While this portfolio is still risky (due to the residual risk, e_P), the systematic risk has been eliminated, and if P is reasonably well-diversified, the remaining nonsystematic risk will be small. Thus the objective is achieved: the manager can take advantage of the 4% alpha without inadvertently taking on market exposure. The process of separating the search for alpha from the choice of market exposure is called *alpha transport*.

This "long-short strategy" is characteristic of the activity of many *hedge funds*. Hedge fund managers identify an underpriced security and then try to attain a "pure play" on the perceived underpricing. They hedge out all extraneous risk, focusing the bet only on the perceived "alpha" (see the box on p. 272). Tracking funds are the vehicle used to hedge the exposures to which they do *not* want exposure. Hedge fund managers use index regressions such as those discussed here, as well as more-sophisticated variations, to create the tracking portfolios at the heart of their hedging strategies.

SUMMARY

1. A single-factor model of the economy classifies sources of uncertainty as systematic (macroeconomic) factors or firm-specific (microeconomic) factors. The index model assumes that the macro factor can be represented by a broad index of stock returns.
2. The single-index model drastically reduces the necessary inputs in the Markowitz portfolio selection procedure. It also aids in specialization of labor in security analysis.
3. According to the index model specification, the systematic risk of a portfolio or asset equals $\beta^2\sigma_M^2$ and the covariance between two assets equals $\beta_i\beta_j\sigma_M^2$.
4. The index model is estimated by applying regression analysis to excess rates of return. The slope of the regression curve is the beta of an asset, whereas the intercept is the asset's alpha during the sample period. The regression line is also called the *security characteristic line*.
5. Optimal active portfolios constructed from the index model include analyzed securities in proportion to their information ratios. The full risky portfolio is a mixture of the active portfolio and the passive market index portfolio. The index portfolio is used to enhance the diversification of the overall risky position.
6. Practitioners routinely estimate the index model using total rather than excess rates of return. This makes their estimate of alpha equal to $\alpha + r_f(1 - \beta)$.
7. Betas show a tendency to evolve toward 1 over time. Beta forecasting rules attempt to predict this drift. Moreover, other financial variables can be used to help forecast betas.

Related Web sites for this chapter are available at www.mhhe.com/bkm

KEY TERMS

single-factor model	residuals	information ratio
single-index model	security characteristic line	tracking portfolio
regression equation	scatter diagram	

PROBLEM SETS

Quiz

1. What are the advantages of the index model compared to the Markowitz procedure for obtaining an efficiently diversified portfolio? What are its disadvantages?
2. What is the basic trade-off when departing from pure indexing in favor of an actively managed portfolio?
3. How does the magnitude of firm-specific risk affect the extent to which an active investor will be willing to depart from an indexed portfolio?
4. Why do we call alpha a "nonmarket" return premium? Why are high-alpha stocks desirable investments for active portfolio managers? With all other parameters held fixed, what would happen to a portfolio's Sharpe ratio as the alpha of its component securities increased?

Problems

5. A portfolio management organization analyzes 60 stocks and constructs a mean-variance efficient portfolio using only these 60 securities.
   a. How many estimates of expected returns, variances, and covariances are needed to optimize this portfolio?
   b. If one could safely assume that stock market returns closely resemble a single-index structure, how many estimates would be needed?

6. The following are estimates for two stocks.

Stock	Expected Return	Beta	Firm-Specific Standard Deviation
A	13%	0.8	30%
B	18	1.2	40

The market index has a standard deviations of 22% and the risk-free rate is 8%.

