguous. However, our model connects several distinct firm characteristics to the same economic mechanism, heterogeneous exposure of assets in place and growth opportunities to IST shocks, through a common set of structural parameters. Thus, by simultaneously reproducing empirical stock return patterns in relation to various firm characteristics in our model, we confirm that its core mechanism is



quantitatively plausible.

### **Tobin's Q**

The firm's average Tobin's  $Q$ , defined as the market value of the firm  $V_f$  over the replacement cost of its capital stock

$$B_{ft} = p_t^I \sum_{j \in \mathcal{J}_{ft}} K_j, \quad (24)$$

is positively related to the ratio of growth opportunities to firm value:

$$Q_{ft} = \frac{V_{ft}}{B_{ft}} = \left(1 - \frac{PVGO_{ft}}{V_{ft}}\right)^{-1} \times \frac{VAP_{ft}}{B_{ft}}. \quad (25)$$

Average  $Q$  is a noisy measure of growth opportunities, since it also depends on the profitability of existing projects through  $VAP/B$ . The relation between  $Q$  and  $PVGO/V$  is positive if cross-sectional differences in growth opportunities, and not the differences in profitability of existing projects, are the dominant source of variation in Tobin's  $Q$  across firms.

### **Investment rate**

The firm's investment rate, measured as the ratio of capital expenditures to the lagged replacement cost of its capital stock,  $B_{ft}$ , is related to the ratio of growth opportunities to firm value. Specifically, a firm's investment over an interval  $[t, t + \Delta]$  is equal to the cumulative capital expenditures

$$INV_{f,t+\Delta} = \int_t^{t+\Delta} p_s^I K^*(z_s) dN_{fs}. \quad (26)$$

In our model, there is both a cross-sectional and a time-series relation between investment and average returns. In the cross-section of firms, firms with more growth opportunities tend to have higher investment rates. Moreover, when a given firm acquires a project, the market revises upward its estimate of the firm's growth opportunities (see equation (8)). Both of these channels imply a positive relation between  $PVGO/V$  and firms' past investment rates. However, project acquisition also increases the value of assets in place, which has an

offsetting effect. The net effect of investment on the relative value of growth opportunities thus depends on the structural parameters and the current state of the firm. In particular, firm investment tends to be positively correlated with  $PVGO/V$  in the cross-section if differences in  $PVGO/V$  among firms are sufficiently large.

This mechanism linking investment rates to risk premia is new, and conceptually different from the mechanisms proposed in other studies. In particular, in Carlson et al. (2004) and Carlson, Fisher, and Giammarino (2006) growth opportunities have higher risk premia than assets in place. Investment converts growth opportunities to assets in place, so following an increase in investment, the same firm has a higher mix of  $VAP/V$  and therefore lower risk premia. In our model, the opposite is true, that is, growth opportunities have *lower* risk premia than assets in place, consistent with the empirical evidence on the value premium. In Zhang (2005) and Bazdrech et al. (2009), operating leverage leads to a negative relation between productivity and systematic risk, as captured by market beta. Consequently, investment – which is increasing in firm productivity – is negatively related to market beta and therefore risk premia.

## Earnings-to-Price

A number of studies in the empirical literature have documented that firm earnings scaled by the market value of equity are related to average returns (e.g., Basu (1977), Fama and French (1992)). To explore this relation in light of our model, note that the value of assets in place increases in the output of current projects

$$VAP_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} a_0 K_j^\alpha + a_1 Y_{ft} \approx a_1 Y_{ft} \quad \text{if } \theta_u \ll 1, \quad (27)$$

$$\text{where } Y_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} u_{jt} K_j^\alpha, \quad (28)$$

where  $a_0$  and  $a_1$  are two positive constants, and  $a_0$  tends to zero as the persistence of the project-specific shocks increases.

In our model firms have no production costs, hence  $Y$  also represents their earnings. Thus, sorting firms on  $Y/V$  is analogous to sorting them on their earnings-to-price ratios in the data. Equation (27) implies that such a sort approximately ranks firms by the ratio of the value of their assets in place to the total firm value,  $VAP/V$ , which is inversely related to  $PVGO/V$ .

Furthermore, a number of studies relate accounting-based measures of profitability, such as return on assets, to future stock returns (see, e.g., Haugen and Baker, 1996). To explore the ability of the model to replicate these relations, we form the equivalent of ROA by scaling  $Y$  by the book value of capital  $B$  above. Intuitively, firms with more productive projects (high  $u$ ) are likely to have high accounting profitability ratios and thus higher share of assets in place to firm value ( $VAP/V$ ).

### Market Beta

Our model implies that a firm's market beta is an increasing function of the share of growth opportunities in firm value. In particular, the market portfolio, defined as the value-weighted portfolio of all consumption and investment firms, is exposed to both the disembodied shock  $x$  and the IST shock  $z$

$$\beta_{Mt}^x = 1, \quad \beta_{Mt}^z = \frac{\alpha}{1-\alpha} \frac{PVGO_{Ct} + V_{It}}{V_{Ct} + V_{It}}, \quad (29)$$

where  $PVGO_{Ct} = \int_{\mathcal{F}} PVGO_{ft} df$  and  $V_{Ct} = \int_{\mathcal{F}} V_{ft} df$  are the total present value of growth opportunities and the total firm value in the consumption sector respectively. A consumption-sector firm's market beta is therefore equal to

$$\beta_{ft}^M = B_{0t} + B_{1t} \frac{PVGO_{ft}}{V_{ft}}, \quad (30)$$

where  $B_{0t} > 0$ ,  $B_{1t} > 0$  are functions of the structural parameters and  $V_{Ct}$ ,  $PVGO_{Ct}$  and  $V_{It}$  only. As a result, cross-sectional differences in market betas are positively related to

cross-sectional differences in growth opportunities.

Equation (30) implies that the relation between market beta and risk premia has the same sign as the price of IST shocks,  $\gamma_z$ , which we estimate to be negative. This negative relation illustrates the failure of the CAPM in our model. Absent any other form of risk heterogeneity in our model, the security market line is downward sloping.

### Idiosyncratic volatility

In our model, the idiosyncratic variance of the firm return equals

$$IVOL_{ft}^2 = \left[ \sigma_u^2 \sum_{j \in \mathcal{J}_{ft}} \frac{1}{u_{jt}} \left( \frac{x_t K_j^\alpha a_1 u_{jt}}{VAP_{ft}} \right)^2 + \delta \sum_{j \in \mathcal{J}_{ft}} \left( \frac{x_t K_j^\alpha A(u_{jt})}{VAP_{ft}} \right)^2 \right] \left( \frac{VAP_{ft}}{V_{ft}} \right)^2 + \left[ \bar{\lambda}_{ft} \left( C(p_{ft}) + B(p_{ft}) \right)^2 + h_e^2 B^2(p_{ft}) \right] \left( \frac{PVG O_{ft}}{V_{ft}} \right)^2, \quad (31)$$

where  $C(p_{ft})$  is the ratio of the NPV of a new project to the firm's PVGO, and  $B(p_{ft})$  captures the uncertainty about the firm's growth opportunities:

$$B(p_{ft}) = \frac{(G_H - G_L) p_{ft} (\lambda_f \lambda_H - \bar{\lambda}_{ft})}{G_L + p_{ft} (G_H - G_L)}; \quad C(p_{ft}) = \frac{(\alpha^{-1} - 1) \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}}}{G_L + p_{ft} (G_H - G_L)}. \quad (32)$$

The relation between idiosyncratic volatility and the share of growth opportunities in firm value is complex. The first term in equation (31) captures fluctuations in the project-specific level of productivity (first part) and the potential decline in firm value due to expiration of existing projects (second part). The second term in (31) also has two parts. The first part captures the effect of project arrival on firm value. The second part reflects changes in firm value due to the arrival of information about the firm's growth prospects.

The sign of the relation between  $PVGO/V$  and firm's idiosyncratic return volatility depends on the relative strength of the various determinants of idiosyncratic return risk. If firms hold sufficiently diversified portfolios of projects, then the first term is likely to be small. In this case, firms with more growth opportunities will have higher idiosyncratic volatility, as

news about future investment opportunities are a dominant source of idiosyncratic risk.

## 4 Data and Calibration

Here we describe the empirical construction of the main variables and model calibration.

### 4.1 Measuring Investment-Specific Shocks

We focus on three measures of capital-embodied technical change directly implied by the model. The construction of these measures closely follows Kogan and Papanikolaou (2011), and we reproduce the key results here for completeness.

The first measure of IST shocks is based on the quality-adjusted price of new capital goods, as in Greenwood et al. (1997, 2000). We use the quality-adjusted price series for new equipment constructed by Gordon (1990) and extended by Cummins and Violante (2002) and Israelsen (2010). We normalize equipment prices by the NIPA consumption deflator, denoting the resulting price series by  $p_t^I$ . We de-trend equipment prices by regressing the logarithm of  $p_t^I$  on a piece-wise linear time trend:

$$p_t^I = a_0 + b_0 \mathbf{1}_{1982} + (a_1 + b_1 \mathbf{1}_{1982}) \cdot t - z_t^I, \quad (33)$$

where  $\mathbf{1}_{1982}$  is an indicator function that takes the value 1 post 1982. The two-piece linear trend accommodates the possibility of a structural break (see e.g. Fisher (2006)). When using the equipment price series to measure investment-specific technology shocks, we approximate them as  $\Delta z_t^I$ .

Our model suggests a factor-mimicking portfolio for IST shocks. In particular, the instantaneous return on a portfolio long firms producing investment goods and short firms producing consumption goods (IMC portfolio) is spanned by the IST shock:

$$R_t^I - R_t^C = E_t[R_t^I - R_t^C] + \frac{\alpha}{1 - \alpha} \beta_{0t} dB_{zt}, \quad (34)$$

where  $\beta_{0t} = (\int_{\mathcal{F}} V_{ft} df) / (\int_{\mathcal{F}} VAP_{ft} df)$  is a term that depends on the share of growth opportunities in the aggregate stock market value. To construct the IMC portfolio in the data, we first classify industries as producing either investment or consumption goods according to the NIPA Input-Output Tables. We then match firms to industries according to their NAICS codes. Gomes, Kogan, and Yogo (Gomes et al.) and Papanikolaou (2011) describe the details of this classification procedure.

## 4.2 Firm Characteristics

We now briefly describe the construction of the firm characteristics that we use in our empirical analysis. Specifically, we measure the investment rate (IK) as the ratio of capital expenditures (capx) to the lagged book value of capital (ppeg). We define Tobin's  $Q$  as the ratio of the market value of common equity (CRSP December market capitalization) plus the book value of debt (dltt) plus the book value of preferred stock (pstkrv) minus inventories (invt) and deferred taxes (txdb) divided by the book value of capital (ppeg). Following common convention, we define the firm's return on assets (ROA) as operating income (ib) divided by lagged book assets (at). Last, we define the firm's earnings-to-price ratio (EP) as the ratio of operating income (ib) plus interest expenses (xint) to the market value of the firm (mkcap + dltt + pstkrv - txdb).

We estimate the firm's market beta ( $BMKT$ ) and IMC beta ( $IMC - BETA$ ) using weekly returns

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^F r_{tw}^F + \varepsilon_{ftw}, \quad w = 1 \dots 52, \quad (35)$$

where  $r_{ftw}$  refers to the log return of firm  $f$  in week  $w$  of year  $t$ , and  $r_{tw}^F \in \{r_{tw}^{mkt}, r_{tw}^{imc}\}$  refers to the log excess return of the market, or IMC portfolio, in week  $w$  of year  $t$ . Thus,  $BMKT_{ft} = \beta_{ft}^{mkt}$  is constructed using information only in year  $t$ .

We also use weekly returns to estimate the firm’s idiosyncratic volatility (*IVOL*)

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^{mkt} r_{tw}^{mkt} + \beta_{ft}^{imc} r_{tw}^{imc} + \varepsilon_{ftw}, \quad w = 1 \dots 52, \quad (36)$$

where  $r_{ftw}^{imc}$  refers to the log return of the IMC portfolio in week  $w$  of year  $t$ . Our measure of idiosyncratic volatility  $IVOL_{ft} = \sqrt{\text{var}_t(\varepsilon_{ftw})}$  is also constructed using information only in year  $t$ . We estimate idiosyncratic volatility from the two-factor specification (36) rather than the market model (35) to ensure that our measure of idiosyncratic variance is not mechanically reflecting variation in IMC betas across firms.

### 4.3 Calibration

Our model features a total of 18 parameters. Table 2 summarizes our parameter choices. Some of these parameters are determined by a priori evidence. In particular, we set the project expiration rate  $\delta$  to 10%, to be consistent with commonly used values for the depreciation rate. We set the interest rate  $r_f$  to 3%, which is close to the historical average real risk-free rate. We pick the price of risk of the IST shock  $\gamma_z = 0.57$  to match the estimate of the price of risk of IST shocks estimated using the cross-section of industry portfolios in Kogan and Papanikolaou (2011). We verify that under this choice, the average return on the value factor HML in the calibrated model matches the historical returns on the value factor constructed using consumption-sector firms.

