Part III 2.a) CAPM

Financial Markets and Investments

Adapted from Bodie, Kane and Marcus' slides, McGraw-Hill/Irwin

CAPM

1. The Model

- It is the equilibrium model that underlies all modern financial economics theory
- Analyse investors behaviour, capital markets, find the prices for market equilibrium
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development

CAPM

1. The Model. Assumptions

- Individual investors are price takers
- Single-period investment horizon
- Investments are limited to traded financial assets
- No taxes and no transaction costs
- Information is costless and available to all investors
- Investors are rational mean-variance optimizers, use Markovitz model
- There are homogeneous expectations:
  - Investors holding period is the same for all
  - All investors have identical expectations on decision making elements, securities, economic environment

CAPM

1. The Model

- Equilibrium Conditions
  - All investors will hold the same portfolio for risky assets: the market portfolio
  - Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value
  - Risk premium on the market depends on the average risk aversion of all market participants
  - Risk premium on an individual security is a function of its covariance with the market

CAPM

1. The Model

- If a representative agent exists then his risky portfolio is the market portfolio and we have a unique CAL, that is the Capital Market Line (CML)

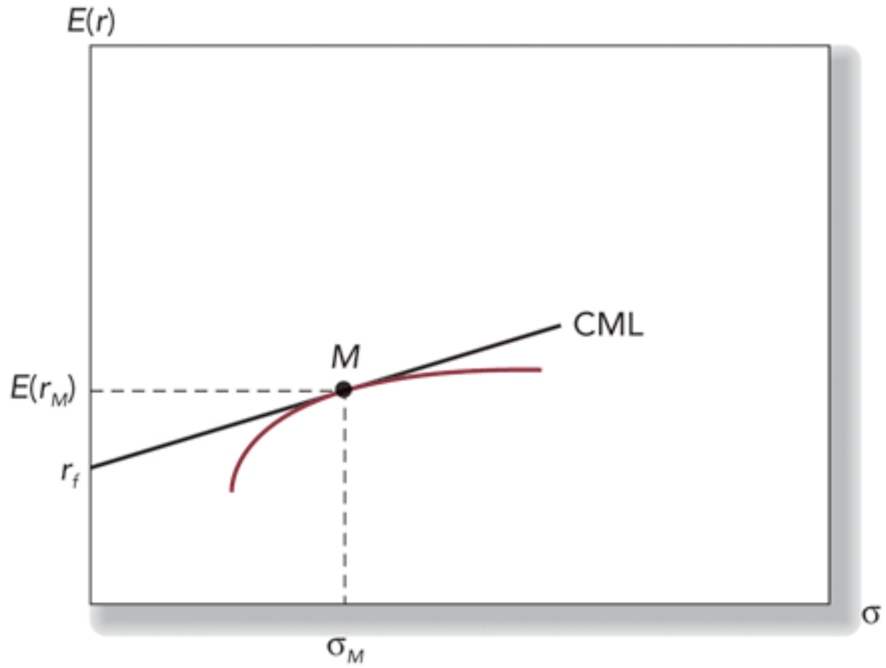

FIGURE 9.1 The efficient frontier and the capital market line

CAPM

1. The Model

- The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor:

The optimal portfolio of a risky portfolio and T-Bills is

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2}$$

Which is,

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

Since the market portfolio is fully diversified, the market risk premium only compensate systematic risk

where σ_M^2 is the variance of the market portolio and

$\overline{A}$ is the average degree of risk aversion across investors

1. The Model

- Return and Risk For Individual Securities
  - The risk premium on individual securities is a function of the individual security's contribution to the risk of the market portfolio
  - An individual security's risk premium is a function of the covariance of returns with the assets that make up the market portfolio

1. The Model

- Using GE Text Example
  - Covariance of GE return with the market portfolio:

$$Cov(r_{GE}, r_M) = Cov\left(r_{GE}, \sum_{k=1}^{n} w_k r_k \right) = \sum_{k=1}^{n} w_k Cov(r_k, r_{GE})$$

  - Therefore, the reward-to-risk ratio for investments in GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE}\left[E(r_{GE}) - r_f\right]}{w_{GE} Cov(r_{GE}, r_M)} = \frac{E(r_{GE}) - r_f}{Cov(r_{GE}, r_M)}$$