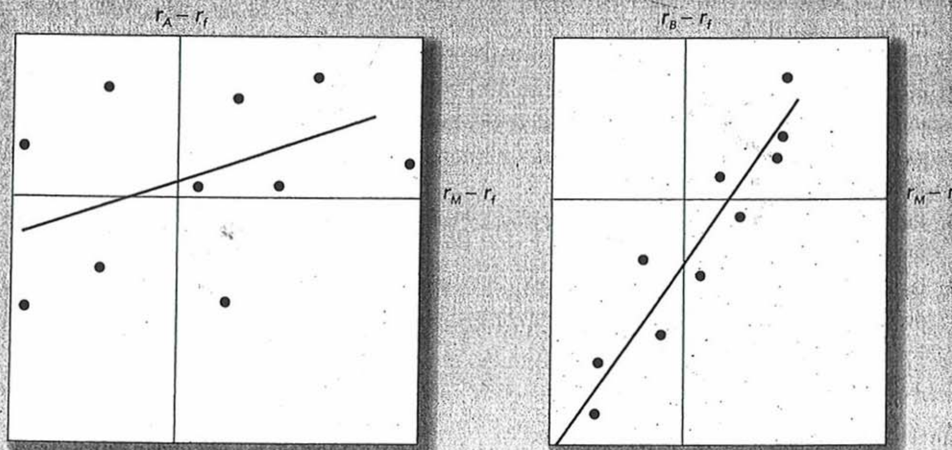
a. What are the standard deviations of stocks *A* and *B*?

b. Suppose that we were to construct a portfolio with proportions:

Stock *A*:	.30
Stock *B*:	.45
T-bills:	.25

Compute the expected return, standard deviation, beta, and nonsystematic standard deviation of the portfolio.

7. Consider the following two regression lines for stocks *A* and *B* in the following figure.



   a. Which stock has higher firm-specific risk?
   b. Which stock has greater systematic (market) risk?
   c. Which stock has higher R^2?
   d. Which stock has higher alpha?
   e. Which stock has higher correlation with the market?

8. Consider the two (excess return) index model regression results for *A* and *B*:

$R_A = 1\% + 1.2R_M$

R-square = .576

Residual standard deviation = 10.3%

$R_B = -2\% + .8R_M$

R-square = .436

Residual standard deviation = 9.1%

a. Which stock has more firm-specific risk?
b. Which has greater market risk?
c. For which stock does market movement explain a greater fraction of return variability?
d. If r_f were constant at 6% and the regression had been run using total rather than excess returns, what would have been the regression intercept for stock *A*?

Use the following data for Problems 9 through 14. Suppose that the index model for stocks A and B is estimated from excess returns with the following results:

$$R_A = 3\% + .7R_M + e_A$$
$$R_B = -2\% + 1.2R_M + e_B$$
$$\sigma_M = 20\%;\ R\text{-square}_A = .20;\ R\text{-square}_B = .12$$

9. What is the standard deviation of each stock?
10. Break down the variance of each stock to the systematic and firm-specific components.
11. What are the covariance and correlation coefficient between the two stocks?
12. What is the covariance between each stock and the market index?
13. For portfolio *P* with investment proportions of .60 in *A* and .40 in *B*, rework Problems 9, 10, and 12.
14. Rework Problem 13 for portfolio *Q* with investment proportions of .50 in *P*, .30 in the market index, and .20 in T-bills.
15. A stock recently has been estimated to have a beta of 1.24:
    a. What will Merrill Lynch compute as the "adjusted beta" of this stock?
    b. Suppose that you estimate the following regression describing the evolution of beta over time:
    $$\beta_t = .3 + .7\beta_{t-1}$$
    What would be your predicted beta for next year?
16. Based on current dividend yields and expected growth rates, the expected rates of return on stocks *A* and *B* are 11% and 14%, respectively. The beta of stock *A* is .8, while that of stock *B* is 1.5. The T-bill rate is currently 6%, while the expected rate of return on the S&P 500 index is 12%. The standard deviation of stock *A* is 10% annually, while that of stock *B* is 11%. If you currently hold a passive index portfolio, would you choose to add either of these stocks to your holdings?
17. A portfolio manager summarizes the input from the macro and micro forecasters in the following table:

Micro Forecasts

Asset	Expected Return (%)	Beta	Residual Standard Deviation (%)
Stock *A*	20	1.3	58
Stock *B*	18	1.8	71
Stock *C*	17	0.7	60
Stock *D*	12	1.0	55

Macro Forecasts

Asset	Expected Return (%)	Standard Deviation (%)
T-bills	8	0
Passive equity portfolio	16	23

a. Calculate expected excess returns, alpha values, and residual variances for these stocks.
b. Construct the optimal risky portfolio.
c. What is Sharpe's measure for the optimal portfolio and how much of it is contributed by the active portfolio?
d. What should be the exact makeup of the complete portfolio for an investor with a coefficient of risk aversion of 2.8?