We select the next set of 15 parameters to approximately match 18 aggregate and firm-specific moments. While all of the model parameters jointly determine its properties, some groups of parameters have particularly strong effect on certain aspects of the model’s behavior, as we discuss below.

We pick the price of the disembodied shock  $\gamma_x$  to match the historical equity premium. We choose the profit margin of investment firms  $\phi = 0.075$  to match the relative size of the consumption and investment sectors in the data.

The parameters governing the projects' cash flows ( $\theta_u = 0.03$ ,  $\sigma_u = 1.25$ ) affect the serial autocorrelation and the cross-sectional distribution of firm-specific profitability and Tobin's Q.

The parameters of the distribution of mean project arrival rates affect the average investment rate and the cross-sectional dispersion of firm characteristics. We model the distribution of mean project arrival rates  $\lambda_f = E[\lambda_{ft}]$  across firms as a uniform distribution  $\lambda_f \sim U[\underline{\lambda}, \bar{\lambda}]$ . The parameters of the distribution of  $\lambda_f$  ( $\underline{\lambda} = 5$ ,  $\bar{\lambda} = 25$ ) affect the average investment rate and the cross-sectional distribution of the investment rate, Tobin's Q, and firm profitability.

The dynamics of the stochastic component of the firm-specific arrival rate ( $\mu_H = 0.05$ ,  $\mu_L = 0.25$ , and  $\lambda_H = 5.1$ ) affects the time-series autocorrelation and cross-sectional dispersion of the firm-specific investment rates.

The parameter governing the precision of the public signal  $\sigma_e = 0.15$  has a strong effect on the correlation between firms' investment and their past stock returns. The returns-to-scale parameter  $\alpha = 0.85$  affects the sensitivity of investment to log Tobin's Q.

We simulate the model at a weekly frequency ( $dt = 1/52$ ) and time-aggregate the data to form annual observations. We estimate the firms' idiosyncratic volatility IVOL and BMKT in simulated data using equations (35-36). We simulate 1,000 samples of 2,000 firms over a period of 100 years. We omit the first half of each simulated sample to eliminate the dependence on initial values. Unless noted otherwise, we report median moment estimates and t-statistics across simulations.

In Table 3, we compare the estimated moments in the data to the median moment estimates and the 5th and 95th percentiles in simulated data. In most cases, the median moment estimate of the model is close to the empirical estimate.



## 5 Results

In this section, we explore the link between the model and the data. First, we document that our model can replicate the observed differences in average returns associated with firm characteristics. Next, we document that portfolios of firms sorted on these characteristics exhibit a significant degree of return comovement. A single common return factor extracted from the pooled cross-section of characteristics-sorted portfolios is related to IST shocks and prices this cross-section. Last, we evaluate the extent to which characteristics forecast returns because they proxy for IST risk exposures.

### 5.1 Firm Characteristics and Risk Premia

Here, we compare the properties of portfolios of firms sorted on characteristics in the model to the data. In order to be consistent with our theoretical model, we restrict our analysis to firms in the consumption-good sector. We describe the details in Appendix A.

#### Portfolios sorted on Tobin's $Q$

Table 4 compares the stock return moments of portfolios sorted on Tobin's  $Q$  in the data (top panel) versus the model (bottom panel). Firm's Tobin's  $Q$  is closely related to the ratio of the market value to the book value of equity, so the results of the top panel largely mimic the findings of the literature on the value premium in stock returns. In particular, there is a declining pattern of average returns across the  $Q$ -sorted portfolios. Furthermore, the high- $Q$  portfolios have higher market betas, implying that the CAPM fails to price this cross-section. The portfolio long the top  $Q$ -decile firms and short the bottom  $Q$ -decile firms has an average return of -8.8% per year and a CAPM alpha of -10.3%. Empirically, high Tobin's  $Q$  portfolios also have higher IMC-betas, which indicates that these portfolios have higher stock return exposure to IST shocks.

The bottom panel of Table 4 shows that our model replicates the above patterns. In

the model, high- $Q$  firms have higher ratios of growth opportunities to firm value. Hence, high- $Q$  firms have lower average returns, higher market betas, higher IMC-betas and higher investment rates than low- $Q$  firms, as we find empirically. These results are in line with Kogan and Papanikolaou (2011), and we reproduce them here for completeness.

### **Portfolios sorted on earnings-to-price**

We present moments of the decile portfolios sorted on the ratio of earnings to firm value ( $EP$ ) in the top panel of Table 5. Consistent with portfolio sorts on the earnings-to-price ratio (e.g. Basu (1977), Fama and French (1992)), firms in the top  $EP$  decile outperform firms in the bottom  $EP$ -decile by 8.9% per year. High- $EP$  firms tend to have lower market betas than low- $EP$  firms and thus the CAPM fails to price this cross-section of returns. The long-short position in the extreme  $EP$ -deciles generates a CAPM alpha of 10.2% per year. In the bottom panel of Table 5, we repeat the same analysis on simulated data. Our model produces similar patterns, with the portfolio long the highest  $EP$  decile and short the lowest decile generating a spread in average returns of 7.2% and the CAPM alpha of 12%. Consistent with our model, high- $EP$  and low- $EP$  portfolios also have large differences in IMC-betas.

For completeness, we also perform sorts on the accounting measure of profitability ( $ROA$ ), both in the data and in the model. As a firm characteristic,  $ROA$  is correlated with the share of growth opportunities in the firm value, just like the  $EP$  ratio, but it is a more noisy proxy for  $PVGO/V$ . As we see in Table 6, the pattern of average returns and firm characteristics is similar in the data and in the model. Sorting on  $ROA$  produces dispersion in average returns and CAPM alphas, but most of this dispersion comes from deciles 1 and 2. Moreover, empirically, the first two deciles also have much higher IMC-betas than the rest, which lines up with the pattern of CAPM alphas across the  $ROA$  portfolios. Our model replicates this pattern reasonably well. This profile of IMC betas suggests that  $ROA$  is most informative about  $PVGO/V$  ( $ROA$  and  $PVGO/V$  are negatively correlated) when earnings

are low relative to book value.

### **Portfolios sorted on investment rate**

The top panel of Table 7 presents the main empirical properties of the decile portfolios sorted on the firms' investment rate (IK). Consistent with the empirical findings of Titman et al. (2004) and Anderson and Garcia-Feijo (2006), firms with high investment rates subsequently earn lower average returns. The extreme decile portfolios (highest- versus lowest-IK firms) have an average return difference of -4.9% annually. Furthermore, the high-IK portfolios have higher market betas than the low-IK portfolios (1.56 for the highest IK-decile vs 0.94 for the lowest decile). As a result, the CAPM misprices the IK portfolios, with a spread in CAPM alphas of -8% between the extreme decile portfolios. High-IK portfolios also have higher betas with the IMC portfolio than the low-IK portfolios. This suggests that the difference in average returns among high- and low-IK firms is partially due to the differences in their exposures to the IST shock.

The bottom panel of Table 7 shows the results in simulated data. The model replicates the two key features of the data. The high-IK firms have lower average returns than low-IK firms, with the 4.8% difference between the extreme deciles. At the same time, high-IK firms have higher market betas than low-IK firms, thus our model replicates the failure of the CAPM to price the IK portfolios.

### **Portfolios sorted on market beta**

In the top panel of Table 8 we illustrate that the security market line is downward sloping, consistent with the findings of Fama and French (1992), Frazzini and Pedersen (2010) and Baker et al. (2011). Contrary to the predictions of the standard CAPM model, sorting firms on their CAPM beta (BMKT) does not produce an increasing pattern of average returns – the highest-BMKT portfolio has 2% lower average return than the low-BMKT portfolio. Consequently, CAPM alphas of the extreme decile portfolios have a spread of -5.7% per year.

The difference in returns on the extreme BMKT decile portfolios is poorly spanned by the market portfolio ( $R^2 = 26\%$ ). Similarly to the sorts on investment rates and earnings-to-price ratios, low-CAPM alpha portfolios (high-BMKT firms) have relatively high exposures to the IST shock, measured by their IMC-betas.

High IMC betas of the high-BMKT firms indicate that these are high-growth firms. Consistent with that view, we find that high-BMKT firms also have higher average investment rates and higher Tobin's  $Q$  than the low-BMKT firms. This positive relation between market beta and either Tobin's  $Q$  or investment rate is consistent with our model, but runs counter to the prediction of production-based asset pricing models relying on operating leverage as their main mechanism Zhang (e.g., 2005).

In the bottom panel of Table 8, we show that our model mimics the empirical properties of BMKT-sorted portfolios. In the model, high-BMKT firms tend to be firms with more growth opportunities, hence they have higher IST-shock exposures. The negative market price of IST shocks implies that high-BMKT firms earn lower returns on average than low-BMKT firms. In particular, the top-minus-bottom decile portfolio has an average return of -5.6% per year and a CAPM alpha of -9%. The fact that the CAPM-alpha spread is larger in the model than in the data is likely an artifact of all firms having the same exposure to the disembodied shock. Relaxing this restriction of the model, for instance by introducing operating leverage, would likely weaken the strong relation between BMKT and PVGO/V in the model.

### **Portfolios sorted on idiosyncratic return volatility**

In the top panel of Table 9 we replicate the empirical findings of Ang et al. (2006, 2009). Sorting firms on portfolios based on idiosyncratic volatility results in significant differences in average returns and CAPM alphas. The top and bottom extreme decile portfolios have an average return difference of -5.9% per year. Furthermore, firms with high idiosyncratic volatility tend to also have higher market betas. Consequently, CAPM alphas are large: the

difference in alphas between the extreme decile portfolios is -10.8%. In the bottom panel of Table 9 we show that our model reproduces the negative relation between IVOL and average returns, as well as the failure of the CAPM. The 10 minus 1 portfolio has an average return of -5.2% and a CAPM alpha of -7.6%.

The model reproduces these findings because firms with high idiosyncratic volatility are richer in growth opportunities and therefore have higher exposure to the IST shock. Consistent with this prediction, in the data, firms with high idiosyncratic volatility have higher betas with the IMC portfolio, higher investment rates and Tobin's  $Q$ .

## 5.2 Return Comovement

In this section we show that portfolios constructed by sorting firms on the five characteristics related to growth opportunities ( $Q$ , EP, IK, BMKT, and IVOL) exhibit substantial return comovement that is not captured by their exposure to the market portfolio. In particular, we first remove the effect of the market factor by regressing annual excess returns on these portfolios on excess returns of the market. We normalize the residuals to unit standard deviation, and extract the first principal component from the normalized residuals in each of the five cross-sections. In addition, we extract the first principal component from a pooled cross-section of twenty portfolios that includes portfolios 1, 2, 9, and 10 from each sort.<sup>13</sup>

As we see in Table 10, there is substantial comovement of firms with similar characteristics. Within each set of ten portfolios, the eigenvalue associated with the first principal component normalized by the sum of all eigenvalues ranges from 31% to 52%. These results show that there are return factors within the cross-sections associated with each firm characteristic. The existence of these return factors is often interpreted as an indication that CAPM alphas associated with various firm characteristics could be generated by the exposure of firms to

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<sup>13</sup>As a robustness check, we have repeated the same analysis using monthly returns and the entire cross-section of fifty portfolios, with similar results.

some source of systematic risk missing from the single-factor market model.

According to our theory, returns of firms with similar characteristics comove because these characteristics are correlated with the firms' IST-shock exposures. Empirically, we find that the common factors extracted from portfolios sorted on each of the five characteristics are highly correlated with each other (we remove the market component from each portfolio return, so the return factors we extract are uncorrelated with the market returns). Table 10 shows that there is substantial comovement among these IK-, EP-, Q-, BMKT-, and IVOL-factors. As we see in Table 11, the first principal component  $PC1$  is essentially the average of long-short portfolios across the IK, EP, Q, BMKT, and IVOL sorts. The magnitude of this common source of comovement is substantial: the normalized eigenvalue associated with the first principal component from the pooled cross-section of twenty portfolios is 33%. The correlation between each individual factor and the first principal component of the pooled cross-section ranges from 47% to 92%. These results indicate the presence of a common source of return variation across the portfolios sorted on various characteristics. For instance, not only do high-IK firms comove more with other high-IK firms, but these firms also comove with low-EP, high-Q, high-BMKT, and to some extent high-IVOL firms.

Most importantly, the common factor in characteristic-sorted portfolio returns is closely related to the IST shocks. We compute correlations between  $PC1$  and the measures of the IST shock in the bottom panel of Table 10. The common factor extracted from the pooled cross-section has correlation 69% with the IMC portfolio and 38% with the price of equipment shock  $\Delta z^I$ .

### 5.3 Asset Pricing Tests

As a first pass, we check whether the common systematic risk factor extracted from all five sets of characteristic-sorted portfolios (the first principal component,  $PC1$ ) prices the returns on each set of portfolios. In particular, we use a two-factor model including the market

portfolio and *PC1*. The *PC1* factor earns a negative risk premium, with the annual Sharpe ratio of -0.51, so that portfolios loading positively on this factor must earn relatively low average returns. Table 13 shows that the two-factor model effectively captures the spreads in average returns in the cross-sections sorted by Q, EP, IK, IVOL, and BMKT. In particular, the factor model generates GRS test p-values (see Gibbons, Ross, and Shanken (1989)) that are greater than 10% in each of the cross-sections.