CAPM

1. The Model

- Using GE Text Example
  - Reward-to-risk ratio for investment in market portfolio:

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(r_M) - r_f}{\sigma_M^2}$$

  - Reward-to-risk ratios of GE and the market portfolio:

$$\frac{E(r_{GE}) - r_f}{Cov(r_{GE}, r_M)} = \frac{E(r_M(-r_f)}{\sigma_M^2}$$

  - And the risk premium for GE:

$$E(r_{GE}) - r_f = \frac{Cov(r_{GE}, r_M)}{\sigma_M^2}\left[E(r_M) - r_f\right]$$

CAPM

1. The Model

- Expected Return-Beta Relationship
  - CAPM holds for the overall portfolio because:

$$E(r_P) = \sum_k w_k E(r_k) \text{ and}$$

$$\beta_P = \sum_k w_k \beta_k$$

  - This also holds for the market portfolio:

$$E(r_M) = r_f + \beta_M \left[E(r_M) - r_f \right]$$

1. The Model

- If Beta is the appropriate risk measure, then we can represent all assets in the space Beta-Return
- Market's Beta is 1

$$E(r_M) = r_f + \beta_M\left[E(r_M) - r_f\right]$$
$$E(r_M) - r_f = \beta_M\left[E(r_M) - r_f\right] \Rightarrow \beta_M = 1$$

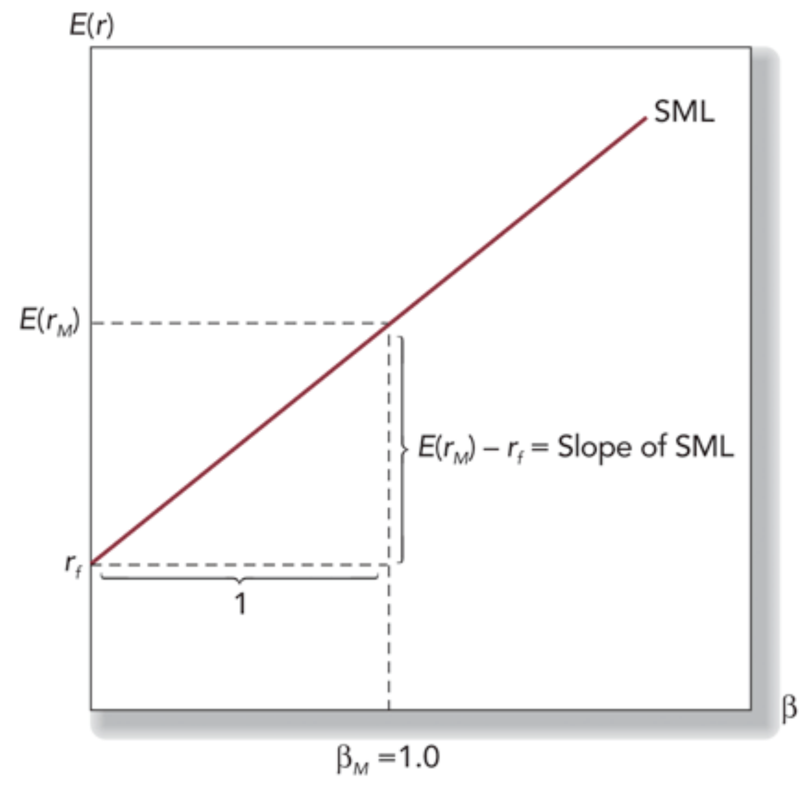

FIGURE 9.2 The security market line

CAPM

1. The Model

- If a given stock is not in the line, there are arbitrage opportunities
- Arbitrageurs will take advantage of them, bringing the stock price to an equilibrium

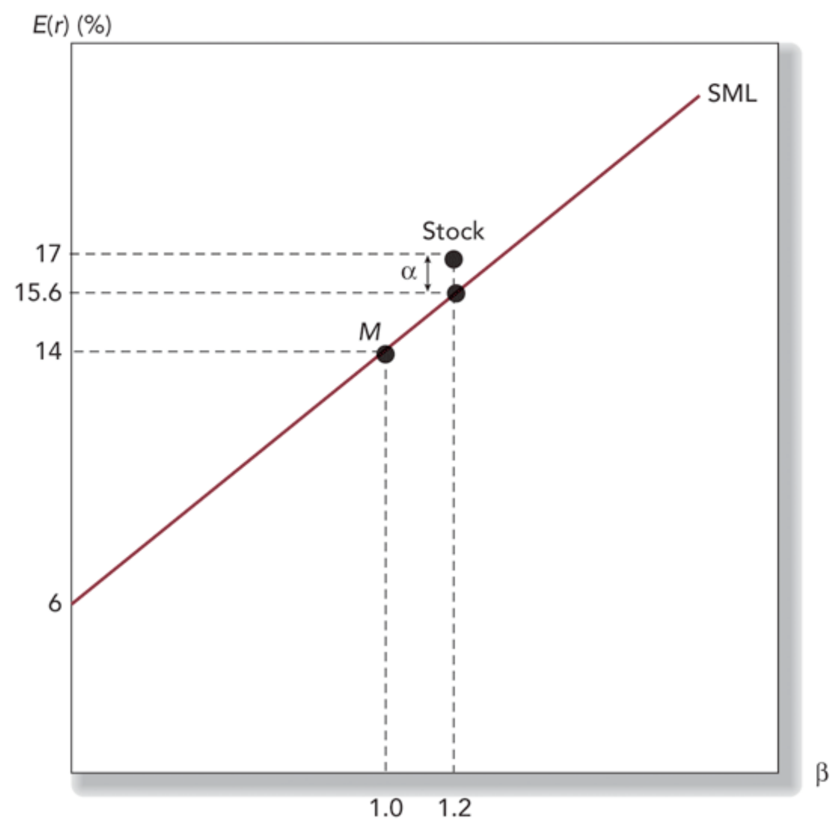

FIGURE 9.3 The SML and a positive-alpha stock

2. CAPM and the Single-Index Model

- The Index Model and Realized Returns
  - To move from expected to realized returns—use the index model in excess return form:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

  - The index model beta coefficient turns out to be the same beta as that of the CAPM expected return-beta relationship

CAPM

2. CAPM and the Single-Index Model

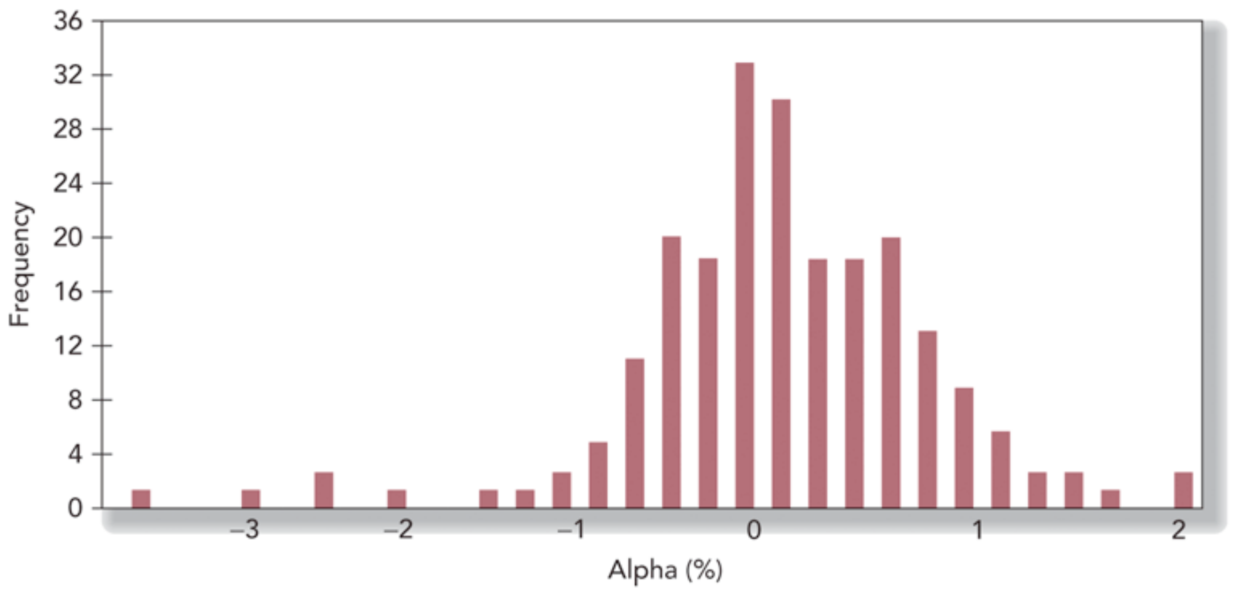

FIGURE 9.4 Estimates of individual mutual fund alphas, 1972–1991

This is a plot of the frequency distribution of estimated alphas for all-equity mutual funds with 10-year continuous records.

Source: Burton G. Malkiel, "Returns from Investing in Equity Mutual Funds 1971–1991," *Journal of Finance* 50 (June 1995), pp. 549–72. Reprinted by permission of the publisher, Blackwell Publishing, Inc.

3. The CAPM and Reality

- Is the condition of zero alphas for all stocks as implied by the CAPM met
  - Not perfect but one of the best available
- Is the CAPM testable
  - Proxies must be used for the market portfolio
- CAPM is still considered the best available description of security pricing and is widely accepted

4. Extensions of the CAPM

- Zero-Beta Model
  - Helps to explain positive alphas on low beta stocks and negative alphas on high beta stocks
- Consideration of labor income and non-traded assets
- Merton’s Multiperiod Model and hedge portfolios
  - Incorporation of the effects of changes in the real rate of interest and inflation

4. Extensions of the CAPM

- A consumption-based CAPM
  - Models by Rubinstein, Lucas, and Breeden
    - Investor must allocate current wealth between today's consumption and investment for the future
- Considering Liquidity
  - Liquidity
  - Illiquidity Premium
  - Research supports a premium for illiquidity.
    - Amihud and Mendelson
    - Acharya and Pedersen

1. The Model

- The liquidity premium increase with the bid-ask spread at a decreasing rate, due the "clientele effect"

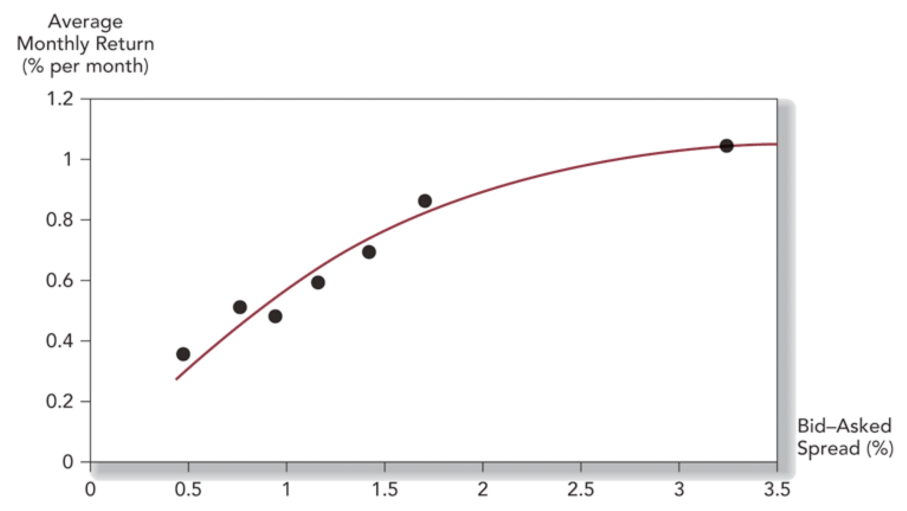

FIGURE 9.5 The relationship between illiquidity and average returns

Source: Derived from Yakov Amihud and Haim Mendelson, "Asset Pricing and the Bid–Ask Spread," *Journal of Financial Economics* 17 (1986), pp. 223–49.

CAPM

4. Extensions of the CAPM

- Three Elements of Liquidity
  - Sensitivity of security's illiquidity to market illiquidity:

$$\beta_{L1} = \frac{Cov(C_i, C_M)}{Var(R_M - C_M)}$$

  - Sensitivity of stock's return to market illiquidity:

$$\beta_{L2} = \frac{Cov(R_i, C_M)}{Var(R_M - C_M)}$$

  - Sensitivity of the security illiquidity to the market rate of return:

$$\beta_{L3} = \frac{Cov(C_i, R_M)}{Var(R_M - C_M)}$$