18. Recalculate Problem 17 for a portfolio manager who is not allowed to short sell securities.
    a. What is the cost of the restriction in terms of Sharpe's measure?
    b. What is the utility loss to the investor ($A = 2.8$) given his new complete portfolio?
19. Suppose that based on the analyst's past record, you estimate that the relationship between forecast and actual alpha is:

$$\text{Actual abnormal return} = .3 \times \text{Forecast of alpha}$$

Use the alphas from Problem 17. How much is expected performance affected by recognizing the imprecision of alpha forecasts?

Challenge Problem

20. Suppose that the alpha forecasts in row 44 of Spreadsheet 8.1 are doubled. All the other data remain the same. Recalculate the optimal risky portfolio. Before you do any calculations, however, use the Summary of Optimization Procedure to estimate a back-of-the-envelope calculation of the information ratio and Sharpe ratio of the newly optimized portfolio. Then recalculate the entire spreadsheet example and verify your back-of-the-envelope calculation.

CFA® PROBLEMS

1. When the annualized monthly percentage rates of return for a stock market index were regressed against the returns for ABC and XYZ stocks over a 5-year period ending in 2008, using an ordinary least squares regression, the following results were obtained:

Statistic	ABC	XYZ
Alpha	−3.20%	7.3%
Beta	0.60	0.97
R^2	0.35	0.17
Residual standard deviation	13.02%	21.45%

Explain what these regression results tell the analyst about risk–return relationships for each stock over the sample period. Comment on their implications for future risk–return relationships, assuming both stocks were included in a diversified common stock portfolio, especially in view of the following additional data obtained from two brokerage houses, which are based on 2 years of weekly data ending in December 2008.

Brokerage House	Beta of ABC	Beta of XYZ
A	.62	1.45
B	.71	1.25

2. Assume the correlation coefficient between Baker Fund and the S&P 500 Stock Index is .70. What percentage of Baker Fund's total risk is specific (i.e., nonsystematic)?
3. The correlation between the Charlottesville International Fund and the EAFE Market Index is 1.0. The expected return on the EAFE Index is 11%, the expected return on Charlottesville International Fund is 9%, and the risk-free return in EAFE countries is 3%. Based on this analysis, what is the implied beta of Charlottesville International?
4. The concept of *beta* is most closely associated with:
    a. Correlation coefficients.
    b. Mean-variance analysis.
    c. Nonsystematic risk.
    d. Systematic risk.
5. Beta and standard deviation differ as risk measures in that beta measures:
    a. Only unsystematic risk, while standard deviation measures total risk.
    b. Only systematic risk, while standard deviation measures total risk.

c. Both systematic and unsystematic risk, while standard deviation measures only unsystematic risk.

d. Both systematic and unsystematic risk, while standard deviation measures only systematic risk.

Go to **www.mhhe.com/edumarketinsight** and click on the *Company* link. Enter the ticker symbol for the stock of your choice and click on the *Go* button. In the *Excel Analytics* section go to the *Market Data* section and get the *Monthly Adjusted Prices* data for the past 4 years. The page will also show monthly returns for your stock and for the S&P 500. Copy the data into an *Excel* worksheet and then do a regression to generate the characteristic line for the stock. (Use the menus for *Tools, Data Analysis, Regression,* input the X range and the Y range, select *New Worksheet Ply* under *Output Options,* and click on *OK.*) Based on the regression results, what is the beta coefficient for your stock?

STANDARD &POOR'S

Next use *Excel* to plot an X-Y Scatter graph of the stock's returns versus the S&P 500's returns. Once the graph is constructed, select one of the data points and right click on it. Choose the *Add Trendline* option and select the *Linear* type. On the *Options* tab, select *Display Equation on Chart.* How does the equation compare with your regression results?

Go back to the main page for your stock's information and select *S&P Stock Reports* from the menu. Choose *Stock Report* from the submenu and when the stock report opens, find the beta coefficient for the firm. How does this beta compare to your results? What are possible reasons for any differences?