One limitation of the tests above is that they use the return factor constructed from the cross-section of test assets, favoring the pricing models in finite samples. More importantly, these tests focus on the pricing properties of the constructed return factor rather than on its economic source. To link the cross-sectional dispersion in average returns directly to dispersion in IST shock exposure, we form an equivalent of the linearized stochastic discount factor (SDF) (11) in our model,

$$m = a - \gamma_x \Delta x - \gamma_z \Delta z. \quad (37)$$

We directly estimate how well the empirical proxies for IST shocks capture the average return patterns in portfolios formed on firm characteristics by analyzing the pricing errors implied by this SDF.

We estimate (37) using the generalized method of moments (GMM). We use the model pricing errors as moment restrictions, namely, we impose that the SDF in equation (37) should price the cross-section of test asset returns. The resulting moment restrictions are

$$E[R_i^e] = -cov(m, R_i^e), \quad (38)$$

where  $R_i^e$  denotes the excess return of portfolio  $i$  over the risk-free rate.<sup>14</sup> We report first-stage

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<sup>14</sup>Since we use portfolio returns in excess of the risk free rate, the mean of the stochastic discount factor is not identified. Without loss of generality, we choose the normalization  $E(m) = 1$ , which leads to the moment restrictions (38). See Cochrane (2005, pages 256-258) for details.

GMM estimates using the identity matrix to weigh moment restrictions, and adjust the standard errors using the Newey-West procedure with a maximum of three lags. As a measure of fit, we report the sum of squared errors from the Euler equations (38). As test assets, we use decile portfolios 1, 2, 9 and 10 from each of the five cross-sections (Q, EP, IK, BMKT and IVOL).

We proxy for IST shocks with the relative price of new equipment,  $\Delta z^I$ . For the neutral technology shock  $x$ , we use the change in the (log) total factor productivity in the consumption sector from Basu, Fernald, and Kimball (2006). We also consider specifications of the SDF based on portfolio returns. In particular, we use a linear combination of the market portfolio with either the IMC portfolio, or the HML portfolio, both of which span the same linear subspace as the two technology shocks  $x$  and  $z$  in the model. We normalize all shocks to unit standard deviation.

We show the results in Table 14. Cross-sectional differences in the IST risk among the test portfolios account for a sizable portion of the differences in their average returns. Column (1) shows that the specification with only the disembodied shock  $x$  produces large pricing errors (4.23%), similar to the CAPM (3.61%). In contrast, adding the equipment-price shock as a proxy for the IST shock in column (2) reduces the pricing errors to 0.92%. Furthermore, adding IMC or HML portfolio returns to the market return in columns (4) and (5) reduces the pricing errors to 0.75% and 1.40% respectively. Figure 1 illustrates the performance of the factor pricing models considered in Table 14, comparing the model-predicted expected returns on the test portfolios to their realized average returns in the data.

The market price of the IST shock in column (2) is negative,  $-1.35$ , and statistically significant, which implies a negative relation between average returns on the characteristic-sorted portfolios and their IST shock exposures. These point estimates are somewhat higher than the numbers implied by our calibration, based on Kogan and Papanikolaou (2011), but the calibrated parameter value for the price of IST shocks is still within the empirical



confidence interval.

## 5.4 Covariances, Characteristics and Risk Premia

In this paper we argue that firm characteristics (Q, IK, EP, BMKT, and IVOL) are correlated with the firms' exposure to IST shocks, and therefore related to risk premia. However, IST shock exposures may be difficult to estimate using stock market data alone. Here, we explore whether these firm characteristics help forecast IST risk exposures in addition to direct measures of IST betas constructed using only stock returns. In the process, we try to disentangle the direct effect of characteristics on average stock returns from their ability to predict IST risk exposures.

### IMC-beta and firm characteristics

IST risk exposures depend on growth opportunities, which are time-varying. We exploit the fact that the IMC portfolio is a factor-mimicking portfolio for IST shocks and use IMC betas as a measure of IST risk exposures. However, these betas are measured with error. To evaluate whether firm characteristics are related to the true IST risk exposures, we regress estimated IMC betas on their own lagged values and on firm characteristics  $X_{ft}$ :

$$\beta_{ft}^{imc} = a + \gamma_t + \beta X_{ft-1} + \rho \beta_{ft-1}^{imc} + e_{ft}, \quad (39)$$

where  $\gamma_t$  is a year fixed effect and  $X \in \{Q, IK, EP, BMKT, IVOL\}$ .

We show the results in the top panel of Table 15. We find that controlling for current IMC betas, each one of the five characteristics has predictive power for the firms' future exposures to IST shocks. The last column of Table 15 shows that in a joint regression almost all firm characteristics are still significant predictors of future IMC betas, with the exception of EP.

Firm characteristics are informative about future IST risk exposures for two reasons. The

first reason is quite general: IMC betas are measured with error. If the measurement error is i.i.d. over time, firm characteristics that are correlated with the true betas are informative, even controlling for the statistical beta estimates. Second, IMC betas change over time, hence past IMC betas, even abstracting from sampling errors, are not sufficient statistics for the future IST exposures. Certain firm characteristics can help estimate changes in IMC betas because they are measured as the end-of-period value in year  $t$ , whereas covariances are measured using data over the entire year  $t$ . Hence, characteristics may contain more up-to-date information about growth opportunities (and hence IST exposure) than realized return covariances.

To illustrate these effects in the model, we replicate the predictive regression (39) in simulated data. We show the results in the bottom panel of Table 15. In simulations, like in the empirical data, firm characteristics help predict future IST shock exposures when controlling for the recent return-based estimates of IMC betas.

### **Risk exposures versus characteristics**

Prior research shows that the firm characteristics we consider in this paper predict future stock returns. We now evaluate the extent to which this relation is driven by the fact that characteristics are correlated with IST shock exposures. Furthermore, we compare the empirical results to the output of the model, in which, controlling for the IST risk exposures, firm characteristics contain no additional information about expected stock returns.

We frame our analysis as a Fama-MacBeth regression, with the right-hand-side variables being i) firm characteristics; and ii) either lagged IMC-betas estimated using weekly return data, or the forecasts of future IMC betas based on (39) using all of the characteristics. We perform the same analysis on data simulated from the model, and show the results in Table 16.

In the first column of the left panel, we see that each of the variables in question (Q, EP,

IK, IVOL, and BMKT) predicts future stock returns at the firm level. In the second column, we see that the firm's market beta is negatively related to future returns, even controlling for each of the other four characteristics. Hence, not only does the CAPM fail, but variation in market betas unrelated to the included characteristics is negatively correlated with future returns. Including the firm's past IMC beta (columns (3) and (4)) helps reduce the point estimate on the firm characteristic, but the effect is marginal in most cases. In contrast, firm's predicted future IMC beta (columns (5) and (6)) substantially reduces the predictive power of firm characteristics. In particular, controlling for market beta, the fitted IMC-beta drives out idiosyncratic volatility and Tobin's Q in predicting returns. Earnings to price (EP) and investment rate (IK) retain statistically significant predictive ability, but the point estimates are reduced by 30 to 70%.

We find that controlling for IST risk exposures, variation in market beta is associated with a positive risk premium. Column (5) in the first panel shows that in the specification containing the fitted IMC-beta and the market beta, the latter enters with a positive and statistically significant risk premium of 5.8% annually, consistent with the magnitude of the equity premium.

We compare the empirical results to simulated data in the right panel of Table 16. Columns (1) and (2) show that characteristics forecast returns when controlling for market betas. In addition, estimated IMC betas are noisy, even in simulated data. Hence, firm characteristics are not driven out by IMC betas estimated using stock returns only, as we see in columns (3) and (4). In contrast, using the IMC beta projected on firm characteristics largely eliminates the relation between firm characteristics and average returns in simulated data.

## 6 Testing the Mechanism

In this section we provide further evidence that Tobin's Q, IK, EP, BMKT and IVOL are correlated with differences in growth opportunities among firms. Since growth opportunities

are not observable directly, we rely on indirect tests of the mechanism. In particular, our model predicts that firms with more investment opportunities should increase investment by a larger amount following a positive IST shock. In addition, firms with more growth opportunities should exhibit an acceleration in output growth following a positive IST shock, as they acquire more capital.

## 6.1 IST Shocks and Comovement in Investment

We compare the investment response to IST shocks as a function of firm characteristics using the following specification:

$$i_{ft} = a_1 + \sum_{d=2}^5 a_d D(G_{f,t-1})_d + b_1 \Delta z_{t-1} + \sum_{d=2}^5 b_d D(G_{f,t-1})_d \Delta z_{t-1} + u_t, \quad (40)$$

where  $i_t$  is the firm's investment rate;  $\Delta z_t \in \{\Delta z^I, R^{imc}, R^{pc1}\}$  is one of the three measures of the IST shock;  $D(G_f)_d$  is a dummy variable that takes the value one if the firm's growth opportunity measure  $G_f \in \{Q_f, IK_f, EP_f, BMKT_f, IVOL_f\}$  belongs to the quintile  $d$  in year  $t - 1$ . We standardize all right-hand side variables to zero mean and unit standard deviation. We account for unobservable time and firm effects by clustering standard errors by firm and year (see Petersen (2009)). To evaluate the ability of the model to quantitatively replicate the data, we also estimate (40) using simulated data from the model.

As we see in Panel A of Table 17, in the model, following a positive IST shock, firms with more growth opportunities (high Q, high IK, low EP, high BMKT and high IVOL) increase investment by more relative to firms with fewer growth opportunities. In panels B to D we explore this prediction of the model in the data, using different measures of the IST shock. We find that in almost all cases, the empirical results support the model's predictions. Furthermore, the magnitude of this effect in the data is substantial: a one-standard deviation  $z$ -shock leads to firms with more growth opportunities increasing investment by 0.31 to 1.56% relative to firms with low growth opportunities. For comparison, the median investment

rate in our sample is 11%. In simulated data, the corresponding magnitudes are a bit larger (1.3-2.4%), but still comparable.

## 6.2 IST Shocks and Comovement in Firm Output Growth

A positive IST shock implies that new projects are more productive relative to existing projects. Therefore, following a positive IST shock, firms with more growth opportunities should experience higher output growth than low-growth firms. To the extent that firm characteristics such as Tobin's Q, IK, EP, BMKT, and IVOL are correlated with  $PVGO/V$ , we expect these characteristics to capture the differential sensitivity of output growth to IST shocks.

We estimate the response of output growth to the IST shock using the following specification:

$$\ln y_{f,t+k} - \ln \bar{y}_{t+k} = a_0 + \sum_{d=2}^5 a_d D(G_{f,t-1})_d + b_1 \Delta z_t + \sum_{d=2}^5 b_d D(G_{ft})_d \Delta z_t + \rho (\ln y_{ft} - \ln \bar{y}_t) + u_{f,t+k}, \quad (41)$$

where  $y_{f,t}$  is firm output, defined as firm sales (sale) plus change in inventories (inv);  $\bar{y}$  is average output across firms;  $G \in \{Q, IK, EP, IVOL, BMKT\}$  is the set of firm characteristics;  $\Delta z_t \in \{\Delta z^I, R_{IMC}\}$  is our measure of IST shocks normalized to unit standard deviation; and  $D(G_{f,t-1})_d$  is a quintile dummy variable. We control for industry fixed effects in the regression and cluster standard errors by firm and by year.

The coefficient of interest is  $b_5(k)$ , which captures the differential impact of an IST shock on  $k$ -period output growth between firms in the top ( $G_5$ ) and the bottom ( $G_1$ ) quintile. For brevity, we present results with the real proxy for IST shocks constructed using the price of equipment ( $\Delta z^I$ ), but we obtain similar results using returns to the IMC portfolio or the common factor PC1.

We estimate equation (41) for horizons of 1 to 6 years. We plot the estimated  $b_5(k)$

coefficients along with the 90% confidence intervals. To evaluate the connection with the model, we perform the same exercise in simulated data and plot the median coefficient estimates  $b_5(k)$ , along with the 5th and 90th percentiles across simulations.

We present results in the top (data) and bottom (model) panels of Figure 2. Our findings are consistent with the mechanism. Specifically, we find that a positive IST shock  $z$  leads to a differential increase in output growth of firms with more growth opportunities, as measured by high Tobin's Q, IK, BMKT, IVOL and low EP, relative to firms with fewer growth opportunities. Furthermore, the magnitude of this effect in the model is comparable to the empirical estimates.

## 7 Additional Predictions

We now discuss the predictions of our model for the behavior of aggregate discount rates. In particular, the same mechanism in our model that links firm characteristics to risk premia also leads to time-varying expected returns to the market portfolio. Furthermore, our assumption that the price of IST shocks is negative implies that risk premia on dividend strips are declining with maturity. Both of these predictions are consistent with empirical evidence.

### 7.1 Return predictability

In our model, expected returns on the market portfolio are time-varying. Even though the model features constant price of risk for the two aggregate shocks  $x$  and  $z$ , the asset composition of the market portfolio varies over time, leading to time-variation in its expected excess return. In particular, applying equation (22) to the market portfolio,

$$\frac{1}{dt}E_t[R_{Mt}] - r_f = \gamma_x \sigma_x + \frac{\alpha}{1 - \alpha} \gamma_z \sigma_z \frac{PVGO_{Ct} + V_{It}}{V_{Ct} + V_{It}}. \quad (42)$$

The equity premium depends on the relative values of growth opportunities in the consumption sector,  $PVGO_C$ , the market values of the consumption sector,  $V_C$ , and the investment sector,

$V_I$ . The fact that discount rates are time-varying implies that, in the model, price-dividend and price-earning ratios predict returns with a negative sign, consistent with the data (see e.g. Campbell and Shiller (1988)). Furthermore, the aggregate Tobin's  $Q$  and investment rate are positively correlated with the ratio of PVGO to  $V$ . Hence, a negative price of risk for the IST shock  $\gamma_z$  implies that both  $Q$  and aggregate investment predict market portfolio returns with a negative sign, as in the data (see, e.g., Cochrane, 1991).

To quantify the degree of return predictability generated by changes in asset composition, we replicate return predictability regressions in simulated data. In particular, we estimate

$$\sum_{h=1}^k r_{M,t+h} = a(k) + b(k) x_t + u_{t+h},$$

where  $r_M$  is the log gross market portfolio return and  $x$  is the log of a predictor variable. As predictors, we use variables that have been empirically linked to subsequent stock returns: a) the aggregate price-dividend ratio; b) the aggregate price-earnings ratio; c) aggregate Tobin's  $Q$ ; and d) aggregate investment rate. We adjust the standard errors using the Newey-West procedure, with a lag length equal to 1.5 times the number of overlapping observations. We show the results for horizons of one to five years in Table 18. The estimated slope  $b(k)$  is negative and statistically significant. In addition, the adjusted  $R^2$  ranges from 7% at the horizon of one year, to 28% at five years. The magnitude of return predictability in the model is in line with the magnitude in the data (see, e.g., Fama and French, 1988).

## 7.2 Term structure of risk premia

van Binsbergen, Brandt, and Koijen (2012) provide evidence that short-term claims on the dividend of the S&P 500 index have substantially higher average returns than the S&P 500 index itself. The authors interpret this finding as evidence that the term-structure of risk premia is downward sloping, and argue that standard asset pricing models have difficulty matching this fact (see, for instance, Campbell and Cochrane, 1999; Bansal and Yaron, 2004).

A downward-sloping term structure of risk premia is consistent with the models of Lettau and Wachter (2007) and Nakamura, Steinsson, Barro, and Ursa (2010). A key feature of both of these models is that negative shocks to dividends are associated with subsequent increases in expected dividend growth. In our model, this pattern of mean reversion arises endogenously. In particular, a positive IST shock leads to firms reducing dividends to pay for increased investment. In future periods, this increased investment bears fruit, leading to higher future dividend growth.

To illustrate how this mechanism leads to a downward-sloping term structure of risk premia, consider the price of a dividend strip, defined as a claim on the time- $T$  aggregate dividend

$$v_t(D_T) = x_t e^{-a_1(T-t)} \int_{\mathcal{F}} \sum_{j \in \mathcal{J}_{ft}} K_j^\alpha df + \bar{\lambda} a_2 x_t z_t^{\frac{\alpha}{1-\alpha}} \left[ \frac{1}{\rho + \delta} (1 - e^{-(\rho+\delta)(T-t)}) - (1 - \phi) e^{-\rho(T-t)} \right], \quad (43)$$

where

$$a_1 = r + \gamma_x \sigma_x + \delta - \mu_x, \quad a_2 = \left( \frac{\alpha}{r_f + \gamma_x \sigma_x + \delta - \mu_X} \right)^{\frac{1}{1-\alpha}} \quad (44)$$

The first term in equation (43), which captures the contribution of existing assets to the time- $T$  dividend, is decreasing in maturity  $T$  due to discounting and has no exposure to the IST shock. The second term captures the contribution to the time- $T$  dividend of projects acquired between time  $t$  and  $T$  (first term in brackets) and the time- $T$  investment expenditures (second term in brackets). This term in (43) is increasing in the maturity of the dividend strip  $T$  and has positive exposure to the IST shock  $z$ . Therefore, a negative price of IST shocks  $\gamma_z$  implies that risk premia are decreasing with maturity  $T$ . To illustrate that our calibration produces a downward-sloping term structure of risk premia for dividend strips, we plot the risk premium associated with dividend strips of maturities  $k = 1/12, \dots, 20$  years in Figure 3.



Furthermore, dividend strips with very short maturities have negative IST exposure, since investment outlays exceed the cash flows generated from future projects. Consequently, return volatility of dividend strips has a U-shaped pattern. In addition, since these short-term maturities have the opposite IST shock exposure from the market portfolio, and the same exposure to the disembodied technology shocks, their market betas are below one.

## 8 Robustness Tests

In this section we perform a number of robustness tests. To conserve space, we briefly summarize the results and refer the reader to the Internet Appendix for details.

First, we explore whether our results are driven primarily by intra- or inter-industry variation in firm characteristics. To clarify the relative importance of these two dimensions of firm heterogeneity, we repeat our portfolio sorts *within* the Fama and French (1997) 17-industry classifications. In this case, we find that our results on comovement and dispersion in risk premia are similar or stronger for all the considered characteristics, with the exception of IVOL. Sorting firms on IVOL within industries produces a significantly smaller spread in average returns (2.2%) and CAPM alphas (4.7%) relative to the unconditional sort. In addition, the first principal component extracted from the cross-section of within-industry sorted firms is very weakly correlated with the other cross-sections. These results suggest that there is substantial intra-industry variation in idiosyncratic volatility that is not related to firms' growth opportunities.

Second, we explore the potential effect of variation in leverage across the characteristic-sorted portfolios. If firm leverage correlated positively with the fraction of growth opportunities in firm value, then equity prices of firms with more growth opportunities would likely have higher exposure to any systematic risk factor, not only to IST shocks. This is an unlikely situation, since prior literature finds leverage to be negatively related to growth opportunities (e.g., Barclay and Morellec (2006)). As we measure growth opportunities somewhat differently,

we directly explore this issue. We compute the median book leverage within each decile portfolio and confirm using our methodology that leverage is, if anything, negatively related to growth opportunities. Thus, our results agree with the prior findings in the literature, and show that differential leverage is not an alternative explanation for our findings on return comovement.

Third, we evaluate whether our results are driven by small firms. We repeat our analysis after eliminating the bottom 20% of firms in terms of market capitalization every year. We find that our results are similar and in some cases stronger on this sub-sample, and thus unlikely to be driven by the smallest firms.

Fourth, we consider an additional empirical proxy for IST shocks – changes in the aggregate investment-to-consumption ratio (as in Kogan and Papanikolaou (2011)). This measure follows from our model, albeit it is a more complex empirical object than the model suggests. We find that using this proxy for IST shocks leads to similar empirical findings.

Last, we focus on a subsample of firms that are more likely to be capital-intensive and hence correspond more closely to the firm in our model. In particular, we perform our empirical analysis excluding the firms that produce services (industries 14-17 according to the Fama and French (1997) 17-industry classification scheme). We find that our results are somewhat stronger in this subsample.

## 9 Conclusion

In this paper we describe the joint dynamics of firm investment decisions, profitability, and stock returns in a structural model in which firms experience shocks to their productivity and investment opportunities. We link several commonly used firm characteristics, such as Tobin's  $Q$ , investment rate, earnings-to-price ratio, market beta, and idiosyncratic volatility to the firm's growth opportunities. As a result, these characteristics are correlated with the firm's exposures to the aggregate investment-specific technology shocks, which helps explain stock

return comovement among firms with similar characteristics, and cross-sectional correlations between the characteristics and average stock returns.

Understanding the economic sources of stock return comovement is important for further progress in our analysis of stock market behavior. From the perspective of asset pricing, this helps better interpret the properties of the empirical factor-based pricing models. From the perspective of macroeconomics, additional progress on this front should promote a more fruitful use of asset pricing data in studies concerned with the sources of aggregate fluctuations. In this paper we aim to contribute on both fronts.

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## A. Data

### Macroeconomic variables

Data on the consumption deflator, consumption of non-durables and services and non-residential investment is from the Bureau of Economic Analysis. Data on the relative price of equipment is from Israelsen (2010). Data on TFP in the consumption sector is from Basu et al. (2006).

### Firm-level variables

Firm-level variables are from Compustat, unless otherwise noted. We summarize their definitions in Table 1.

### Sample

We omit firms with fewer than 50 weekly stock-return observations per year, firms producing investment goods, financial firms (SIC codes 6000-6799) and utilities (SIC codes 4900-4949). In our investment regressions we also exclude firms with missing values of CAPEX (Compustat item capx), PPE (Compustat item ppent), Tobin's Q, firms in their first three years following the first appearance in Compustat, and firms with negative book values. Our sample contains 6,832 firms and 63,295 firm-year observations and covers the 1965-2008 period.

### Portfolio construction

**HML portfolio** We construct a  $2 \times 3$  sort, sorting firms first on their market value of equity (CRSP December market capitalization) and then on their ratio of Book-to-Market (see above for more details). We construct the value factor ( $HML$ ) as  $1/2(SV - SG) + 1/2(LV - LG)$ , where  $SG, SV, LG$  and  $LV$  refer to the corner portfolios.

**IMC portfolio** We follow Gomes, Kogan, and Yogo (Gomes et al.) and Papanikolaou (2011) and classify firms as investment or consumption producers based on the U.S. Department of Commerce's National Income and Product Account (NIPA) tables. We classify industries based on the sector to which they contribute the most value. We use the 1997 Input-Output tables to classify NAICS industries into investment or consumption producers. We include common shares (shrcd=10,11) of all firms traded in NYSE, AMEX and NASDAQ (exchcd=1,2,3).

**Portfolios sorted on characteristics** We sort firms annually into 10 value-weighted portfolios based on the past value of characteristic. We estimate  $\beta^{imc}$  using weekly returns. We include common shares (shrcd=10,11) of all firms traded in NYSE, AMEX and NASDAQ (exchcd=1,2,3). We restrict the sample to firms producing consumption goods, and



exclude financial firms (SIC6000-6799) and utilities (SIC4900-4949). When using accounting variables for characteristics, we rebalance the portfolios on June of every calendar year. When using moments of stock returns (MBETA, IVOL), we rebalance at the end of every year.

# Tables

**Table 1: Definitions**

Variable	Data	Model
Investment (I)	capx	$x z^{-1} K_f^*$
Capital (K)	ppeg	$z^{-1} x \sum_{j \in \mathcal{J}^f} K_j$
Book Assets (A)	at	$z_t^{-1} x_t \sum_{j \in \mathcal{J}_t^f} K_j$
Operating Cash Flows (CF)	dp + item ib	$\sum_{j \in \mathcal{J}^f} y_j$
Payout	DIV+REP	$\sum_{j \in \mathcal{J}_t^f} y_{jt} - x z^{-1} K_f^*$
Market-to-Book (M/B)	V / EC	V / K
Tobin's Q (Q)	(V + EP + D - INVT - T) / K	V / K
Market Capitalization (V)	CRSP December market cap	$V_f$
Dividends (DIV)	dvc +dvp	-
Share Repurchases	prstk	-
Book Debt (D)	dltt	-
Book Preferred Equity (EP)	pstkrv	-
Book Common Equity (EC)	ceq	-
Inventories (INVT)	inv	-
Deferred Taxes (T)	txdb	-
R&D Expenditures (R&D)	xrd	-
Cash Holdings (CASH)	che	-

Table 1 shows the definitions of the variables used in empirical analysis and model simulations.

**Table 2: Parameters**

Parameter	Symbol	Value
<i>Technology</i>		
Growth rate of X-shock	$\mu_x$	0.50%
Volatility of $x$ -shock	$\sigma_x$	7.00%
Growth rate of IST shock	$\mu_Z$	0.30%
Volatility of IST shock	$\sigma_Z$	4.20%
Mean-reversion parameter of project-specific shock	$\theta_u$	0.03
Volatility of project-specific shock	$\sigma_u$	1.25
<i>Production</i>		
Project DRS parameter	$\alpha$	0.85
Profit margin of investment firms	$\phi$	7.5%
Depreciation rate of capital	$\delta$	10%
<i>Learning</i>		
Noise in public signal	$\sigma_e$	15%
<i>Investment</i>		
Maximum long-run project arrival rate	$\bar{\lambda}$	25
Minimum long-run project arrival rate	$\underline{\lambda}$	5
Project arrival rate in high-growth state	$\lambda_H$	5.100
Transition probability into high-growth state	$\mu_H$	0.050
Transition probability into low-growth state	$\mu_L$	0.250
<i>Stochastic discount factor</i>		
Risk-free rate	$r$	3%
Price of risk of $x$ -shock	$b_x$	1.77
Price of risk of IST shock	$b_z$	0.57

Table 2 shows the parameters in model calibration.

**Table 3: Calibration**

Moment	Data	Model		
		Median	5%	95%
<i>Aggregate moments, real variables</i>				
Mean of aggregate dividend growth	0.025	0.025	-0.048	0.076
Volatility of aggregate dividend growth	0.118	0.088	0.055	0.221
Volatility of aggregate investment growth	0.157	0.155	0.100	0.228
<i>Aggregate moments, asset prices</i>				
Mean excess return of market portfolio	0.059	0.061	0.034	0.092
Volatility of market portfolio return	0.161	0.145	0.100	0.199
Mean return of HML portfolio	0.035	0.038	0.024	0.047
Volatility of HML portfolio	0.141	0.066	0.041	0.090
Relative market capitalization of I- and C-sector	0.149	0.138	0.060	0.211
<i>Firm characteristics, time-series moments</i>				
Median firm investment rate	0.116	0.100	0.041	0.177
Correlation between investment and lagged stock returns	0.177	0.180	0.068	0.249
Correlation between investment and lagged Tobin's $Q$	0.280	0.280	0.198	0.387
Serial correlation of return on assets	0.825	0.841	0.821	0.899
Serial correlation of firm investment rate	0.478	0.524	0.421	0.604
<i>Firm characteristics, cross-sectional dispersion (IQR)</i>				
Firm investment rate	0.157	0.107	0.048	0.180
Cashflows-to-Capital	0.234	0.234	0.203	0.298
Tobin's $Q$	3.412	2.327	1.339	4.280
$\beta^{imc}$	0.990	0.691	0.477	0.937
Firm size relative to average size	0.830	0.778	0.725	0.931

Table 3 compares sample moments to moments in simulated data. Stock return moments are estimated over the sample 1963-2008. The moments of investment growth are estimated using the series on real private nonresidential investment in equipment and software. Moments of firm-specific variables are estimated using Compustat data over the 1963-2008 period, where we report time series moments of the investment rate and cash flows over capital, Tobin's  $Q$  and IMC-beta. Moments of dividend growth are from the long sample in Campbell and Cochrane (1999). We construct the value factor (HML) in the consumption sector as  $1/2(LV - LG) + 1/2(SV - SG)$  where  $LV$ ,  $LG$ ,  $SV$ ,  $SG$  refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. We exclude firms producing investment goods, financial firms (SIC6000-6799), and utilities (SIC4900-4949).

**Table 4: Portfolios sorted on Tobin's  $Q$**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	10.26 (4.43)	9.48 (5.01)	7.43 (3.91)	5.74 (2.82)	4.19 (1.80)	4.27 (2.10)	5.76 (2.21)	5.13 (1.91)	4.94 (1.93)	1.47 (0.58)	-8.79 (-3.26)
$\sigma(\%)$	19.06	19.74	16.87	16.62	18.13	17.29	17.35	20.85	20.31	24.94	20.75
$\beta^{mkt}$	0.86 (7.25)	0.92 (7.27)	0.82 (8.82)	0.82 (10.24)	0.90 (16.87)	0.89 (20.49)	0.87 (15.79)	1.06 (13.78)	0.99 (15.13)	1.15 (9.61)	0.29 (1.66)
$\alpha(\%)$	5.98 (3.25)	4.90 (3.68)	3.35 (3.22)	1.66 (1.61)	-0.32 (-0.35)	-0.20 (-0.26)	1.43 (1.16)	-0.17 (-0.14)	-0.00 (-0.00)	-4.29 (-2.25)	-10.27 (-3.64)
$R^2(\%)$	65.33	69.79	75.77	78.14	79.96	86.69	81.02	83.85	76.71	69.02	6.55
$\beta^{imc}$	0.07	0.17	0.12	0.22	0.21	0.31	0.15	0.41	0.31	0.68	0.61
$I/K$	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.21	0.28	
$Q$	0.29	0.53	0.72	0.95	1.27	1.77	2.56	4.07	7.57	23.83	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	10.12 (6.44)	8.31 (4.87)	7.36 (4.05)	6.71 (3.50)	6.16 (3.05)	5.70 (2.69)	5.32 (2.43)	5.03 (2.25)	4.81 (2.16)	4.85 (2.35)	-5.27 (-5.71)
$\sigma(\%)$	11.23	12.30	13.16	13.98	14.78	15.53	16.14	16.47	16.51	15.82	6.89
$\beta^{mkt}$	0.72 (21.96)	0.80 (33.01)	0.86 (46.04)	0.92 (60.03)	0.97 (66.17)	1.02 (63.02)	1.06 (56.91)	1.08 (54.01)	1.08 (51.91)	1.03 (46.87)	0.31 (7.71)
$\alpha(\%)$	5.61 (11.73)	3.30 (8.95)	1.98 (6.49)	0.99 (3.89)	0.12 (0.39)	-0.63 (-2.50)	-1.24 (-4.29)	-1.66 (-5.30)	-1.88 (-5.89)	-1.51 (-4.94)	-7.13 (-10.83)
$R^2(\%)$	89.60	94.29	96.07	96.94	97.27	97.25	97.11	96.92	96.80	96.08	53.34
$\beta^{imc}$	0.50	0.57	0.64	0.70	0.78	0.84	0.90	0.93	0.93	0.86	0.36
$I/K$	0.08	0.08	0.09	0.10	0.10	0.11	0.11	0.12	0.13	0.15	
$Q$	1.81	2.21	2.56	2.92	3.31	3.71	4.10	4.51	5.01	5.89	

Table 4 shows characteristics for the 10 portfolios of firms sorted on Tobin's  $Q$  (see Table 1 for variable definitions). The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 5: Portfolios sorted on the earnings-to-price ratio (EP)**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	2.28 (0.72)	1.65 (0.84)	2.23 (1.13)	5.35 (2.56)	7.34 (3.67)	6.65 (3.47)	7.89 (3.51)	8.23 (3.33)	8.95 (3.98)	11.19 (4.25)	8.91 (3.00)
$\sigma(\%)$	25.72	16.90	16.24	16.99	18.25	17.27	17.86	19.05	19.76	20.96	19.72
$\beta^{mkt}$	1.21 (10.72)	0.86 (12.44)	0.85 (15.67)	0.85 (12.41)	0.90 (10.86)	0.88 (13.09)	0.91 (10.10)	0.92 (10.37)	0.81 (4.85)	0.97 (9.59)	-0.25 (-1.62)
$\alpha(\%)$	-3.80 (-2.24)	-2.63 (-2.67)	-2.00 (-3.02)	1.11 (1.72)	2.85 (2.40)	2.26 (3.10)	3.36 (2.95)	3.64 (2.88)	4.88 (2.50)	6.35 (3.15)	10.15 (3.51)
$R^2(\%)$	72.25	83.28	88.35	81.05	78.41	83.92	83.54	75.21	55.15	68.98	5.11
$\beta^{imc}$	0.87	0.31	0.30	0.08	0.17	0.24	0.17	0.18	-0.04	0.18	-0.69
IK	0.13	0.12	0.11	0.11	0.11	0.11	0.10	0.11	0.12	0.13	
$Q$	2.20	2.11	1.99	1.80	1.62	1.41	1.34	1.22	1.00	0.44	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	2.97 (0.99)	3.75 (1.41)	4.32 (1.75)	4.91 (2.15)	5.49 (2.57)	6.11 (3.06)	6.82 (3.66)	7.62 (4.43)	8.60 (5.48)	10.18 (7.47)	7.21 (3.93)
$\sigma(\%)$	20.48	18.69	17.55	16.53	15.59	14.65	13.66	12.63	11.48	9.85	13.33
$\beta^{mkt}$	1.36 (26.04)	1.25 (32.56)	1.18 (40.53)	1.11 (51.63)	1.05 (66.04)	0.99 (78.22)	0.92 (63.71)	0.84 (44.29)	0.76 (30.22)	0.62 (17.48)	-0.74 (-9.53)
$\alpha(\%)$	-5.76 (-7.65)	-4.28 (-7.86)	-3.27 (-7.81)	-2.24 (-7.09)	-1.27 (-5.28)	-0.24 (-1.42)	0.90 (3.66)	2.18 (7.25)	3.71 (9.67)	6.18 (11.69)	11.94 (10.47)
$R^2(\%)$	92.30	94.52	95.84	96.59	97.12	97.25	96.88	95.69	92.83	83.49	64.23
$\beta^{imc}$	1.22	1.09	0.99	0.91	0.83	0.75	0.66	0.58	0.49	0.35	-0.90
$I/K$	0.13	0.12	0.12	0.11	0.11	0.10	0.09	0.08	0.07	0.07	
$Q$	4.27	4.26	4.11	3.86	3.61	3.27	2.97	2.69	2.47	2.26	

Table 5 shows characteristics for the 10 portfolios of firms sorted on the earnings-to-price ratios (see Table 1 for variable definitions). The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 6: Portfolios sorted on return-to-assets (ROA)**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	1.56 (0.28)	3.10 (0.91)	4.44 (1.52)	4.49 (1.71)	5.59 (2.60)	5.70 (2.86)	5.41 (2.61)	6.22 (3.15)	4.56 (2.23)	6.21 (2.45)	4.66 (0.88)
$\sigma(\%)$	42.17	30.22	23.95	19.96	17.36	18.18	18.17	17.26	17.16	18.93	32.94
$\beta^{mkt}$	1.69 (8.74)	1.43 (7.41)	1.15 (9.38)	0.93 (8.55)	0.86 (12.92)	0.93 (14.88)	0.95 (19.48)	0.89 (12.26)	0.88 (16.35)	0.94 (14.33)	-0.75 (-3.56)
$\alpha(\%)$	-6.89 (-1.47)	-4.07 (-1.73)	-1.29 (-1.00)	-0.14 (-0.09)	1.30 (1.22)	1.04 (1.02)	0.66 (0.85)	1.79 (1.86)	0.16 (0.20)	1.51 (1.32)	8.40 (1.65)
$R^2(\%)$	52.06	73.15	74.28	69.84	79.06	85.15	88.51	85.50	85.02	80.15	16.75
$\beta^{imc}$	1.43	1.01	0.31	0.30	0.12	0.31	0.23	0.24	0.19	0.33	-1.10
IK	0.14	0.11	0.10	0.09	0.10	0.10	0.10	0.12	0.14	0.17	
Q	3.76	1.15	0.85	0.87	0.89	0.88	1.04	1.39	2.20	4.85	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	3.79 (1.80)	5.12 (2.73)	6.69 (3.27)	6.98 (3.58)	7.09 (3.74)	7.13 (3.83)	7.07 (3.84)	7.03 (3.83)	6.97 (3.86)	7.11 (4.19)	3.32 (2.13)
$\sigma(\%)$	18.79	16.00	14.66	14.01	13.65	13.46	13.37	13.40	13.31	12.72	7.70
$\beta^{mkt}$	1.25 (31.44)	1.07 (47.93)	0.98 (56.28)	0.94 (59.10)	0.91 (56.51)	0.90 (54.62)	0.89 (51.39)	0.89 (47.16)	0.88 (44.00)	0.83 (34.89)	-0.42 (-7.90)
$\alpha(\%)$	-4.21 (-5.79)	-1.77 (-2.26)	0.37 (1.47)	0.94 (3.75)	1.22 (4.74)	1.34 (5.06)	1.34 (4.92)	1.30 (4.48)	1.32 (4.17)	1.81 (4.65)	6.02 (6.80)
$R^2(\%)$	94.40	96.46	96.86	96.78	96.64	96.53	96.37	96.17	95.80	93.71	60.24
$\beta^{imc}$	1.05	0.83	0.73	0.68	0.64	0.63	0.62	0.61	0.60	0.55	-0.49
IK	0.10	0.09	0.09	0.09	0.08	0.09	0.09	0.09	0.10	0.11	
Q	3.31	2.98	2.79	2.75	2.84	2.96	3.18	3.44	3.89	4.93	

Table 6 shows characteristics for the 10 portfolios of firms sorted on profitability (see Table 1 for variable definitions). The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 7: Portfolios sorted on the investment rate (IK)**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	7.46 (3.00)	6.98 (3.42)	7.25 (4.40)	7.08 (3.73)	6.98 (3.29)	6.00 (2.64)	5.74 (2.45)	4.07 (1.67)	3.28 (1.30)	2.51 (0.65)	-4.94 (-1.42)
$\sigma(\%)$	20.60	17.04	15.15	16.47	16.33	17.81	18.72	19.50	23.00	33.30	24.86
$\beta^{mkt}$	0.94 (7.79)	0.82 (10.05)	0.73 (10.61)	0.83 (12.24)	0.85 (15.75)	0.91 (12.85)	0.93 (15.96)	0.99 (14.18)	1.16 (14.69)	1.56 (9.52)	0.62 (2.91)
$\alpha(\%)$	2.77 (1.40)	2.87 (2.16)	3.61 (4.13)	2.92 (4.38)	2.70 (2.87)	1.43 (1.70)	1.09 (0.88)	-0.89 (-0.89)	-2.52 (-2.02)	-5.27 (-1.85)	-8.04 (-2.41)
$R^2(\%)$	67.02	75.49	75.24	82.93	88.70	85.42	80.01	83.89	82.52	70.80	20.13
$\beta^{imc}$	0.13	0.14	0.07	0.13	0.24	0.30	0.23	0.35	0.67	1.16	1.03
IK	0.03	0.05	0.07	0.09	0.11	0.13	0.16	0.21	0.30	0.64	
$Q$	0.96	0.78	0.91	1.03	1.19	1.45	1.73	2.08	2.83	4.54	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	9.41 (5.95)	8.13 (4.57)	7.24 (3.77)	6.53 (3.20)	5.97 (2.78)	5.54 (2.48)	5.52 (2.48)	6.10 (2.99)	6.34 (3.45)	4.61 (2.30)	-4.81 (-7.85)
$\sigma(\%)$	11.51	13.03	14.09	14.95	15.67	16.18	15.97	14.64	13.39	14.45	4.41
$\beta^{mkt}$	0.75 (27.22)	0.87 (44.25)	0.94 (61.23)	1.00 (70.66)	1.05 (63.66)	1.09 (56.21)	1.07 (45.82)	0.99 (52.66)	0.90 (56.02)	0.98 (53.67)	0.22 (7.10)
$\alpha(\%)$	4.52 (11.11)	2.49 (8.34)	1.12 (4.41)	0.04 (-0.02)	-0.85 (-3.34)	-1.49 (-5.05)	-1.44 (-4.42)	-0.30 (-1.00)	0.50 (1.90)	-1.71 (-6.81)	-6.23 (-13.88)
$R^2(\%)$	91.98	95.73	96.81	97.13	97.11	96.87	96.60	96.84	96.81	97.11	53.01
$\beta^{imc}$	0.48	0.62	0.72	0.80	0.87	0.92	0.92	0.79	0.67	0.78	0.29
$I/K$	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.14	0.25	
$Q$	2.36	2.81	3.21	3.60	3.96	4.22	4.36	4.31	2.90	2.97	

Table 7 shows characteristics for the 10 portfolios of firms sorted on investment rate (see Table 1 for variable definitions). The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).



**Table 8: Portfolios sorted on the market beta (BMKT)**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	5.81 (2.51)	6.40 (3.04)	5.42 (2.72)	6.73 (3.77)	5.92 (3.13)	5.08 (2.65)	6.02 (2.43)	5.04 (1.95)	3.98 (1.19)	3.84 (0.98)	-1.97 (-0.52)
$\sigma(\%)$	18.20	16.21	17.50	15.60	16.86	16.62	20.16	21.02	25.47	30.98	26.47
$\beta^{mkt}$	0.71 (5.19)	0.64 (5.42)	0.78 (5.33)	0.76 (13.67)	0.80 (10.39)	0.83 (10.02)	1.06 (18.41)	1.10 (24.42)	1.27 (14.65)	1.46 (11.75)	0.75 (4.26)
$\alpha(\%)$	2.24 (1.11)	3.21 (2.09)	1.53 (1.32)	2.93 (2.72)	1.90 (1.59)	0.95 (0.87)	0.74 (0.76)	-0.44 (-0.43)	-2.39 (-1.64)	-3.47 (-1.48)	-5.71 (-1.87)
$R^2(\%)$	49.70	50.28	64.13	76.94	73.46	80.12	88.95	88.09	81.13	72.10	25.93
$\beta^{imc}$	0.10	-0.13	-0.00	0.12	0.05	0.11	0.44	0.57	0.78	1.14	1.05
$I/K$	0.09	0.09	0.09	0.09	0.10	0.11	0.12	0.13	0.15	0.17	
$Q$	1.04	1.02	1.07	1.13	1.23	1.32	1.48	1.62	1.98	2.61	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	9.59 (6.50)	8.63 (5.38)	7.81 (4.54)	7.14 (3.92)	6.53 (3.37)	6.01 (2.93)	5.50 (2.54)	5.07 (2.21)	4.63 (1.90)	4.04 (1.56)	-5.55 (-4.06)
$\sigma(\%)$	10.66	11.66	12.53	13.32	14.13	14.88	15.65	16.49	17.33	18.33	9.84
$\beta^{mkt}$	0.68 (21.22)	0.76 (29.69)	0.83 (39.88)	0.88 (53.09)	0.94 (67.40)	0.99 (71.61)	1.05 (63.28)	1.10 (52.33)	1.15 (41.13)	1.22 (31.87)	0.54 (9.21)
$\alpha(\%)$	5.21 (10.71)	3.74 (9.56)	2.49 (7.63)	1.46 (5.34)	0.48 (1.91)	-0.38 (-1.78)	-1.21 (-4.77)	-2.00 (-6.26)	-2.78 (-6.73)	-3.77 (-6.95)	-8.98 (-10.14)
$R^2(\%)$	87.92	92.50	94.89	96.17	96.78	97.12	97.05	96.72	96.01	94.66	63.49
$\beta^{imc}$	0.40	0.47	0.54	0.60	0.67	0.73	0.80	0.86	0.93	1.02	0.65
$I/K$	0.05	0.07	0.08	0.08	0.09	0.10	0.11	0.11	0.11	0.12	
$Q$	2.40	2.55	2.73	2.96	3.16	3.41	3.63	3.80	3.91	3.96	

Table 8 shows characteristics for the 10 portfolios of firms sorted on market beta. The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 9: Portfolios sorted on idiosyncratic return volatility (IVOL)**

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	5.02 (2.45)	5.96 (2.60)	6.40 (3.23)	7.28 (2.89)	5.84 (1.99)	5.60 (1.72)	3.59 (0.97)	3.42 (0.87)	1.01 (0.20)	-0.84 (-0.14)	-5.86 (-0.95)
$\sigma(\%)$	15.35	17.38	17.82	20.69	25.59	26.55	30.27	32.75	39.33	42.44	37.05
$\beta^{mkt}$	0.77 (12.71)	0.92 (44.95)	0.93 (18.91)	1.07 (19.60)	1.29 (15.81)	1.26 (10.53)	1.45 (10.81)	1.53 (9.59)	1.67 (9.23)	1.76 (8.20)	0.98 (3.97)
$\alpha(\%)$	1.14 (1.12)	1.34 (1.65)	1.73 (2.10)	1.92 (2.69)	-0.63 (-0.42)	-0.68 (-0.30)	-3.67 (-1.41)	-4.24 (-1.43)	-7.34 (-1.71)	-9.64 (-1.90)	-10.78 (-1.83)
$R^2(\%)$	82.67	91.64	89.14	86.86	82.69	72.53	74.67	70.99	58.41	55.65	22.85
$\beta^{imc}$	0.07	0.27	0.32	0.41	0.77	0.81	0.98	1.04	1.55	1.45	1.37
$I/K$	0.10	0.11	0.11	0.12	0.13	0.14	0.15	0.16	0.15	0.12	
$Q$	0.93	1.19	1.23	1.28	1.38	1.43	1.65	1.88	1.95	2.25	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	9.61 (6.45)	8.27 (4.86)	7.56 (4.15)	7.02 (3.68)	6.57 (3.31)	6.15 (2.99)	5.76 (2.71)	5.39 (2.47)	4.99 (2.25)	4.37 (1.95)	-5.24 (-5.38)
$\sigma(\%)$	10.90	12.51	13.36	13.99	14.49	14.92	15.33	15.65	15.88	15.85	7.03
$\beta^{mkt}$	0.70 (21.74)	0.83 (35.65)	0.89 (49.38)	0.94 (65.35)	0.98 (76.75)	1.01 (70.48)	1.03 (60.14)	1.05 (49.43)	1.07 (42.20)	1.07 (35.09)	0.36 (6.99)
$\alpha(\%)$	5.06 (10.78)	2.92 (8.65)	1.79 (6.44)	0.97 (3.96)	0.29 (1.06)	-0.32 (-1.65)	-0.89 (-3.48)	-1.40 (-4.63)	-1.89 (-5.33)	-2.50 (-6.01)	-7.56 (-10.68)
$R^2(\%)$	89.17	94.78	96.29	96.90	97.13	97.09	96.93	96.65	96.15	95.53	54.79
$\beta^{imc}$	0.41	0.56	0.63	0.69	0.74	0.79	0.82	0.85	0.87	0.89	0.48
$I/K$	0.04	0.06	0.07	0.08	0.08	0.09	0.10	0.11	0.13	0.15	
$Q$	2.63	3.09	3.33	3.50	3.63	3.74	3.83	3.84	3.74	3.12	

Table 9 shows characteristics for the 10 portfolios of firms sorted on idiosyncratic volatility. The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio,  $\beta_t^{mkt}$ , and the investment minus consumption portfolio (defined in Appendix A),  $\beta_t^{imc}$ . Estimation is done at annual frequencies in both the model and the data. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 10: Return comovement**

	Cross-sections						Eigenvalues
	IK	EP	IVOL	BMKT	Q	ALL	$\lambda_1 / \sum \lambda_i$
IK							35.5
(p-value)							(0.00)
-EP	72.2						35.6
(p-value)							(0.00)
IVOL	40.7	22.4					51.5
(p-value)							(0.00)
BMKT	81.0	63.9	53.7				41.0
(p-value)							(0.00)
Q	76.7	67.0	39.7	63.8			30.8
(p-value)							(0.00)
ALL (IK, EP, IVOL, BMKT, Q)	92.0	77.9	46.8	89.7	74.2		33.2
(p-value)	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)		(0.00)
IMC	61.3	60.8	54.0	67.5	69.6	68.6	
$\Delta z^I$	31.9	35.2	14.2	49.3	19.0	38.4	

Table 10 shows return comovement across the five decile portfolio sorts, on IK, EP, IVOL, BMKT, and Q (see Table 1 for variable definitions). We extract the first principal component from standardized return residuals from a market model regression. We normalize the sign of the first principal component so that it loads positively on portfolio 10 with the exception of the *EP* sort, where it loads negatively on portfolio 10. In addition, we extract the first principal component from a pooled cross-section of 20 portfolios that includes portfolios 1, 2, 9 and 10 from each sort. We show the correlation matrix of these principal components, along with their correlations with IMC, HML and the real proxy for the IST shock  $\Delta z^I$  (Appendix A contains the details of portfolio construction). We compute *p*-values based on 10,000 permutations, where we randomly and independently permute the time-series order of each cross-section.

**Table 11: Loadings of 1st PC on corner portfolios**

Portfolio	1	2	9	10
IK	-0.244	-0.261	0.248	0.243
E/V	0.201	0.016	-0.276	-0.230
Q	-0.291	-0.223	0.074	0.216
BMKT	-0.233	-0.334	0.245	0.239
IVOL	-0.237	-0.071	0.190	0.120

Table 11 reports loadings of the first principal component of the pooled cross-section on the extreme decile portfolios sorted on IK, EP, BMKT and IVOL (see Table 1 for variable definitions). Estimation is done at annual frequencies. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 12: Correlation between characteristic decile assignments**

Sort	Data				Model			
	Q	IK	EP	BMKT	Q	IK	EP	BMKT
Q								
IK	38.1				19.3			
EP	-63.6	-20.6			-47.4	-27.5		
BMKT	20.5	19.3	-14.3		40.4	25.6	-73.2	
IVOL	12.3	11.3	-11.5	21.3	12.7	44.3	-49.2	39.1

Table 12 reports correlations between portfolio decile assignments across the characteristics Q, IK, EP, BMKT, and IVOL (see Table 1 for variable definitions). The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 13: Pricing the characteristics-sorted portfolios with a common factor**

Investment rate												
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo	GRS
$\alpha$	-0.96	0.03	0.53	0.29	1.66	1.05	0.64	-0.82	0.56	0.37	1.32	0.668 (0.89)
$\beta^{mkt}$	(-0.63)	(0.03)	(0.91)	(0.40)	(1.64)	(1.00)	(0.55)	(-0.61)	(0.46)	(0.16)	(0.62)	
$\beta_z$	0.94	0.82	0.73	0.83	0.85	0.91	0.93	0.99	1.16	1.56	0.62	
$R^2$	(10.78)	(12.66)	(27.40)	(25.03)	(18.57)	(14.72)	(14.42)	(13.56)	(23.74)	(12.00)	(5.16)	
	-0.29	-0.22	-0.24	-0.20	-0.08	-0.03	-0.03	0.01	0.24	0.44	0.73	
	(-6.48)	(-6.68)	(-10.91)	(-6.78)	(-2.36)	(-0.95)	(-0.53)	(0.12)	(5.13)	(5.54)	(10.01)	
	79.87	86.34	91.37	92.91	90.29	85.60	80.24	83.89	89.55	82.03	75.75	
Earnings to Price												
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo	GRS
$\alpha$	-0.28	-2.49	-1.96	-0.63	1.23	0.74	1.37	1.10	0.17	2.89	3.17	1.303 (0.21)
$\beta^{mkt}$	(-0.12)	(-1.82)	(-2.48)	(-0.52)	(0.87)	(0.93)	(1.34)	(0.80)	(0.11)	(2.05)	(1.41)	
$\beta_z$	1.21	0.86	0.85	0.85	0.90	0.88	0.91	0.92	0.81	0.97	-0.25	
$R^2$	(12.99)	(12.24)	(17.20)	(16.47)	(13.78)	(17.86)	(17.49)	(17.57)	(10.52)	(12.92)	(-2.66)	
	0.27	0.01	0.00	-0.13	-0.13	-0.12	-0.15	-0.20	-0.37	-0.27	-0.54	
	(2.85)	(0.21)	(0.10)	(-2.38)	(-2.07)	(-2.77)	(-3.80)	(-5.09)	(-6.27)	(-10.57)	(-5.83)	
	79.59	83.31	88.36	85.15	81.48	86.93	88.43	82.16	77.43	79.68	54.24	
Market beta												
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo	GRS
$\alpha$	-1.64	-1.71	-2.37	1.16	-0.83	-0.80	1.23	0.66	1.11	1.57	3.21	0.947 (0.57)
$\beta^{mkt}$	(-0.93)	(-2.13)	(-2.51)	(1.08)	(-0.97)	(-0.72)	(1.22)	(0.56)	(0.92)	(0.67)	(1.27)	
$\beta_z$	0.71	0.64	0.78	0.76	0.80	0.83	1.06	1.10	1.27	1.46	0.75	
$R^2$	(5.97)	(12.20)	(9.40)	(17.32)	(16.61)	(13.91)	(19.05)	(25.19)	(18.26)	(15.55)	(6.94)	
	-0.30	-0.38	-0.30	-0.14	-0.21	-0.14	0.04	0.09	0.27	0.39	0.69	
	(-6.97)	(-11.87)	(-7.24)	(-4.01)	(-4.05)	(-2.56)	(0.96)	(2.80)	(4.01)	(5.87)	(8.16)	
	67.55	86.40	83.59	81.95	83.76	84.44	89.19	89.16	88.53	82.47	70.47	
Idiosyncratic volatility												
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo	GRS
$\alpha$	-0.81	0.88	1.44	2.14	1.51	1.71	-1.46	-0.76	-1.13	-5.25	-4.44	0.688 (0.88)
$\beta^{mkt}$	(-0.82)	(1.10)	(1.46)	(1.76)	(0.97)	(0.76)	(-0.60)	(-0.28)	(-0.26)	(-1.26)	(-0.94)	
$\beta_z$	0.77	0.92	0.93	1.07	1.29	1.26	1.45	1.53	1.67	1.76	0.98	
$R^2$	(18.56)	(34.90)	(18.74)	(19.99)	(15.87)	(10.49)	(11.79)	(10.54)	(9.14)	(8.24)	(4.24)	
	-0.15	-0.04	-0.02	0.02	0.17	0.19	0.17	0.27	0.48	0.34	0.49	
	(-5.09)	(-1.18)	(-0.61)	(0.28)	(3.15)	(2.76)	(2.92)	(4.22)	(2.72)	(2.97)	(3.75)	
	89.01	91.92	89.24	86.90	85.42	75.71	76.77	75.41	68.18	59.84	34.32	
Tobin's Q												
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo	GRS
$\alpha$	1.76	1.77	0.73	-0.44	-2.30	-0.70	0.21	0.57	0.93	-0.42	-2.18	0.647 (0.91)
$\beta^{mkt}$	(1.48)	(1.16)	(0.61)	(-0.40)	(-2.52)	(-0.69)	(0.15)	(0.35)	(0.56)	(-0.21)	(-1.00)	
$\beta_z$	0.86	0.92	0.82	0.82	0.90	0.89	0.87	1.06	0.99	1.15	0.29	
$R^2$	(12.24)	(11.63)	(13.70)	(13.28)	(14.91)	(16.46)	(16.81)	(13.45)	(12.86)	(12.92)	(3.83)	
	-0.33	-0.24	-0.20	-0.16	-0.15	-0.04	-0.09	0.06	0.07	0.30	0.63	
	(-11.16)	(-3.93)	(-4.74)	(-3.72)	(-5.41)	(-1.50)	(-2.13)	(1.10)	(0.75)	(3.75)	(7.32)	
	84.52	79.62	85.23	84.39	84.63	87.02	82.96	84.34	77.53	78.47	66.13	

Table 13 reports portfolio alphas and betas with respect to the two-factor model that includes the market portfolio and the first principal component of the pooled cross-section (PC1, normalized to standard deviation of 10%) of Q, IK, EP, BMKT and IVOL portfolios. See Table 1 for variable definitions. Estimation is done at annual frequencies. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 14: Stochastic discount factor**

Factor price	(1)	(2)	(3)	(4)	(5)
$\Delta x$	0.75 [0.04, 1.46]	-0.77 [-1.80, 0.25]			
$R^{mkt}$			0.21 [0.02, 0.40]	0.52 [0.31, 0.73]	0.37 [0.17, 0.57]
$\Delta z^I$		-1.35 [-2.24, -0.46]			
$R^{imc}$				-0.71 [-1.06, -0.36]	
$-R^{hml}$					-0.65 [-0.98, -0.32]
SSQE (%)	4.23	0.92	3.61	0.75	1.40

Table 14 reports empirical estimates of  $\gamma_x$  and  $\gamma_z$  from the model SDF:  $m = a - \gamma_x \Delta x - \gamma_z \Delta z$ . We sort firms on Q, IK, EP, BMKT and IVOL (see Table 1 for variable definitions), and use portfolios 1, 2, 9, and 10 from each sort, resulting in a total of 20 portfolios. See Appendix A and main text for more details. We use annual data in the 1965-2008 period and report first-stage estimates and the sum of squared errors (SSQE) along with 90%-confidence intervals around the point estimates computed using the Newey-West procedure with 3 lags.

Table 15: IMC-beta and Characteristics

	Data					
	IK	EP	BMKT	IVOL	Q	ALL
BIMC(t)						
BIMC(t-1)	0.224 (10.11)	0.231 (10.26)	0.156 (9.35)	0.211 (9.93)	0.219 (10.09)	0.131 (9.07)
IK(t-1)	0.475 (9.49)					0.204 (5.81)
EP(t-1)		-0.202 (-3.34)				-0.004 (0.03)
BMKT(t-1)			0.300 (7.77)	0.200 (6.14)		0.260 (7.30)
IVOL(t-1)				4.753 (9.14)	3.526 (7.20)	3.563 (7.03)
log Q(t-1)					0.097 (8.66)	0.045 (4.99)
	Model					
BIMC(t)	IK	EP	BMKT	IVOL	Q	ALL
BIMC(t-1)	0.610 (104.47)	0.149 (28.33)	0.505 (54.60)	0.565 (98.17)	0.506 (82.35)	0.179 (24.91)
IK(t-1)	0.061 (4.02)					-0.392 (-33.22)
EP(t-1)		-5.817 (-249.20)				-0.398 (-35.21)
BMKT(t-1)			0.757 (98.14)	0.151 (16.15)		-5.061 (-139.69)
IVOL(t-1)				4.552 (38.81)	1.569 (27.57)	0.071 (14.10)
log Q(t-1)					0.128 (51.03)	-4.889 (-133.34)
					0.066 (42.50)	0.035 (32.39)
						0.452 (9.04)
						0.030 (30.52)

Table 15 reports results of regressing a firm's IMC-beta (BIMC) estimated using equation (35) on lagged estimates of IMC-beta and firm characteristics. Estimation is done at annual frequencies. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 16: Fama-MacBeth regressions: Data versus Model**

	A. Market Beta											
	DATA						MODEL					
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
BMKT(t-1)	-0.025 (-2.38)		-0.008 (-1.06)		0.054 (3.68)		-0.060 (-4.24)		-0.004 (-0.86)		0.001 (0.46)	
BIMC(t-1)			-0.021 (-2.17)						-0.046 (-3.67)			
$\widehat{BIMC}(t)$					-0.245 (-4.79)						-0.078 (-4.22)	
	B. Investment rate											
	DATA						MODEL					
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
BMKT(t-1)		-0.019 (-1.92)		-0.004 (-0.62)		0.052 (3.48)		-0.060 (-4.08)		-0.005 (-0.91)		0.008 (1.03)
BIMC(t-1)			-0.022 (-2.45)	-0.018 (-1.98)					-0.050 (-3.72)	-0.046 (-3.46)		
$\widehat{BIMC}(t)$					-0.126 (-4.09)	-0.228 (-4.49)					-0.079 (-4.49)	-0.077 (-4.03)
IK(t-1)	-0.130 (-3.23)	-0.115 (-3.04)	-0.115 (-3.00)	-0.112 (-2.95)	-0.068 (-3.54)	-0.042 (-2.30)	-0.078 (-4.95)	-0.052 (-2.44)	-0.037 (-2.14)	-0.030 (-1.12)	-0.009 (-1.12)	-0.009 (-1.04)
	C. Earnings-to-Price											
	DATA						MODEL					
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
BMKT(t-1)		-0.024 (-2.43)		-0.012 (-1.44)		0.040 (2.94)		-0.007 (-2.39)		-0.001 (-0.23)		0.001 (0.36)
BIMC(t-1)			-0.023 (-2.60)	-0.018 (-1.97)					-0.007 (-2.08)	-0.007 (-1.53)		
$\widehat{BIMC}(t)$					-0.128 (-4.22)	-0.200 (-4.10)					-0.081 (-3.15)	-0.084 (-3.18)
EP(t-1)	0.252 (5.62)	0.233 (5.77)	0.229 (5.84)	0.224 (5.85)	0.190 (5.67)	0.163 (5.44)	0.501 (4.32)	0.419 (4.38)	0.406 (4.45)	0.406 (4.44)	0.025 (1.16)	0.011 (0.90)
	D. Idiosyncratic volatility											
	DATA						MODEL					
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
BMKT(t-1)		-0.019 (-2.07)		-0.014 (-1.79)		0.027 (2.78)		-0.053 (-4.00)		-0.003 (-0.66)		0.002 (0.76)
BIMC(t-1)			-0.017 (-2.17)	-0.011 (-1.81)					-0.044 (-3.64)	-0.041 (-3.52)		
$\widehat{BIMC}(t)$					-0.095 (-3.85)	-0.162 (-3.36)					-0.074 (-4.05)	-0.075 (-3.98)
IVOL(t-1)	-1.407 (-3.28)	-1.207 (-2.83)	-1.213 (-2.87)	-1.164 (-2.77)	-0.821 (-1.94)	-0.438 (-1.35)	-0.209 (-4.41)	-0.183 (-5.16)	-0.182 (-3.70)	-0.032 (-3.67)	-0.031 (-1.51)	-0.031 (-1.35)
	E. Tobin's Q											
	DATA						MODEL					
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
BMKT(t-1)		-0.016 (-1.74)		-0.004 (-0.47)		0.046 (2.45)		-0.046 (-3.90)		-0.004 (-0.80)		0.001 (0.22)
BIMC(t-1)			-0.019 (-2.38)	-0.017 (-1.88)					-0.040 (-3.48)	-0.038 (-3.29)		
$\widehat{BIMC}(t)$					-0.119 (-3.83)	-0.205 (-3.33)					-0.074 (-3.92)	-0.074 (-3.93)
log Q(t-1)	-0.028 (-5.41)	-0.025 (-5.50)	-0.024 (-5.34)	-0.024 (-5.41)	-0.015 (-3.46)	-0.009 (-1.69)	-0.011 (-4.52)	-0.008 (-5.41)	-0.008 (-5.81)	-0.008 (-5.81)	-0.003 (-2.32)	-0.002 (-2.22)

Table 16 reports results of Fama and MacBeth (1973) regressions using lagged firm characteristics, lagged point estimates of IMC beta (BIMC) estimated using equation (35) and contemporaneous fitted values of IMC beta ( $\widehat{BIMC}$ ) using the regression model in the last column of Table 15. Estimation is done at annual frequencies, with the stock return on fiscal year  $t$  defined as the stock return from June of calendar year  $t$  to May of calendar year  $t + 1$ . The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).



**Table 17: Investment response to IST shock: Model versus Data**

$IK_{ft}$	A. Model					B. Data ( $\Delta z^I$ )				
	Q	IK	EP	BMKT	IVOL	Q	IK	EP	BMKT	IVOL
$\Delta z_{t-1}$	2.48 (4.64)	1.92 (4.45)	4.16 (5.27)	2.48 (4.84)	1.94 (4.43)	0.37 (1.83)	0.23 (1.02)	1.14 (2.17)	0.42 (2.19)	0.41 (1.28)
$D(G_f)_2 \times \Delta z_{t-1}$	0.13 (1.25)	0.64 (3.32)	-0.59 (-2.66)	0.15 (1.21)	0.59 (3.72)	0.03 (0.22)	0.16 (1.54)	-0.47 (-1.04)	0.06 (0.42)	0.21 (0.98)
$D(G_f)_3 \times \Delta z_{t-1}$	0.39 (2.00)	1.17 (4.54)	-1.05 (-4.03)	0.47 (2.69)	0.98 (4.85)	0.15 (1.10)	0.48 (4.01)	-0.43 (-1.04)	-0.06 (-0.35)	0.36 (1.57)
$D(G_f)_4 \times \Delta z_{t-1}$	0.72 (3.60)	1.52 (5.40)	-1.61 (-4.89)	0.79 (3.94)	1.46 (5.34)	0.54 (2.00)	0.67 (3.66)	-0.67 (-1.55)	0.33 (1.59)	0.64 (2.14)
$D(G_f)_H \times \Delta z_{t-1}$	1.50 (4.78)	2.30 (6.60)	-2.39 (-5.27)	1.37 (4.81)	2.44 (5.97)	0.78 (2.48)	0.89 (2.62)	-0.78 (-1.65)	0.94 (2.52)	0.31 (1.27)
$IK_{ft}$	C. Data (IMC)					D. Data (PC1)				
	Q	IK	EP	BMKT	IVOL	Q	IK	EP	BMKT	IVOL
$\Delta z_{t-1}$	0.69 (3.53)	0.65 (3.53)	2.04 (3.95)	0.91 (5.40)	0.63 (2.21)	-0.23 (-0.84)	0.17 (0.77)	1.14 (2.00)	-0.10 (-0.29)	0.04 (0.13)
$D(G_f)_2 \times \Delta z_{t-1}$	0.15 (1.65)	0.14 (1.27)	-0.96 (-2.15)	-0.25 (-1.80)	0.28 (1.61)	0.24 (2.03)	-0.08 (-0.65)	-1.03 (-1.89)	-0.07 (-0.44)	0.05 (0.36)
$D(G_f)_3 \times \Delta z_{t-1}$	0.32 (2.46)	0.13 (0.85)	-1.26 (-2.41)	-0.10 (-0.77)	0.35 (1.66)	0.40 (2.92)	0.13 (0.93)	-1.18 (-2.20)	0.03 (0.18)	0.38 (0.19)
$D(G_f)_4 \times \Delta z_{t-1}$	0.54 (2.91)	0.38 (1.72)	-1.49 (-3.18)	-0.00 (0.00)	0.32 (0.88)	0.54 (2.28)	0.13 (1.24)	-1.22 (-2.35)	0.20 (0.71)	0.50 (1.68)
$D(G_f)_H \times \Delta z_{t-1}$	0.49 (1.42)	1.06 (3.53)	-1.56 (-3.43)	0.74 (2.28)	0.86 (3.19)	0.77 (2.34)	0.84 (1.92)	-1.47 (-2.92)	1.13 (3.07)	0.82 (2.53)

Table 17 shows the differential response of investment of firms with different characteristics  $G_f \in \{Q, IK, EP, BMKT, IVOL\}$  on measures of the IST shock  $\Delta z \in \{\Delta z^I, R^{imc}, PC1\}$ , normalized to unit standard deviation. We show the estimated coefficients  $b_1 \dots b_5$  from equation (40), along with  $t$ -statistics computed using standard errors clustered by firm and year. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

**Table 18: Return predictability in the model**

Horizon (k) (years)	a. Price/Dividend			b. Price/Earnings		
	Slope	$t$	$R^2$	Slope	$t$	$R^2$
1	-0.10	-2.04	0.07	-0.14	-2.02	0.07
2	-0.19	-2.47	0.13	-0.27	-2.42	0.13
3	-0.26	-2.76	0.19	-0.38	-2.70	0.19
4	-0.33	-2.97	0.24	-0.47	-2.94	0.24
5	-0.40	-3.18	0.28	-0.57	-3.15	0.28
Horizon (k) (years)	c. Tobin's Q			d. Investment rate		
	Slope	$t$	$R^2$	Slope	$t$	$R^2$
1	-0.11	-1.81	0.06	-0.06	-1.93	0.06
2	-0.22	-2.29	0.12	-0.12	-2.32	0.12
3	-0.32	-2.65	0.17	-0.18	-2.56	0.16
4	-0.39	-2.82	0.21	-0.22	-2.70	0.20
5	-0.47	-3.02	0.25	-0.27	-2.84	0.23

Table 18 reports results of return predictability regressions in simulated data. In particular, we report coefficients  $b(k)$  from

$$\sum_{h=1}^k r_{M,t+h} = a(k) + b(k)x_t + u_{t+h}$$

$r_M$  is log gross return on the market portfolio;  $x$  is the log aggregate price dividend ratio (panel a), aggregate price-earnings ratio (panel b), aggregate Tobin's  $Q$  (panel c), and aggregate investment rate (panel d). Standard errors are computed using the Newey-West procedure with the number of lags equal to 1.5 times the return horizon. Point estimates,  $t$ -statistics, and the  $R^2$  are median values across 1,000 independent simulations of the model. For each simulated sample we start with 100 years of simulate data and omit the first 50 years to eliminate the effect of initial conditions.

Figure 1: Empirical versus model-implied risk premia

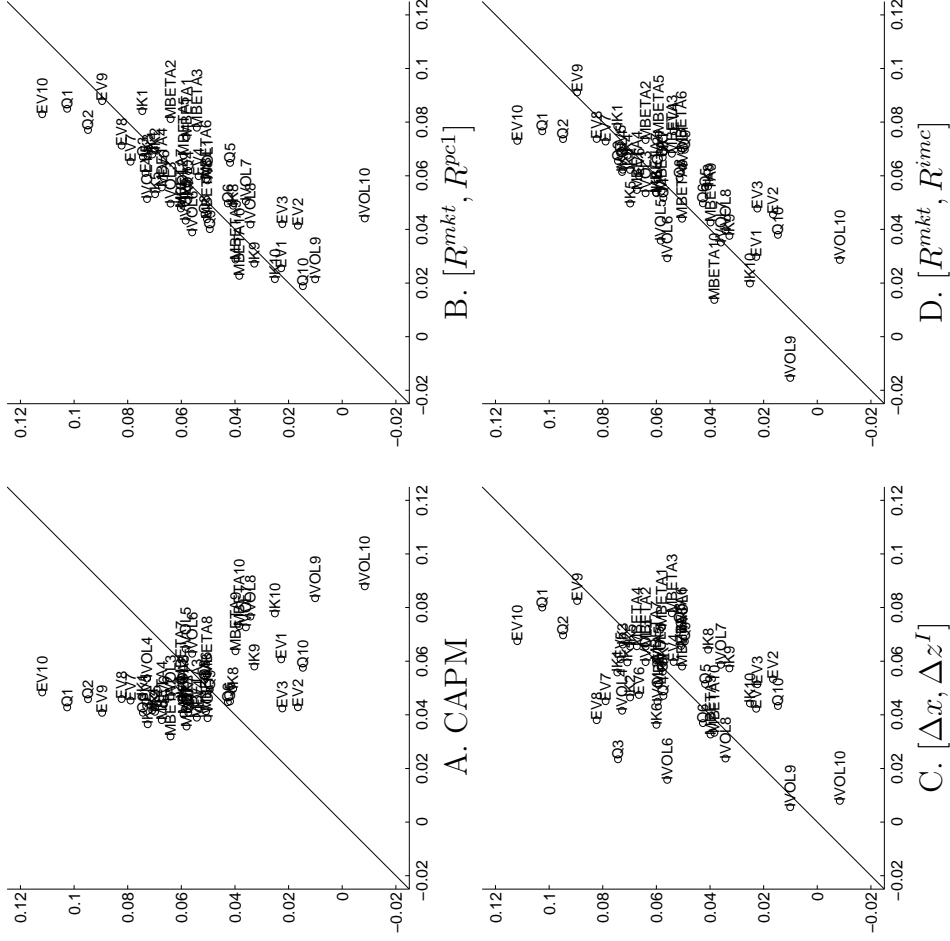


Figure 1 plots predicted expected returns versus realized average returns from four asset pricing models: A) the CAPM; B) the two factor model with the market portfolio and PCI; C) the SDF implied by the model using two real proxies for the IST shock (column 2 of Table 14); and D) the SDF implied by the model using portfolio returns (column 5 of Table 14). We consider the pooled cross-section of IK, EP, Q, BMKT, and IVOL portfolios. See Table 1 for variable definitions.

Figure 2: Investment Shock and Firm Output Comovement

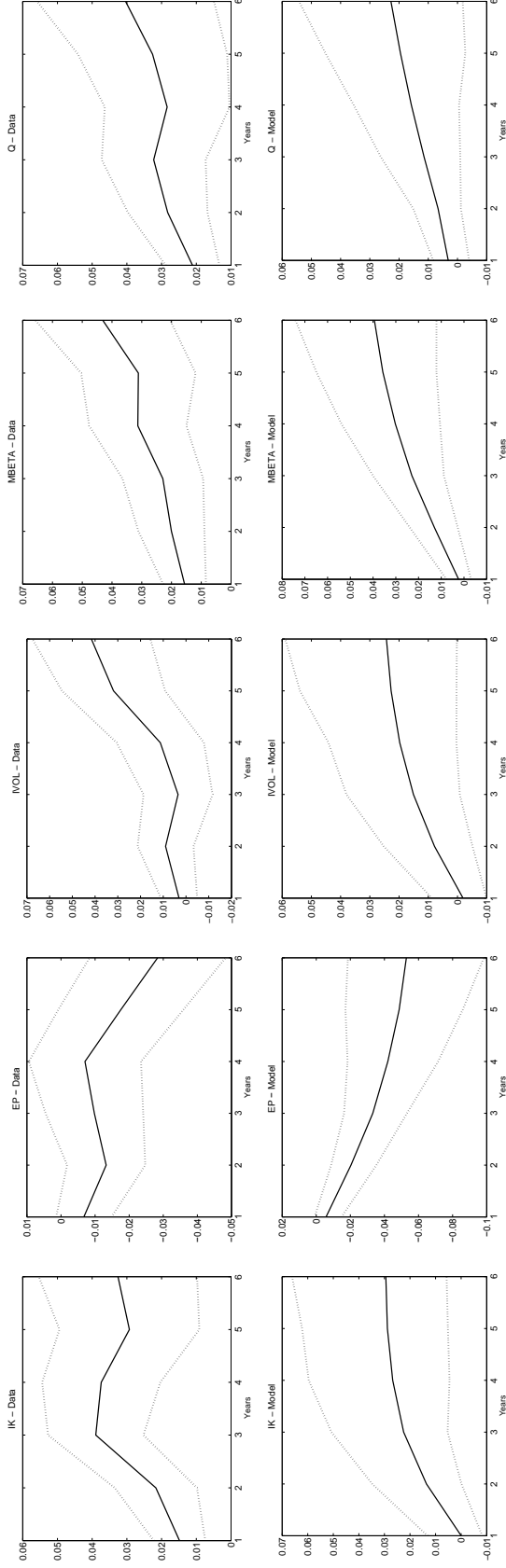


Figure 2 plots the differential response of sales growth on the IST shock between firms with high and low IK, EP, Q, IVOL or BMKT (see Table 1 for variable definitions). Specifically, we plot the estimated  $b_5(k)$  coefficient from the regression

$$\ln y_{f,t+k} - \ln \bar{y}_{t+k} = b_0 + b_1 \Delta z_t^I + \sum_{d=2}^5 b_d(k) D(G_{f,t})_d \Delta z_t^I + \rho (\ln y_{ft} - \ln \bar{y}_t) + u_{f,t+k},$$

where  $y_{f,t}$  is firm output (sales plus change in inventories);  $\Delta z_t$  is the measure of the IST shock extracted from the price of equipment, normalized to unit standard deviation;  $\bar{y}$  is average output across firms;  $G \in \{Q, IK, EP, IVOL, BMKT\}$ ; and  $D(G_{f,t})_d$  is a quintile dummy variable. Top panel shows estimated responses in the data, bottom panel shows median estimated responses in the model, along with the 5th and 95th percentile estimate across simulations.

Figure 3: Risk premium of dividend strips

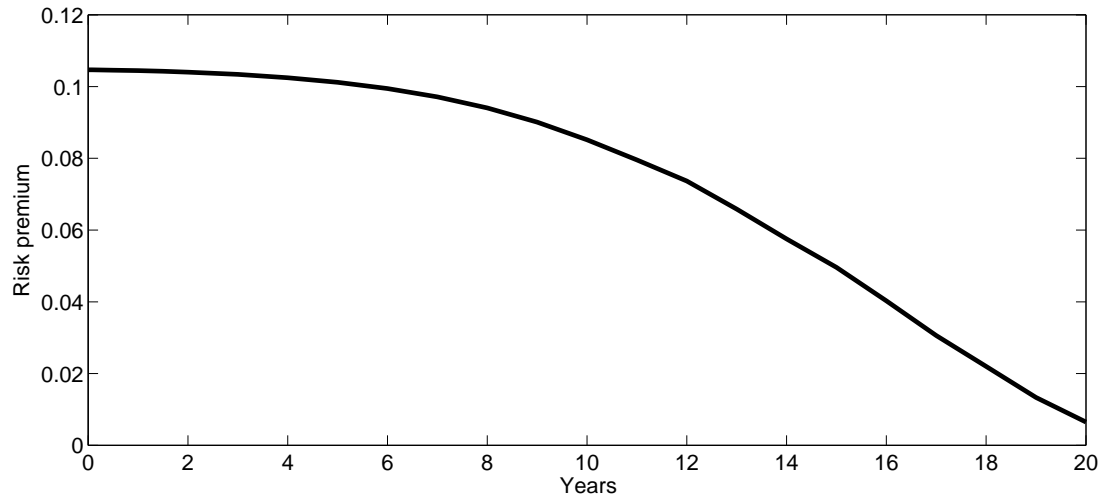


Figure 3 plots the expected excess returns on aggregate dividend strips in the model, with tenors from 0 to 20 years.