E-Investments

Beta Estimates

Go to **http://finance.yahoo.com** and click on *Stocks* link under the *Investing* tab. Look for the *Stock Screener* link under *Research Tools.* The *Java Yahoo! Finance Screener* lets you create your own screens. In the *Click to Add Criteria* box, find *Trading and Volume* on the menu and choose *Beta.* In the *Conditions* box, choose < = and in the *Values* box, enter *1.* Hit the *Enter* key and then request the top 200 matches in the *Return Top_Matches* box. Click on the *Run Screen* button.

Select the *View Table* tab and sort the results to show the lowest betas at the top of the list by clicking on the *Beta* column header. Which firms have the lowest betas? In which industries do they operate?

Select the *View Histogram* tab and when the histogram appears, look at the bottom of the screen to see the *Show Histogram for* box. Use the menu that comes up when you click on the down arrow to select *beta.* What pattern(s), if any, do you see in the distributions of betas for firms that have betas less than 1?

SOLUTIONS TO CONCEPT CHECKS

1. *a.* Total market capitalization is 3,000 + 1,940 + 1,360 = 6,300. Therefore, the mean excess return of the index portfolio is

$$\frac{3{,}000}{6{,}300} \times 10 + \frac{1{,}940}{6{,}300} \times 2 + \frac{1{,}360}{6{,}300} \times 17 = 9.05\% = .0905$$

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Suppose that another portfolio, portfolio E, is well diversified with a beta of .6 and expected return of 8%. Would an arbitrage opportunity exist? If so, what would be the arbitrage strategy?

6. Assume that both portfolios A and B are well diversified, that $E(r_A) = 12\%$, and $E(r_B) = 9\%$. If the economy has only one factor, and $\beta_A = 1.2$, whereas $\beta_B = .8$, what must be the risk-free rate?

7. Assume that stock market returns have the market index as a common factor, and that all stocks in the economy have a beta of 1 on the market index. Firm-specific returns all have a standard deviation of 30%.

   Suppose that an analyst studies 20 stocks, and finds that one-half have an alpha of 2%, and the other half an alpha of −2%. Suppose the analyst buys \$1 million of an equally weighted portfolio of the positive alpha stocks, and shorts \$1 million of an equally weighted portfolio of the negative alpha stocks.

   a. What is the expected profit (in dollars) and standard deviation of the analyst's profit?

   b. How does your answer change if the analyst examines 50 stocks instead of 20 stocks? 100 stocks?

8. Assume that security returns are generated by the single-index model,

$$R_i = \alpha_i + \beta_i R_M + e_i$$

where R_i is the excess return for security i and R_M is the market's excess return. The risk-free rate is 2%. Suppose also that there are three securities A, B, and C, characterized by the following data:

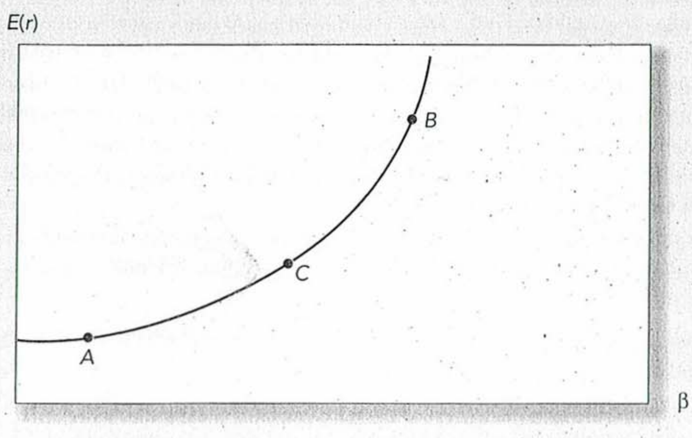
Security	β_i	$E(R_i)$	$\sigma(e_i)$
A	0.8	10%	25%
B	1.0	12%	10%
C	1.2	14%	20%

   a. If $\sigma_M = 20\%$, calculate the variance of returns of securities A, B, and C.

   b. Now assume that there are an infinite number of assets with return characteristics identical to those of A, B, and C, respectively. If one forms a well-diversified portfolio of type A securities, what will be the mean and variance of the portfolio's excess returns? What about portfolios composed only of type B or C stocks?

   c. Is there an arbitrage opportunity in this market? What is it? Analyze the opportunity graphically.

9. The SML relationship states that the expected risk premium on a security in a one-factor model must be directly proportional to the security's beta. Suppose that this were not the case. For example, suppose that expected return rises more than proportionately with beta as in the figure below.



a. How could you construct an arbitrage portfolio? (*Hint:* Consider combinations of portfolios *A* and *B*, and compare the resultant portfolio to *C*.)

b. Some researchers have examined the relationship between average returns on diversified portfolios and the β and β^2 of those portfolios. What should they have discovered about the effect of β^2 on portfolio return?

10. Consider the following multifactor (APT) model of security returns for a particular stock.

Factor	Factor Beta	Factor Risk Premium
Inflation	1.2	6%
Industrial production	0.5	8
Oil prices	0.3	3

a. If T-bills currently offer a 6% yield, find the expected rate of return on this stock if the market views the stock as fairly priced.

b. Suppose that the market expected the values for the three macro factors given in column 1 below, but that the actual values turn out as given in column 2. Calculate the revised expectations for the rate of return on the stock once the "surprises" become known.

Factor	Expected Rate of Change	Actual Rate of Change
Inflation	5%	4%
Industrial production	3	6
Oil prices	2	0

11. Suppose that the market can be described by the following three sources of systematic risk with associated risk premiums.

Factor	Risk Premium
Industrial production (*I*)	6%
Interest rates (*R*)	2
Consumer confidence (*C*)	4

The return on a particular stock is generated according to the following equation:

$$r = 15\% + 1.0I + .5R + .75C + e$$

Find the equilibrium rate of return on this stock using the APT. The T-bill rate is 6%. Is the stock over- or underpriced? Explain.

12. As a finance intern at Pork Products, Jennifer Wainwright's assignment is to come up with fresh insights concerning the firm's cost of capital. She decides that this would be a good opportunity to try out the new material on the APT that she learned last semester. She decides that three promising factors would be (i) the return on a broad-based index such as the S&P 500; (ii) the level of interest rates, as represented by the yield to maturity on 10-year Treasury bonds; and (iii) the price of hogs, which are particularly important to her firm. Her plan is to find the beta of Pork Products against each of these factors by using a multiple regression and to estimate the risk premium associated with each exposure factor. Comment on Jennifer's choice of factors. Which are most promising with respect to the likely impact on her firm's cost of capital? Can you suggest improvements to her specification?

Challenge Problems

13. Assume a universe of n (large) securities for which the largest residual variance is of an order not larger than $n\sigma_M^2$. Construct as many different weighting schemes as you can that generate well-diversified portfolios.

14. Derive a more general (than the numerical example in the chapter) demonstration of the APT security market line:

a. For a single-factor market.

b. For a multifactor market.

15. Small firms will have relatively high loadings (high betas) on the SMB (small minus big) factor.
    a. Explain why.
    b. Now suppose two unrelated small firms merge. Each will be operated as an independent unit of the merged company. Would you expect the stock market behavior of the merged firm to differ from that of a portfolio of the two previously independent firms? How does the merger affect market capitalization? What is the prediction of the Fama-French model for the risk premium on the combined firm? Do we see here a flaw in the FF model?

CFA® PROBLEMS

1. Jeffrey Bruner, CFA, uses the capital asset pricing model (CAPM) to help identify mispriced securities. A consultant suggests Bruner use arbitrage pricing theory (APT) instead. In comparing CAPM and APT, the consultant made the following arguments:
    a. Both the CAPM and APT require a mean-variance efficient market portfolio.
    b. Neither the CAPM nor APT assumes normally distributed security returns.
    c. The CAPM assumes that one specific factor explains security returns but APT does not.

    State whether each of the consultant's arguments is correct or incorrect. Indicate, for each incorrect argument, why the argument is incorrect.
2. Assume that both *X* and *Y* are well-diversified portfolios and the risk-free rate is 8%.

Portfolio	Expected Return	Beta
X	16%	1.00
Y	12	0.25

    In this situation you would conclude that portfolios *X* and *Y*:
    a. Are in equilibrium.
    b. Offer an arbitrage opportunity.
    c. Are both underpriced.
    d. Are both fairly priced.
3. A zero-investment portfolio with a positive alpha could arise if:
    a. The expected return of the portfolio equals zero.
    b. The capital market line is tangent to the opportunity set.
    c. The Law of One Price remains unviolated.
    d. A risk-free arbitrage opportunity exists.
4. According to the theory of arbitrage:
    a. High-beta stocks are consistently overpriced.
    b. Low-beta stocks are consistently overpriced.
    c. Positive alpha investment opportunities will quickly disappear.
    d. Rational investors will pursue arbitrage consistent with their risk tolerance.
5. The arbitrage pricing theory (APT) differs from the single-factor capital asset pricing model (CAPM) because the APT:
    a. Places more emphasis on market risk.
    b. Minimizes the importance of diversification.
    c. Recognizes multiple unsystematic risk factors.
    d. Recognizes multiple systematic risk factors.
6. An investor takes as large a position as possible when an equilibrium price relationship is violated. This is an example of:
    a. A dominance argument.
    b. The mean-variance efficient frontier.
    c. Arbitrage activity.
    d. The capital asset pricing model.

7. The feature of arbitrage pricing theory (APT) that offers the greatest potential advantage over the *simple* CAPM is the:
   a. Identification of anticipated changes in production, inflation, and term structure of interest rates as key factors explaining the risk–return relationship.
   b. Superior measurement of the risk-free rate of return over historical time periods.
   c. Variability of coefficients of sensitivity to the APT factors for a given asset over time.
   d. Use of several factors instead of a single market index to explain the risk–return relationship.
8. In contrast to the capital asset pricing model, arbitrage pricing theory:
   a. Requires that markets be in equilibrium.
   b. Uses risk premiums based on micro variables.
   c. Specifies the number and identifies specific factors that determine expected returns.
   d. Does not require the restrictive assumptions concerning the market portfolio.

STANDARD &POOR'S

Go to **www.mhhe.com/edumarketinsight** and link to *Industry*. From the pull-down menu link to the *Air Freight and Logistics* industry and click on *Go!*. Review the latest *S&P Industry Survey*. What are the current major risk factors that affect this industry? Which of these factors would you expect to be priced, that is, to command a significant risk premium? Now find the latest *S&P Industry Survey* for the *Biotechnology* sector. What risk factors does this industry face? Which of these factors are likely to affect the firm in the long term and which are likely to change over time?

E-Investments

Unanticipated Inflation

One of the factors in the APT model specified by Chen, Roll, and Ross is the percent change in unanticipated inflation. Who gains and who loses when inflation change? Go to **http://hussmanfunds.com/rsi/infsurprises.htm** to see a graph Inflation Surprise Index and Economists' Inflation Forecasts.

SOLUTIONS TO CONCEPT CHECKS

1. The GDP beta is 1.2 and GDP growth is 1% better than previously expected. So you will increase your forecast for the stock return by 1.2 × 1% = 1.2%. The revised forecast is for an 11.2% return.
2. With these lower risk premiums, the expected return on the stock will be lower:

$$E(r) = 4\% + 1.2 \times 4\% + (-.3) \times (-2\%) = 9.4\%$$

3. *a.* This portfolio is not well diversified. The weight on the first security does not decline as n increases. Regardless of how much diversification there is in the rest of the portfolio, you will not shed the firm-specific risk of this security.
   b. This portfolio is well diversified. Even though some stocks have three times the weight as other stocks ($1.5/n$ versus $.5/n$), the weight on all stocks approaches zero as n increases. The impact of any individual stock's firm-specific risk will approach zero as n becomes ever larger.
4. The SML says that the expected return on the portfolio should be $4\% + (⅓)(10 - 4) = 6\%$. The return actually expected is only 5%, implying that the stock is overpriced and that there is an arbitrage opportunity. Buy \$1 of a portfolio that is ⅔ invested in T-bills and ⅓ in the market. The return on this portfolio is $\frac{2}{3} r_f + \frac{1}{3} r_M = \frac{2}{3} \times 4\% + \frac{1}{3} r_M$. Sell \$1 of portfolio G. The net return on the combined position is: