Do Production and Trading Activities Mix?

by

Karan Bhanot and Antonio S. Mello[1]

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[1] Department of Finance, College of Business Administration, The University of Texas at San Antonio, 6900 North Loop, 1604 West, San Antonio, TX-78249, and Department of Finance, School of Business, The University of Wisconsin at Madison, 975 University Avenue, Madison, WI 53706-1323. Emails: karan.bhanot@utsa.edu and amello@bus.wisc.edu.

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Abstract

Commodity producing firms often supplement profits in their production operations by trading in the commodity that they produce. We ask whether these firms should keep their production and trading activities together in one corporate entity, or divide them into separate entities. We highlight two effects that follow from a joint firm: the impact of the strength of the shared balance sheet and the possibility that negative shocks from one operation contaminate the value of the healthy division. The volatility of commodity prices, firm leverage and market depth of the traded contract are key inputs that influence the choice between separate and joint operations. Trading operations are subsidized by the more stable cash flows from the production operations when they share a joint balance sheet, an issue that must be accounted for in compensation for the trading arm.

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Many commodity producing companies engage in trading activities that go beyond the sphere of the firm's typical production operations. For example, the list of the top ten energy trading firms in 2008 included names such as British Petroleum, Shell and Electricité de France each of which also produce the underlying commodity. There are many other recent examples where production and trading operations are housed in the same entity. The initial public offering of Glencore in 2011 reveals that the commodity trader owns a complex web of production assets and logistics businesses. In January 2012, Glencore agreed to a merger with a mining company (Xstrata) that resulted in the largest natural resource company in the world with a chain of businesses from mining to refining, storage, shipping as well as trading of basic commodities. Before its collapse in the fall of 2001, Enron was a trading firm with extensive distribution and production capabilities.

Companies organize their production and trading activities in different ways. In some cases, the trading (or production) is used to complement and support a company's production (trading) business. Air Canada, for example, operates a supply chain of fuel storage depots, pipelines and docks so that it can buy and sell refined jet fuel. In other instances trading operates as a profit center within the firm with a clear mandate to expand the overall earnings of the company. Also, trading is sometimes a stand-alone unit with no discernible relationship to the production business, and with its own financial structure. A recent article by CNBC ("The Glue in Delta's Possible Refinery Deal" by Kate Kelly on 4/11/12) reports on a decision by Delta Airlines to buy a refining company jointly with the investment bank JP Morgan so that trading and production is a joint venture.

The question of having production and trading integrated (common balance sheet) or separated (different balance sheets) has gained relevance in the US and Europe after the passing by Congress of the Dodd-Frank financial reform act of 2010 as well as the European Markets Infrastructure Regulation (EMIR) respectively. Regulatory developments in the US and elsewhere have prompted an examination of corporate structures and sparked a vigorous debate about the advantages and disadvantages of separation of traditional businesses from trading operations - the Volker rule [2] , as well as on the end user exemption and capital requirements for trading arms of non-financial firms.

The scope of trading activities in companies can vary across firms. For example, trading arm activities could include the purchase and sale of the commodity that allows production operations to swiftly adapt to changing market conditions. Trading operations could also engage in hedging commitments, intermediation of buyers and sellers (market making), and proprietary trading to arbitrage temporary price discrepancies. Similarly, the availability of information from operations and close relationships with consumers and logistics providers enables a company to identify trading opportunities and guarantee attractive sales margins.

Information flows between trading to production are an important factor on how the two activities should be combined. It is hard to argue against having production and trading working together in the presence of such information flows. We choose a different perspective and focus on the risk management arguments to explain under what conditions is it preferable to form separate entities for the production and trading activities of the firm or to integrate them under the same legal and financial entity. Our first purpose is to address the concerns voiced by investors and regulators alike worried about the effects of risk taking and the potential damaging consequences to markets when large players experience financial distress. Although banks are the focal point of the debate on financial reform, some large non-financial companies dominate many commodities and energy markets, and display a larger appetite for risk than that of leading Wall Street banks.[3] Second, we want to provide measures that link risk to capital ratios, by solving for the optimal capital structure of the firm and its different activities.

In this paper, separation means that each unit is legally and financially independent. A separate unit could be, for example, a joint venture or a fully owned subsidiary under the holding company, but where the parent firm has no recourse to its assets and cash flow, due to a legally enforceable agreement that protects debt holders and other stake holders (counterparties) of the

[2] The Volcker rule intends to severely limit proprietary trading in US banks.

[3] "Research reveals that Glencore could have lost a daily $42.5m last year on average when measured by the so-called "value-at-risk" measure, much more than the average $25.7m put at risk each day in 2010 in commodities trading by Goldman Sachs, Morgan Stanley, Barclays Capital and JPMorgan." See Javier Blair, "Glencore's risk appetite outstrips Wall St's", May 2, 2011

subsidiary. Integration, on the other hand, means that trading is closely tied to the production business of the firm, and plays a role that enables the firm to optimize its allocation of capital and generate value condensed in a joint and single profit function.

We analyze a firm that decides how much capital it allocates to physical production operations and to its trading activities. Combining production and trading facilitate the marketing of the commodity in the open market. When production lacks flexibility to absorb any shocks to demand, the cheapest avenue the firm has to adjust its position is through trading. Thus trading adds value by complementing production when a rapid adjustment in supply is not possible for the production side, because of a lag between decisions, investment and change in output. Thus, the speed of adjustment is a key differentiator of trading from production.

Next we analyze a stand-alone trading operation that engages in proprietary trading, and is set up as a profit center to take advantage of temporary price deviations. In this case the trading operation is not connected to production, but is more like a hedge fund trading the commodity to capture profits that arise from its informational advantage.

To clarify the benefits and costs of combining production and trading activities and to determine the optimal relative size of each activity we analyze a firm that engages in production as well as in proprietary trading. We then compare this joint firm with the two activities as separate.

In the joint firm, two effects are highlighted. The first relates to the fact that the trading unit can use the strength of the balance sheet of the production unit to generate larger trading positions and revenues. The second effect is that a shock to one activity might contaminate and destroy value in the other activity.

In a joint firm the cash flows from both production and trading are fungible and are pooled. This implies that the firm's equity holders are able to fund larger losses. In addition equity holders are willing to fund larger losses than a standalone trading operation because bankruptcy (or exit) would result in a loss of the continuation value of the production side of operations if the firm were to go bankrupt due to losses in trading. Furthermore, when profits from production are more stable than those from trading, the trading arm perceives that the risk of trading is co-insured by the equity holders of the joint firm. This makes the trading arm more aggressive in terms of taking larger positions than an equivalent stand-alone trading firm. Other things equal, a separated trading operation will have to operate with higher funding costs and higher capital in such instances when production operations are very profitable.

The second effect of combining operations is that it exposes each activity to good and bad outcomes in the other activity. Losses in production with a corresponding fall in cash flows and firm value limit the ability of the trading operation to take on optimal positions. It is also possible that a big trading loss may force the firm to go bankrupt even if the production arm still has value as a standalone entity.

We find that if the weight in trading exceeds a certain amount, the costs of integration can outweigh the benefits. Trading can add value to production insofar as it stays below a threshold that depends on leverage, on the risk controls, the depth of the market for the traded asset, and on the relative volatilities of production versus trading. The analysis shows that if cash flows from a trading operation are very volatile, it is preferable to keep the operation separate from production. With a relatively straightforward extension, it is possible to offer suggestions as to whether the decision to have a joint versus a separate production and trading company depends, to a certain extent, on how trading is executed, whether in illiquid and more specialized financial contracts (OTC markets), or more liquid markets that require posting of margins.

The next part of the paper analyzes the possible subsidies that a trading operation might capture when it shares a balance sheet with production. It is possible for the trading operations to be expanded on the basis of the debt capacity of the full balance sheet. This is a result of the less volatile cash flows from production as well as the impact that the value of the production assets have on the threshold barrier at which equity holders exit the firm.[4]

Our initial analysis is for firms financed by equity only. We discuss the impact of leverage on the decision to combine the trading and production operations in the last part of the paper. Leverage allows a trading firm to exploit additional trading opportunities until a point where the deadweight costs of distress exceed potential ability to exploit market price deviations. A similar outcome follows in a joint firm, though the impact is less so than a standalone trading

[4] The closest paper to ours is Parsons (2008), but whereas Parsons describes in great detail many of the arguments in our paper, we present a formal model and are therefore able to quantify many of the findings in Parsons' work.

firm. Again, in the case of a joint firm, debt constrains the positions of equity holders because it increases the likelihood of distress and correspondingly reduces the benefits of integration.

The paper is organized as follows- Section I provides a model of a pure production firm. Section II adds trading operations that complement production such that demand shocks are hedged. Section III provide a model of a pure trading operation. Section IV analyzes a joint firm where production and trading are combined. Section V discusses the impact of leverage, Section VI has empirical implications and Section VII concludes.

I. The production firm

We first consider a firm that produces a physical commodity. The firm is owned and operated by its equity holders. Equity holders decide the initial capital structure of the firm, and whether to provide additional funding to continue the operations of the firm. They do so to maximize the value of the equity in the firm. The capital market is complete. The firm has a given production capacity, *q*. The earnings of the firm denoted by $\delta_{pr,t}$, are a function of shocks to the commodity price and input costs (assuming that the firm produces and sells *q* units each period):

$$\delta_{pr,t} = \delta_{pr,t-1} + \sigma_{pr}\varepsilon_{pr,t} \tag{1}$$

where the subscript *pr* denotes production. The changes in earnings $\Delta\delta_{pr,t}$ are assumed to be independent, identically and symmetrically distributed: $\varepsilon_{pr,t} \sim N(0,1), \quad E(\varepsilon_{pr,t}, \varepsilon_{pr,t+1}) = 0$, and the volatility σ_{pr} is a constant. There is a default-free asset that pays a gross constant interest of *R* per period. The present value of these unlevered earnings from producing the commodity is given by:

$$V_{pr}^{U} = \mathrm{E}\left[\sum_{t=1}^{\infty} \frac{\delta_{pr,t}}{R^t}\right] = \left(\frac{\delta_{pr,0}}{R-1}\right) \tag{2}$$

The firm continues without limit unless losses are large enough so that equity holders do not find it optimal to finance losses (which occurs when the continuation value is zero and the loss equals

$\left(\frac{\delta_{pr,0}}{R-1}\right)$. For a production firm, the equity value at time 0, E_{pr}^{U}, in this setting equals and is given by (superscript U denotes an unlevered firm):

$$E_{pr}^{U} = \mathrm{E}\left[\sum_{t=1}^{t^*} \underbrace{\left(\frac{\delta_{pr,t}}{R^t}\right)}_{\substack{net\ flows\ to \\ equity\ holders}}\right] \tag{3}$$

The bankruptcy occurs at a random time t^* where the boundary is set endogenously to maximize equity value, and occurs as noted earlier when the continuation value of equity is zero.

II. Integrating Trading to Complement Production

We assume that the production firm in Section I has a fixed capacity q per period, and sells the entire output each period in the market. However, in practice, demand for a firm's product may be more than or less than the amount of production. Take, for example, the case of an electric power producer that operates generators to supply power to a municipality. When demand for power increases because of an unexpected change in weather, the firm is bound to supply power to its customers even though it is running at full capacity. The demand for the firm's product could also be less than the amount produced. The unsold inventory may dissipate or sold at a discount thus creating a potential drag on earnings. Such dissipation of inventory is evident for the case of power that cannot be stored easily.

A firm with no production flexibility is unable to accommodate demand shocks. The firm might decide to hold inventories or operate with spare capacity to make up for sudden increases in demand. However, in some instances the cost to build or switch capacity can be considerable, and especially so for a financially constrained firm. One avenue the firm has to rapidly adjust its response to demand shocks is via purchases and sales through a trading unit. This can be done by trading directly in the goods market the commodity, or trading financial claims on the commodity. Presumably the trading unit develops the capability to understand the statistical regularities of the demand from the many company customers as well as the patterns of volatility. It knows the marketplace in terms of who is sourcing the commodity and who is selling the commodity, where the commodity is being sold and at what price. Trading complements production by purchasing or selling output in cases of sudden changes in customer demand.

When the cost of *not* supplying customers is large or the revenue dissipation from unsold inventory is large, these complementary trading activities add to the value of the firm by avoiding such costs. Note that a rapid adjustment in supply is not possible for the production side in many instances because of a lag between decisions, investment and change in output. *Thus, it is the speed of adjustment that differentiates trading from production*.

Consider the commodity produced by a production firm (Section I) that has a spot price S_t. Assume that changes in the commodity price are independent, identically and symmetrically distributed: $S_t = S_0 + \sigma_S \varepsilon_{S,t}, \varepsilon_{S,t} \sim N(0,1)$ $\mathrm{E}(\varepsilon_{S,t}\varepsilon_{S,t+1}) = 0$, and σ_S is a constant. The earnings of the production firm $\delta_{pr,t}$ and commodity price S_t are not perfectly correlated: $corr(\varepsilon_{pr,t}, \varepsilon_{S,t}) = \rho < 1$. Let F_t denote the market price of a forward contract on the commodity at time *t* with maturity *t*+1.

The demand uncertainty can be captured by assuming that the demand for the firm's output at time *t*+1 is not perfectly known in advance at time *t* even though the firm does receive an imperfect signal (labeled *i*) for next period demand: $q + \underbrace{\sigma_q \varepsilon_{q,t+1}}_{surprise} + \underbrace{\sigma_i \varepsilon_{i,t+1}}_{signal}$ where: $\varepsilon_{q,t+1} \sim N(0,1)$, $\varepsilon_{i,t+1} \sim N(0,1)$, $E(\varepsilon_{i,t}, \varepsilon_{i,t+1}) = 0$, $E(\varepsilon_{q,t}, \varepsilon_{q,t+1}) = 0$, σ_q, σ_i are constants. The firm observes the second component (indexed *i*) at time *t* but not the first component.

When the firm agrees to sell to its customers any excess quantity demanded (over and above the production capacity *q*), it implicitly has a short position in the commodity. Of this implicit short position it can hedge the observed signal part (*i*) by entering into: $\theta_{hf,t} = \sigma_i \varepsilon_{i,t+1}$ forward contracts (where the subscript *hf* denotes hedging demand) using its trading arm. If the demand shock is negative the firm implicitly sells the requisite number of forward contracts to offset the decrease in demand for which it has receives a signal. The position in the forward contract yields a net inflow in the next period equal to the number of contracts times the profit from each unit in the position: $\theta_{hf,t}(S_{t+1}-F_t)$. If the excess demand is sold at the forward price, the net impact on earnings from the signal component is zero.

The earnings of a production firm that uses trading to complement production is given by the sum of the earnings of the production firm that operates at full capacity plus the net profits on the traded positions due to hedging, and cash flows from spot market purchases or sales for the unhedged portion of net demand. Assume that the cash flows from the sale or purchase of the excess unhedged component of demand in the open market is a cost factor $\eta > 0$ times the unhedged demand: $\eta(\sigma_q \varepsilon_{q,t+1})$. In other words, if the firm does not hedge it bears a cost in terms of partial dissipation of inventory or purchases that impose a net cost on the firm[5]:

$$\delta_{hf,t+1} = \delta_{pr,t+1} + \theta_{hf,t}(S_{t+1}-F_t) - \eta\sigma_q\varepsilon_{q,t+1} \qquad (6)$$

The benefit of trading thus accrues from the profits if any from the forward position and the costs (cash flow volatility) avoided by hedging a part of the excess net demand. Absence of hedging increases the volatility of earnings and the possibility that the firm may discontinue operations due to large losses. The equity value for a firm that uses trading to accommodate changes in demand for its products now includes the adjustment for trading (using equation (4)):

$$E_{hf}^{L} = \mathrm{E}\left[\sum_{t=1}^{t^*} \underbrace{\left(\frac{\delta_{hf,t}}{R^t}\right)}_{\substack{net\ flows\ to \\ equity\ holders}}\right] \qquad (7)$$

Thus, Sections I and II demonstrate how trading complements production by supplying customers any excess quantity demanded or selling the excess inventory in the open market. In this setting trading adds value and its scope is determined by the nature of the demand shocks.

III. Arbitrage Trading to Exploit Temporary Deviation in Prices

[5] The cost can take the form of making clients angry that result in market share loss (reputational cost). In our paper, this cost and the fact that the trading firm is unable to perfectly observe innovations in demand result in a violation of the Modigliani-Miller irrelevance proposition.

To analyze the impact of trading for arbitrage purposes we need a model for a pure trading firm. We consider a standalone trading firm whose motive is to exploit temporary deviations from the fundamental value of a traded asset. The asset traded by the firm is based on the commodity produced by the production firm (Section II). Such deviations are a result of demand shocks attributable to noise traders. The trading firm takes positions to profit from such deviations, but bears the risk that the fundamental value of the asset that is based on the price of the underlying commodity could move adversely during that period.

As in section II, the firm trades in a forward contract written on the commodity with spot price S_t. Without loss of generality, we assume that the fundamental value of the forward contract is the risk-neutral expected value of the commodity in the next period: $F_t^* = aS_t = \mathrm{E}(S_{t+1}) = S_t$. Thus the total convenience yield equals the risk-free rate. Two types of traders participate in the market for the forward contracts - noise traders and value traders. In each period noise traders receive a liquidity shock. This causes noise traders to trade a quantity $\sigma_n \varepsilon_{n,t}$ of the forward contract where $\varepsilon_{n,t} \sim N(0,1)$, $\mathrm{E}(\varepsilon_{n,t}, \varepsilon_{n,t+1}) = 0$ and σ_n is a constant. Noise trader shocks are uncorrelated to commodity price shocks: $\mathrm{E}(\varepsilon_{S,t}, \varepsilon_{n,t+1}) = 0$.

Noise traders cause the equilibrium price of the forward contract to diverge from its fundamental price. Value based traders respond to exploit the divergence of the market price due to the presence of noise traders. The value traders submit demand schedules of the form: $-\frac{1}{\beta}(F_t - F_t^*)$, with $\beta \geq 0$ specifying the quantity they will trade as a function of price. The aggregate flow of trades in the interval $(t, t+1)$ therefore equals: $d_t = -\frac{1}{\beta}(F_t - F_t^*) + \sigma_{n,t}\varepsilon_{n,t}$.

The equilibrium market price is determined by the market clearing condition: $d_t = 0$, or $-\frac{1}{\beta}(F_t - F_t^*) + \sigma_{n,t}\varepsilon_{n,t} = 0$ and is given by:

$$F_t = F_t^* + \beta \sigma_n \varepsilon_{n,t} \tag{8}$$

The trading firm exploits deviations from the fundamental price by taking a position θ_t each period. The trades are done in imperfectly illiquid markets, so to characterize the price impact of trades in a simple way, we assume a monopolistic trading firm even though the results carry through for many firms. The aggregate order flow is $d_t = -\frac{1}{\beta}(F_t - F_t^*) + \sigma_{n,t}\varepsilon_{n,t} + \theta_t$ and the equilibrium market clearing price using equation (10) is $F_t = F_t^* + \beta(\sigma_n \varepsilon_{n,t} + \theta_t)$.

An unconstrained trading firm can finance any shortfalls and interim losses. Given that the contract converges to the underlying commodity price on the settlement day, the problem each period reduces to:

$$\underset{\theta_t}{Max}\ \theta_t^{U,u}\left(F_{t+1}(\theta_t^{U,u} = 0, \varepsilon_{n,t} = 0) - F_t\right) \tag{9}$$

where $\theta_t^{U,u}$ is the trading position of an unlevered and unconstrained trader (identified by the superscript (U,u). Using $F_t = F_t^* + \beta(\sigma_n \varepsilon_{n,t} + \theta_t)$ we have:

$$\underset{\theta_t}{Max}\ \mathrm{E}\left[\theta_t^{U,u}\left[\left(S_0 + \sigma_S \varepsilon_{S,t+1}\right) - \left(F_t^* + \beta\left(\sigma_n \varepsilon_{n,t} + \theta_t^{U,u}\right)\right)\right]\right]$$

$$\underset{\theta_t}{Max}\ \mathrm{E}\left[\theta_t^{U,u}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_t^{U,u}\right)\right)\right]\right]$$

Taking the first derivative of the equation above and setting it equal to zero gives the optimal size of the trading position:

$$\theta_t^{U,u} = -\frac{1}{2}\left(\sigma_n \varepsilon_{n,t}\right) \tag{10}$$

Therefore, the per period profits at this optimal trading position in the next period (t) are:

$$\begin{aligned}\Pi_{t+1} &= \theta_t^{U,u}\left(\sigma_S \varepsilon_{S,t+1} - \beta\left(\sigma_n \varepsilon_{n,t} + \theta_t^{U,u}\right)\right)\\ &= \left(-\frac{1}{2}\sigma_n \varepsilon_{n,t}\right)\left(\sigma_S \varepsilon_{S,t+1} - \beta\left(\frac{1}{2}\sigma_n \varepsilon_{n,t}\right)\right)\\ &= \frac{1}{4}\beta\sigma_n^2 \varepsilon_{n,t}^2 - \frac{1}{2}\sigma_S \sigma_n \varepsilon_{S,t+1}\varepsilon_{n,t}\end{aligned} \tag{11}$$

And the equity value of an unconstrained trading firm is:

$$E_{tr}^{U,u} = \sum_{t=1}^{\infty}\left[\frac{1}{R^t}\mathrm{E}\left[\frac{1}{4}\beta\sigma_n^2 \varepsilon_{n,t}^2 - \frac{1}{2}\sigma_S \sigma_n \varepsilon_{S,t+1}\varepsilon_{n,t}\right]\right] = \frac{1}{4}\frac{\beta\sigma_n^2}{(R-1)} \tag{12}$$

where the subscript tr refers to a trading firm. The variance of profits at time t for the optimal trading position at this time is:

$$\sigma_\Pi^2 = Var\left(\theta_t^{U,u}\left(F_{t+1} - F_t\right)\right) = Var\left(\frac{1}{2}\sigma_S \sigma_n \varepsilon_{S,t+1}\varepsilon_{n,t}\right) = \left(\frac{1}{2}\sigma_S \sigma_n \varepsilon_{n,t}\right)^2 \tag{13}$$

The unconstrained trading firm is a useful reference. However, in practice a trading firm faces constraints that include the initial amount of capital raised and margin requirements by counterparties. Therefore, the trading firm may not be able to take on the optimal unconstrained position and achieve the first best value in equation (13). Next, we characterize the constraints faced by a trading firm when it operates with its own capital, and when it uses leverage to finance its trading positions.

The Constrained Trading Firm

In an unconstrained firm described above, equity holders are willing to fund losses to keep the firm operational. Suppose instead that cash allocated to trading is consistent with a value at risk target set by the equity holders. If losses exceed this threshold level, equity holders exit the firm. We could instead set the exit boundary based on other considerations and that does not impact our results.

Denote the amount of cash allocated by equity holders to trading activities at time *t* as M_0. Equity holders withdraw any profits each period if the amount of cash in the firm is augmented because of trading profits (when $M_t > M_0$) or contribute funds to maintain the amount M_0 when trading operations sustain losses. This continues unless losses are high enough so that equity holders decide to stop trading, and abandon the firm. $M_{tr,B}$ is the (endogenous) exiting barrier at which the firm stops trading.

Each period the sequence of events is as follows:

1. The shocks to the traded asset ($\varepsilon_{S,t}$) are realized.
2. The opening net cash balance (M_t) is determined by the last period cash ($M_{t-1} = M_0$) plus accumulated interest and any trading profits on positions initiated in last period and any payments to creditors.
3. The firm determines if the cash balance if losses exceed the *VaR* constraint: $M_t < 0$. If yes, the firm stops trading.
4. If the firm keeps trading, (when $M_t > 0$) equity holders receive a payout equal to the profit from trading plus any interest on cash held in the account. This brings M_t down to

M_0. In case of losses, equity holders may need to contribute additional funds, a margin call equal to $M_0 - M_t$ to keep the firm going.

5. The shocks to noise traders ($\varepsilon_{n,t}$) are realized.
6. The equity holders decide θ_t subject to the *VaR* constraint.

Thus, for an unlevered and constrained trading firm at time *t* (denoted by superscript (*U,c*)) the problem for equity holders at time *t* is to choose a trading strategy after the realization of noise shocks that maximizes the flows to equity holders (where the subscript *tr* refers to a trading firm):

$$E_{tr,t}^{U,c} = \underset{\theta_{tr,t}^{U,c}}{Max} \underset{M_{t+1}>0}{\mathrm{E}} \left[\underbrace{\frac{\Pi_{t+1} + (R-1)M_0}{R}}_{profits\ \ plus\ \ interest} + \underbrace{\frac{E_{tr,t+1}^{U,c}}{R}}_{\substack{continuation \\ value}} \right] \tag{14}$$

where:

Opening Cash: $M_{t+1} = RM_0 + \Pi_{t+1}$,

Trading profit: $$\Pi_{t+1} = \theta_{tr,t}^{U,c} \left[\left(\sigma_S \varepsilon_{S,t+1} \right) - \left(\beta \left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c} \right) \right) \right] , \tag{15}$$

VaR constraint: $$VaR_t \left[(100-n)\% \right] = M_0 , \tag{16}$$

and $E_{tr,t+1}^{U,c}$ is the continuation value of equity when it follows the optimal strategy and the time *t* expectation of the continuation value $\mathrm{E}\left(E_{tr,t+1}^{U,c}\right) = Y_{tr,t+1}^{U,c}$. Equation (16) is the firm's own limit on risk taking in the form of a value-at-risk constraint. Risk management measures like *VaR* attempt to constrain risk taking by specifying a cut-off loss level that will be only exceeded *n*% of the time over a single period (e.g., *n*=95%). Such a constraint is equivalent to specifying a trading position so that a shock to commodity prices $\varepsilon_{S,t+1}$ results in a loss that exceeds the cutoff only with a probability of (*100-n*)% (characterized in the next paragraph - an upper limit on the variable defined as ϕ). Therefore a firm may be constrained from trading because of this *VaR* constraint (the constraint equation (16) is binding). If however the constraint is not binding the solution to problem (14) is an interior solution. We can approximate equation (14) (see Appendix for proof):

$$E_{tr,t}^{U,c} \cong \frac{1}{R} \underset{\theta_{tr,t}^{U,c}}{Max} \left[\underbrace{-\theta_{tr,t}\beta\left(\sigma_n\varepsilon_{n,t} + \theta_{tr,t}\right)}_{profits \quad from \quad trading} + \underbrace{(R-1)M_t}_{\substack{interest \quad on \\ cash \;\; balances}} + Y_{tr,t+1}^{U,c} \right] - \underbrace{N(\phi)\left[Y_{tr,t+1}^{U,c} - M_0\right]}_{continuation \;\; value} \qquad (17)$$

where $N(.)$ is the cumulative normal density function $\phi = \dfrac{-RM_0 + \theta_{tr,t}\beta\left(\sigma_n\varepsilon_{n,t} + \theta_{tr,t}\right)}{\theta_{tr,t}\sigma_S}$.

Equation (17) comprises two parts – the first part in brackets gives the cash flows to equity holders from trading, the interest on cash balances and the continuation value of equity. The second part is the bankruptcy costs that depend on the traded positions. The *VaR* constraint imposed by the firm is effectively a restriction that specifies highest absolute value of ϕ. In the setting described here the optimal position increases profits but at the same time it also increases the bankruptcy probability and bankruptcy costs.

Proposition 1: *The probability of a loss in continuation value of a trading firm-*

(i) *Increases with the trading position* $\theta_{tr,t}^{U,c}$.

(ii) *Increases with volatility of the commodity price* σ_S *when* $RM_0 > \theta_{tr,t}^{U,c}\beta\left(\sigma_n\varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)$.

(iii) *Increases as market liquidity decreases (* β *increases) when* $\sigma_S\sigma_n > \theta_{tr,t}^{U,c}$.

Proof: See Appendix

If the extent of the noise trader shock is small, the optimal position is close to the unconstrained position (in equation (12)) when the *VaR* constraint is not binding. If however the extent of the noise shock requires the firm to take on a larger position, the first order condition of equity in equation (17) with respect to the trading position gives:

$$\left[\underbrace{-\beta\left(\sigma_n\varepsilon_{n,t} - 2\theta_{tr,t}^{U,c}\right)}_{\substack{incremental \;\; profits \\ from \quad trading}} \right] - \underbrace{\frac{\partial\phi}{\partial\theta_{tr,t}^{U,c}} n(\phi)\left[Y_{tr,t+1}^{U,c} - M_0\right]}_{incremental \;\; loss \;\; in \;\; continuation \;\; value} = 0 \qquad (18)$$

where *n*(.) is the normal density function. The solution to (18) is denoted $\theta_{tr}^{U,c}$. A higher position increases profits but makes it more likely that the firm may exit and lose the continuation value. Thus, the optimal trading position is given by $Min\left(\theta_{tr}^{U,c}, \theta_{VaR}\right)$ where θ_{VaR} is the upper limit on positions given by the *VaR* constraint in equation (16). Equation (18) is easily solved with numerical methods. Note that the first term in equation (18) is the maximization problem facing an unconstrained firm. The second term captures the impact of constraints on the trading firm on the trading position.

Figure 1(a) illustrates the impact of the trading position on the probability of default for two levels of market depth. In each of these cases the *VaR* constraint is not binding. As expected a larger position leads to a higher probability of default. Figures 1(b) and 1 (c) illustrate the optimal position as a function of market depth and the volatility of the commodity price. Figure 1(b) shows that a higher beta results (lower depth) in a higher deviation from the equilibrium price, and a correspondingly larger optimal position given the level of the noise shock. Figure 1(c) illustrates the optimal positions in trading as a function of the commodity price volatility. Again a larger commodity price volatility reduces the optimal position because the firm curtails its trading activity to reduce the chance of a loss in continuation value.

In sum, the ability of a trading firm to exploit price deviations depends on the market depth, volatility of the commodity price as well as the binding constraints due to *VaR* requirements.[6]

IV. A Firm that Produces and Trades

To clarify the benefits and costs of combining production and trading activities and to determine the impact of the relative size of each activity we need a model of a firm that engages in production (Section I) as well as in trading (Section III). The firm produces the physical commodity and also trades to exploit temporary deviations in prices. Recall that $corr(\varepsilon_{pr,t}, \varepsilon_{S,t}) = \rho < 1$ and also that $\mathrm{E}(\varepsilon_{S,t}, \varepsilon_{n,t+1}) = 0$. Therefore, trading cannot perfectly hedge production cash flows and vice versa.

[6] We do not have margin requirements that vary with the financial health of the company. Many contracts contain provisions whereby additional margin must be provided should the company's credit rating drop. With the contingent capital requirements a company has to stand ready to provide if it is going to be business.

When the firm integrates both production and trading there is a joint profit function and a single capital structure that funds both activities. By contrast, when the two activities are separate, each activity has its own value function and capital structure, as described in the previous sections. Our objective is to analyze the impact of trading operations when they are a part of a production firm.

Recall that the trading arm can rapidly adjust positions based on new information whereas an investment in production capacity is a multi-year position that cannot be rapidly adjusted. Thus a trading arm can capitalize more easily on both short term price and demand discrepancies than a production arm.

Suppose that the firm's policy is to allocate a fixed proportion (denoted *x*) of the firm's initial capital (J_0) to trading activities and the balance to production. The capital $M_0 = xJ_0$ is allocated to trading activities, where $0 \le x \le 1$ while $V_{pr,0} = (1-x)J_0$ is allocated to production activities. The proportion of cash allocated to trading is consistent with a value at risk target set by the equity holders.

Earnings from production and the profits from trading are pooled together:

$$\delta_{jo,t} = \underbrace{(1-x)\delta_{pr,t}}_{production} + \underbrace{\left(\Pi_t + (R-1)M_0\right)}_{trading\ \ plus\ interest\ \ income} \tag{19}$$

where *jo* denotes the joint firm. Each period the opening cash balance (M_t) is now determined by the cash last period ($M_{t-1} = M_0 = xJ_0$) plus cash flows from production, the interest on cash balances and any trading profits. The sequence of events is similar to that outlined in Section III. Each period the shocks to commodity prices and production are realized. This determines the opening cash M_t for the trading section and the value of production. If the firm realizes gains from trading and production, these cash flows are paid out as dividends to equity holders. Losses are funded by equity holders to bolster capital back to its initial position $M_0 = xJ_0$. Then, new positions are initiated when shocks to noise traders are realized. The optimization facing equity holders can therefore be written as (where $E_{jo}^{U,c}$ is the equity value, the subscript *jo* denotes a joint firm and the superscript (U,c) denotes an unlevered and constrained firm):

$$E_{jo}^{U,c} = \underset{\theta_{jo,t}^{U,c}}{Max} \underset{J_{t+1}>J_B}{\mathrm{E}} \left[\frac{(1-x)\delta_{pr,t+1} + \left(\Pi_{t+1} + (R-1)M_0\right)}{R} + \frac{E_{jo,t+1}^{U,c}}{R} \right] \tag{20}$$

Opening Cash: $M_{t+1} = xJ_0 + \left((1-x)\delta_{pr,t+1} + \left(\Pi_{t+1} + (R-1)xJ_0\right)\right)$

Trading profit: $\Pi_{t+1} = \theta_{jo,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{jo,t}^{U,c}\right)\right)\right]$,

VAR constraint: $Var_t\left[(100-n)\%\right] = xJ_0$,

Exit: $t^* = Min(t : J_t < 0), \quad J_t = \underbrace{(1-x)V_{pr,t}}_{\substack{production \\ part\ of\ firm}} + \underbrace{M_t}_{\substack{cash\ in \\ trading \\ account}}$,

Here the continuation value of equity is $\mathrm{E}\left(E_{jo,t+1}^{U,c}\right) = Y_{jo,t+1}^{U,c}$ and it comprises both the future payoffs from production assets payoffs as well as the payoffs from trading (strictly stationary under our assumptions). The exit barrier is exogenous is set to where the asset value declines to zero. The probability of reaching this barrier depends on both the value of the production assets as well as the cash generated from trading. In this setting, interim losses are funded by equity holders unless the firm incurs large losses and equity holders are unwilling to contribute additional funds because the firm has breached the exit barrier. A firm may instead choose to close or reduce the speculative component of a trading operation when the trading arm losses are larger than the *VaR* limit set by equity holders. In other words, it is possible that equity holders fund losses only until that limit even though the production operations continue. Our results to do not change under this alternate scenario with trading discontinued on breaching the *VaR* constraint. We can approximate equation (20) (see Appendix for proof):

$$E_{jo,t}^{U,c} \cong \frac{1}{R} \underset{\theta_t}{Max} \left[\underbrace{(1-x)\delta_{pr,t} - \theta_t\, \beta\left(\sigma_n \varepsilon_{n,t} + \theta_t\right)}_{profits\ \ from\ \ production\ \ and\ \ trading} + \underbrace{(R-1)M_0}_{\substack{interest\ on \\ cash\ balances}} + \underbrace{Y_{jo,t+1}^{U,c}}_{\substack{continuation \\ value}} \right] - \underbrace{G(\varphi)\left[Y_{jo,t+1}^{U,c} - xJ_0\right]}_{loss\ in\ continuation\ value} \tag{21}$$

where $\varphi = \left(-M_0 R - \delta_0 (1-x)\left(1/R + 1\right) + \theta_{jo,t}^{U,c}\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{jo,t}^{U,c}\right)\right)$, $G(0, \sigma_{jo})$ is the cumulative normal density function with mean 0 and the volatility of the joint firm cash flows are given by

$\sigma_{jo}^2 = (1-x)^2 \sigma_{pr}^2 + \theta_{jo,t}^{U,c\,2} \sigma_S^2 + 2\rho(1-x)\theta_{jo,t}^{U,c}\sigma_{pr}\sigma_S$. When an interior solution exists a first order condition for equation (21) gives (where *g* is the normal probability density function):

$$\left[\underbrace{-\beta\left(\sigma_n \varepsilon_{n,t} - 2\theta_{jo,t}^{U,c}\right)}_{\substack{incremental\ \ profits \\ from\ \ trading}}\right] - \underbrace{\frac{\partial \phi}{\partial \theta_t} g(\varphi)\left[Y_{jo,t+1}^{U,c} - xJ_0\right]}_{incremantal\ loss\ in\ continuation\ value} = 0 \qquad (22)$$

The optimal trading strategy weighs the benefits and costs of trading via its impact on cash available and the possibility of bankruptcy as in the case of the trading firm. The trading strategy and the bankruptcy barrier are chosen jointly to maximize equity value. The solution to (22) is $\theta_{jo}^{L,c}$. Then, given *x*, the optimal trading position is given by $Min\left(\theta_{jo}^{L,c}, \theta_{VaR}\right)$.

Co-insurance and Contamination effects

In our previous analysis we have fixed *x* of the firm's initial capital (J_0). The question is whether and how the capital should be split up and allocated to separate firms-production and trading, or should the same amount of capital fund a joint firm that houses production and trading under one entity.

In the joint firm, two effects must be considered in deciding how capital should be allocated between the production and trading activities. The first effect follows from the fact that the trading unit can use the strength of the balance sheet of the production unit to generate more trading as well as avoid bankruptcy.[7] The second effect of having a joint firm is that a shock to one activity might contaminate and destroy value in the other activity.

Consider the first effect. In a joint firm the cash flows from both production and trading are fungible and are pooled - recall that $\delta_{jo,t} = (1-x)\delta_{pr,t} + \left(\Pi_t + (R-1)M_0\right)$. This implies that a firm's equity holders are able to fund larger losses because losses in the trading arm are funded by the more stable cash flows from production (opening cash equals profits from trading plus production cash flows). When profits from production are more stable than those from trading,

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[7] Leland (2007) examines the financial benefits of merging diverse businesses to reduce the overall risk of default.

the trading arm perceives that the risk of trading is co-insured by the equity holders of the joint firm because of the benefit of pooling. This makes the trading arm more aggressive in terms of *taking larger positions* than an equivalent stand-alone trading firm. Suppose for illustration the extreme case where production revenues have zero volatility. Now traders perceive that a continuous flow of $\delta_{pr,t}$ can be used to offset some losses.

Another dimension to this aspect is that production operations have a continuation value of $(1-x)V_{pr,t} = \left(\frac{(1-x)\delta_{pr,t}}{R-1}\right)$, trading losses have to be larger than $(1-x)V_{pr,t} + M_0R$ in order for the firm to declare bankruptcy. Equity holders have an incentive to keep the firm running even if trading arm losses exceed the *VaR* limit set by the firm. Thus equity holders are willing to fund larger losses than a standalone trading operation because bankruptcy would result in a loss of the continuation value of the production side of operations of the firm. This is apparent in the computation of the bankruptcy cost in the computation of $\varphi = \left(-M_0R - \delta_0(1-x)(1/R+1) + \theta_t\,\beta(\sigma_n\varepsilon_{n,t} + \theta_t)\right)$ where the distance to default is computed from the exit boundary that comprises the value of both production assets and cash from trading. Thus losses from trading have to offset the value of the production arm completely before equity holders decide to stop operations. Other things equal, a separated trading operation will have to operate with higher funding costs and higher capital in such instances when production operations are very profitable.

Proposition 2: *Given the amount of capital in trading (x), a joint trading firm with stable production revenues takes on a larger trading position than a standalone trading firm with an equivalent amount of capital when the following condition is satisfied.*

$$\frac{\partial\phi}{\partial\theta}n(\phi)\left[Y^{U,c}_{tr,t+1} - M_0\right] > \frac{\partial\phi}{\partial\theta}g(\varphi)\left[Y^{U,c}_{jo,t+1} - xJ_0\right]$$

Proof: See Appendix

Figure 2 is a graphical depiction of the optimal position of traders in a joint firm versus those in a combined firm, for the same level of capital. The positions in the joint firm are close to the amount dictated by margin requirements and are larger than those of the separate firm for a given noise level shock (a boundary solution). This continues until the fraction in trading becomes large so that it is offset by the incremental bankruptcy costs to a larger extent for the separate firm than for the joint firm.

The second effect of combining operations is that it exposes each activity to good and bad outcomes in the other activity- a negative shocks in one activity might contaminate the operations of the other activity. Suppose, for example that the production arm experiences a negative shock, with a corresponding decline in cash flows and firm value. This limits the ability of the trading operation to take on optimal positions. Therefore, a decline in the value of production operations can have a contamination effect on the value of the trading part of the business. Similarly, it is also possible that a big trading loss may force the firm to go bankrupt even if the production arm still has value as a standalone entity.

Remark 1: *Negative shocks to production (trading) can reduce the value of the trading (production) operation*.

To assess the total impact of combining production and trading, Figure 3 compares the change in firm value when the firm is integrated and when it is separated into two entities. In each case, we compute the change in firm value of the combined entity compared to the non integrated entity when the amount of capital available is fixed.

Figure 3a shows the impact of *VaR* constraints on the value addition from a joint operation relative to separate firms. When the constraint is tighter or when the *VaR* is low, given a price shock the firm is unable to fully exploit the trading opportunities. The difference between a standalone firm and joint operation is smaller at the outset for low values of x when the constraint is binding because both operate with a position that is close to a boundary solution. As more capital is allocated (larger x), the joint firm can exploit trading opportunities to a larger extent. Now the volatility of cash flows coupled with the exit barrier constrains the standalone trading firm more. These constraints are apparent especially the case when *VaR* requirements are lower and the firm is able to take on larger positions with a more limited amount of capital.

Figure 3b is a graphical depiction of the impact of liquidity on the amount in trading. The figure reveals that in a less liquid market (high beta makes the price impact due to a noise shock is higher) the profits from trading are higher. These can in turn be exploited to a larger extent by a joint firm than a standalone firm.

In sum, if the weight in trading exceeds a certain amount, the costs of integration outweigh the benefits. Hence trading can add value to production insofar as it stays below a threshold that depends on leverage- the feedback effect and the bankruptcy costs-, on the margin requirements in trading, the depth of the market for the traded asset (an important aspect given that many commodity trading firms tend to deal in illiquid markets), and on the relative volatilities of production versus trading. The analysis shows that if a trading operation is not very volatile (is an ancillary operation), it is preferable to keep it integrated in the same firm with production; the converse is true for a trading operation whose returns are more volatile.

The subsidy from production to trading

Our previous analysis allows us to answer the question- what is the right amount of capital needed to fund a trading unit organized as a profit center? A different and perhaps more difficult task is to measure profitability of trading when trading and production share the same balance sheet. Indeed, it is common tendency to underestimate the risk and capital requirements associated with proprietary trading, leading to an exaggeration of its profitability (see Parsons, (2008)). This leads to an oversimplification of how trading operations contribute to value.

The above analysis reveals that a trading arm that is housed with a stable and profitable production arm can take on larger trading positions as a result of the equity holders willingness to absorb losses in trading to preserve the production assets.

Increased revenues in the trading arm from the higher positions relative to a standalone trading operation on average are a result of the subsidy from the production side to the trading side of operations. This is because an independent trading unit would face more erratic capital demands and a higher relative bankruptcy barrier.

To get a handle on the extent of the subsidy, one avenue is to compute the Sharpe-ratio for each activity and compute the excess return that a trading activity must generate in order to compensate for the higher volatility. First consider the production activity of a firm as outlined in Section I. The return on production in a given period is equal to (using equation (2)):

$$\tilde{r}_{pr,0} = \frac{\delta_{pr,1} + V_{pr,1}^{U}}{V_{pr,0}^{U}} - 1 \tag{22}$$

where $V_{pr,1} = \left(\dfrac{1}{R-1}\right)\left(\delta_{pr,1}\right)$, $\delta_{pr,1} = \left(\delta_{pr,0} + \sigma_{pr}\varepsilon_{pr}\right)$

The ratio of expected return to standard deviation of returns is correspondingly equal to:

$$\phi_{pr} = \frac{E(\tilde{r}_{pr,0})}{Std.\,Dev\,(\tilde{r}_{pr,0})} = \frac{R-1}{\left(\dfrac{\sigma_{pr}}{V_0}\right)} = \frac{\delta_{pr,0}}{\sigma_{pr}} \tag{23}$$

We can compute the corresponding Sharpe ratio for the trading arm. Given a noise shock, the expected return and variance of the trading arm are the sum of the return on the capital in the trading account (the margin amount earns a risk-free rate) plus the risk-adjusted return on the trading position (we approximate the computation insofar as the expected return and variance are not conditioned on the probability of bankruptcy).

$$\begin{aligned}\phi_{tr} &= (R-1) + \frac{\mu_{\Pi}}{\sigma_{\Pi}} = \frac{E\left(\theta^{L,c}_{jo,t-1}\left[\left(\sigma_S\varepsilon_{S,t}\right) - \left(\beta\left(\sigma_n\varepsilon_{n,t-1} + \theta^{L,c}_{jo,t-1}\right)\right)\right]\right)}{St.\,Dev.\left(\theta^{L,c}_{jo,t-1}\left[\left(\sigma_S\varepsilon_{S,t}\right) - \left(\beta\left(\sigma_n\varepsilon_{n,t-1} + \theta^{L,c}_{jo,t-1}\right)\right)\right]\right)} \\ &= (R-1) + \frac{-\theta^{L,c}_{jo,t-1}\beta\left[\sigma_n\varepsilon_{n,t-1} + \theta^{L,c}_{jo,t-1}\right]}{(\theta^{L,c}_{jo,t-1})\left[\sigma_S\right]}\end{aligned} \tag{24}$$

If the two Sharpe ratios are unequal, there is a benefit that accrues to one arm at a cost to the other. This subsidy from production to trading is given by equating equations (23) and (24):

$$\phi_{pr} = \phi_{tr} + \frac{Subsidy}{\sigma_{\Pi}} \tag{25}$$

The analysis implies that when a trading arm of a joint firm is subsidized by the production side of operations, the compensation to the traders must account for the manner in which the strength of the joint balance sheet is exploited by the trading arm of the operations. There should be a cost of capital charge equivalent to the subsidy provided by the rest of the firm to the trading operations. If the profits from trading larger positions that merely result from combining the two activities are fully captured by the traders, then traders are using the balance sheet of the firm to their own benefit. This assumes increasing importance in the current environment where many production firms have large trading operations and the incentives to trade must be properly aligned with the costs incurred by the rest of the firm.

V. Impact of Leverage

This section discusses the impact of leverage in the setting discussed earlier. Equity holders receive a payout equal to the earnings plus any interest on cash positions, net of coupon payments. In case of losses, equity holders may need to contribute additional funds to maintain the initial capital. If the firm incurs large losses so that the cash balance is below the bankruptcy barrier, the firm goes bankrupt. Equity holders are unwilling to contribute additional funds when the continuation value of equity is lower than the additional contribution needed to keep the firm viable. In such cases, if a firm has a positive cash balance remaining, it distributes the cash to existing debt holders after paying any legal or other bankruptcy costs. For simplicity, we assume that debt is sold only once at time 0, and that it has infinite maturity and a constant coupon flow denoted C_{pr}, C_{tr} and C_{jo} for the three types of firms – production, trading and joint. The deadweight costs of bankruptcy are (1-α_{pr}), (1-α_{tr}) and (1-α_{jo}) respectively where $0 < \alpha_{pr,tr,jo} < 1$ is the fraction of the asset value that debt holders receive in the event of bankruptcy (equity holders receive nothing).

Levered trading firm

The problem facing a levered trading firm is similar to that in equation (14), except that the coupon C_{tr} needs to be subtracted before any payments are made to the equity holders each period. The profit net of interest payments then equals: $\Pi_{t+1} = \theta_t^{L,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_t^{L,c}\right)\right)\right] - C_{tr}$, where $\left|\dfrac{\sigma_S \varepsilon_{S,t+1}}{\beta\sigma_n \varepsilon_{n,t}} > \beta\theta_t^{L,c}\right|$. Naturally $\theta_{tr}^{L,c}$ is affected by the debt payment C_{tr}, that reduces the cash necessary to comply with the margin requirement. Leverage allows the firm to trade more aggressively to exploit price deviations.

Figure 4 is a graphical depiction of the value generated for equity holders of a constrained firm that adds capital via the sale of debt. The incremental value added to equity from additional debt first increases, and subsequently declines. As the firm raises new capital with debt sales, it increases the amount of capital available for investment (a higher M_0). This increased capital allows the firm to take on a larger trading position when the price deviation is high and increase the expected profit: $E\left(\theta_{tr,t-1}^{L,c}\left[\left(\sigma_S \varepsilon_{S,t}\right)-\left(\beta\left(\sigma_n \varepsilon_{n,t-1}+\theta_{tr,t-1}^{L,c}\right)\right)\right]\right)$. However, this also increases the variance of the profits because of the fundamental risk of the trading position from the variance of the commodity price: $Var\left(\theta_{tr,t-1}^{L,c}\left[\left(\sigma_S \varepsilon_{S,t}\right)-\left(\beta\left(\sigma_n \varepsilon_{n,t-1}+\theta_{tr,t-1}^{L,c}\right)\right)\right]\right)=(\theta_{tr,t-1}^{L,c})^2\left[\sigma_S^2\right]$. The increased variance combined with the higher exit barrier (because of leverage) increases the probability of bankruptcy. Bankruptcy costs offset the increased profits to a larger extent as more debt is added. Thus additional debt beyond a point is value decreasing because the firm optimally reduces its positions. Now the cost incurred from the coupon on additional debt (risky debt coupon cost is larger than R-1) is not offset by increased profits.

Additional debt beyond the optimal amount reduces the ability of the firm to exploit trading opportunities in all states of the world. The firm will optimally mitigate the feedback effect of leverage by reducing the amount traded. The feedback effect of leverage therefore depends on how far the firm is from the bankruptcy boundary. The presence of debt reduces the optimal trading position for the entire range. For medium level of noise trader shocks, a firm with more debt is constrained to a larger extent than it is when the noise shock is large. This occurs because the return relative to risk for large shocks is high enough to offset bankruptcy costs. In other words if the extent of the price deviation is high, the corresponding return is high when prices revert to fundamentals. Therefore a firm can take on a larger position because the high returns justifies the larger bankruptcy cost risk taken on by the firm. Note that bankruptcy represents the losses to equity holders from the future profits of a going concern (the continuation value of equity). For large shocks, the firm takes on the maximum position possible which is determined by the margin constraint. However, for large amounts of debt the firm optimally reduces its position.

In reality, the restrictions on leverage are considerably higher than those modeled above, because a high investment grade credit rating is required to run a trading operation, while it is not necessary to continue as a physical producer. Without an investment grade credit rating, counterparties are unwilling to trade with the company or they demand very high levels of collateral/margin.

Levered joint firm

As in the case of a trading firm the equity holders of a joint firm receive a payout equal to the production profits, profits from trading plus any interest on cash positions net of coupon payments. Thus the trading positions are constrained because of a lower degree of cash availability to fund trading losses (lower coinsurance) as well as a higher bankruptcy barrier. In this setting negative shocks to production are more likely to constrain trading operations. In the presence of debt, negative shocks to production and the debt burden reduce the coinsurance effect because of pooling of cash flows. In addition the presence of debt increases the bankruptcy barrier. Thus leverage constrains the ability of the joint firm to take risks in terms of exploiting arbitrage opportunities. As the firm puts more of its capital into trading (parameter x), the benefits of integration are initially greater than the costs, because the incremental deadweight costs from the feedback effect of leverage are lower than the benefits from the increased revenue from larger positions and the fact that these revenues are not perfectly correlated with production shocks. The optimal x is larger when the traded asset is less volatile. When there is less debt, a firm benefits more by integrating production and trading, and a higher proportion of the firm capital is optimally tied up in trading. However for a high level of debt as trading activity increases, the benefits of having production jointly with trading decline rapidly.

VI. Empirical Implications and Remarks

Our discussion can be of use to explain whether a trading business is integrated or a separate entity from production. Among the factors determining the structure are the relative size of the trading operation, average earnings attributable to trading, the extent of leverage used by traders, the volatility of trading revenues, and several characteristics such as the depth of the market and the existence of margin requirements. Thus, a compilation of data on firms that have spun off their trading arm versus those that have kept it integrated, together with this set of variables, should have explanatory power.

One practical implication of our study is the insight into the way separation of a trading operation can influence the overall credit risk of a company. Consider, for example, a company that has a large proportion of its capital allocated to trading. It is possible that a spin-off may increase firm value and enhance the credit rating of the separate entities. This would occur because the separate trading entity may be able to generate more trading revenues if its balance sheet is clean, and consequently increase its value. Also, the production arm revenues of the separate entity are less volatile in the absence of a trading operation. Our results make it possible to estimate the extent to which integration or separation changes the overall firm value, the probability of default, and consequently the credit rating of the company. This analysis is of relevance to credit rating agencies and counterparties that constantly need to analyze and monitor the implications of trading on the possibility of financial distress.

VII. Summary and conclusions

Many natural resources and large manufacturing firms speculate in the commodities that are either an input to or an output of, their production process.. The speculation could also be motivated by macroeconomic considerations. Trading can be legally and financially separated from the production activities or integrated within the same firm. The decision is complex and involves many factors, such as economies of scale and scope, information spillovers, risk management considerations, as well as managerial compensation. In this paper we focus on the risk management argument to explain under what conditions it is optimal to form separate entities for the production and trading activities of the firm or integrate them under the same legal and financial entity?

From a risk management perspective, the choice needs to consider the benefits of diversification from uncorrelated profits in trading and production and the ability and extent to which each unit can make use of the strength of the balance sheet of the joint entity. The strength of the balance sheets is used to generate additional business as well as to reduce funding costs but bears the risk of contamination of a healthy part of the firm from another unit that suddenly becomes financially crippled.

We find that integrating a high-risk trading business with production is of benefit up to when a relatively low proportion of capital is tied up in trading.

The arguments in favor of separation come from the very different cash flow profiles of the production and trading, and how this affects the difficulty of merging in one firm activity that requires very different capital structures. Trading operations that are particularly sensitive to changes in credit rating that affect the ability to generate trading business should be spun off in order to avoid potential contamination of the balance sheet that a negative shock to the production side of the firm would cause. Similarly, risky trading activities present a serious potential value destruction to stable and profitable production operations.

The paper makes a contribution to the on-going debate about the advantages and disadvantages of separation of traditional businesses from trading operations - the “Volker rule”, as well as on the end user exemption and capital requirements for trading arms of non-financial firms.

References

Fehle, F., and S. Tysplakov, 2003, Dynamic risk management: Theory and evidence, *Journal of Financial Economics* 78, 3-47.

Ho, T. S. Y., and H. R. S. Stoll, 1983, The dynamics of dealer markets under competition, *Journal of Finance* 38, 1053-1074.

Kyle, A. S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

Leland, H. E., 2007, On purely financial synergies and the optimal scope of the firm: Implications for mergers, spinoffs, and structured finance, *Journal of Finance* 62, 765-808.

Manaster, S., and S. C. Mann, 1996, Life in the pits: Competitive market making and inventory control, *Review of Financial Studies* 1996, 953-975.

Parsons, J., 2008. Do Trading and Power Operations Mix? The Case of Constellation Energy Group 2008. *MIT CEEPR working paper 08-014.*

Appendix:

Proof of equation (17):

Using $M_{t+1} = RM_0 + \Pi_{t+1}$, $\Pi_{t+1} = \theta_{tr,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)\right)\right]$ and $\mathrm{E}\left(E_{tr,t+1}^{U,c}\right) = Y_{tr,t+1}^{U,c}$ we get

$$E_{tr,t}^{U,c} = \underset{\theta_t}{Max} \underset{M_{t+1}>0}{\mathrm{E}}\left[\theta_{tr,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)\right)\right] + (R-1)M_0 + E_{tr,t+1}^{U,c}\right]/R$$

$$= \underset{\theta_{tr,t}^{U,c}}{Max}\left(\begin{array}{l} \mathrm{E}\left[\theta_{tr,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)\right)\right] + (R-1)M_0 + \left(Y_{tr,t+1}^{U,c}\right)\right]/R \\ \qquad - \underset{M_{t+1}<0}{\mathrm{E}}\left[\theta_{tr,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)\right)\right] + (R-1)M_0 + Y_{tr,t+1}^{U,c}\right]/R \end{array}\right).$$

Here the noise shock $\varepsilon_{n,t}$ is known at time t and expected changes in the commodity price $\mathrm{E}\left(\varepsilon_{S,t+1}\right) = 0$. The condition that $\theta_{tr,t}^{U,c}\left[\left(\sigma_S \varepsilon_{S,t+1}\right) - \left(\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)\right)\right] + RM_0 + \left(Y_{tr,t+1}^{U,c}\right) = 0$ gives

$\phi = \varepsilon_{S,t+1} = \dfrac{-RM_0 + \theta_{tr,t}^{U,c}\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)}{\theta_{tr,t}^{U,c}\sigma_S}$. Also, $\dfrac{\int_{-\infty}^{d} xf(x) - d\int_{-\infty}^{d} f(x)}{\int_{-\infty}^{d} xf(x)} \le h(d)$ where $f(x)$ is the standard normal density, h is a function and d<0. The approximation $\int_{-\infty}^{d} xf(x) \approx d\int_{-\infty}^{d} f(x)$ can be easily evaluated using numerical methods. As d becomes more negative the approximation improves. Using $\mathrm{E}\left(\varepsilon_{S,t+1}\right) = 0$ and $\int_{-\infty}^{\phi} \varepsilon_{S,t+1} f(x) \approx \phi\int_{-\infty}^{\phi} f(x)$ we get:

$$E_{tr,t}^{U,c} \cong \frac{1}{R}\underset{\theta_{tr,t}^{U,c}}{Max}\left[\underbrace{-\theta_{tr,t}\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}\right)}_{profits\ from\ trading} + \underbrace{(R-1)M_t}_{\substack{interest\ on \\ cash\ balances}} + Y_{tr,t+1}^{U,c}\right] - \underbrace{N(\phi)\left[Y_{tr,t+1}^{U,c} - M_0\right]}_{bankruptcy\ costs}$$ where $N(.)$ is the cumulative normal density function.

Proof of Proposition 1:

We are given that $N(.)$ is the cumulative normal density function and $\phi = \dfrac{-RM_0 + \theta_{tr,t}^{U,c}\beta\left(\sigma_n \varepsilon_{n,t} + \theta_{tr,t}^{U,c}\right)}{\theta_{tr,t}^{U,c}\sigma_S}$. An increase in $N(\phi)$ implies an increase in the probability of exit. Evaluating each derivative (suppressing the superscript (U,c)) gives: $\frac{\partial \phi}{\partial \theta_t} = \frac{\beta + \frac{M_0}{\theta_t^2}}{\sigma_s} > 0$,

$\frac{\partial \phi}{\partial \sigma_S} = \frac{RM_0 - \theta_t \, \beta(\sigma_n \varepsilon_{n,t} + \theta_t)}{\theta_t \, \sigma_S^2} > 0$, when $RM_0 > \theta_t \, \beta(\sigma_n \varepsilon_{n,t} + \theta_t)$ and $\frac{\partial \phi}{\partial \beta} = \frac{\sigma_S \sigma_n + \theta_t}{\sigma_S} > 0$ when $\sigma_S \sigma_n > \theta_t$.

Proof of equation (22):

The proof follows the logic used in the Proof for equation (17). We need to compute the probability so that the combined shock from trading and production is such that the firm value declines to the exit point (suppressing superscripts):

$$(1-x)\sigma_{pr}\varepsilon_{pr,t+1} + \theta\sigma_S\varepsilon_{S,t+1} = \left(-M_0 R - \delta_0(1-x)/(R-1) + \theta_t \, \beta(\sigma_n \varepsilon_{n,t} + \theta_t)\right).$$

The variable on the left hand side $((1-x)+1/(R-1))\sigma_{pr}\varepsilon_{pr,t+1} + \theta_t \sigma_S \varepsilon_{S,t+1}$ has mean 0 and variance $\sigma_{jo}^2 = (1-x)^2\sigma_{pr}^2 + \theta^2\sigma_S^2 + 2\rho(1-x)\theta\sigma_{pr}\sigma_S$.

$$\underset{J_{t+1}>J_B}{\mathrm{E}}\left[\frac{(1-x)\delta_{pr,t+1} + (\Pi_{t+1} + (R-1)M_0)}{R} + \frac{E_{jo,t+1}^{U,c}}{R}\right]$$
$$= \mathrm{E}\left[\frac{(1-x)\delta_{pr,t} + (\Pi_{t+1} + (R-1)M_0)}{R} + \frac{E_{jo,t+1}^{U,c}}{R}\right] - \underset{J_{t+1}<J_B}{\mathrm{E}}\left[\frac{(1-x)\delta_{pr,t+1} + (\Pi_{t+1} + (R-1)M_0)}{R} + \frac{E_{jo,t+1}^{U,c}}{R}\right]$$

Using: $\delta_{pr,t} = \delta_{pr,0} + \sigma_{pr}\varepsilon_{pr,t}$, trading flows of $\Pi_{t+1} = \theta_t \left[(\sigma_S \varepsilon_{S,t+1}) - (\beta(\sigma_n \varepsilon_{n,t} + \theta_t))\right]$ and using the approximation $\int_{-\infty}^{\varphi} x f(x) \approx \varphi \int_{-\infty}^{\varphi} f(x)$ we get the desired result.

Proof of Proposition 2:

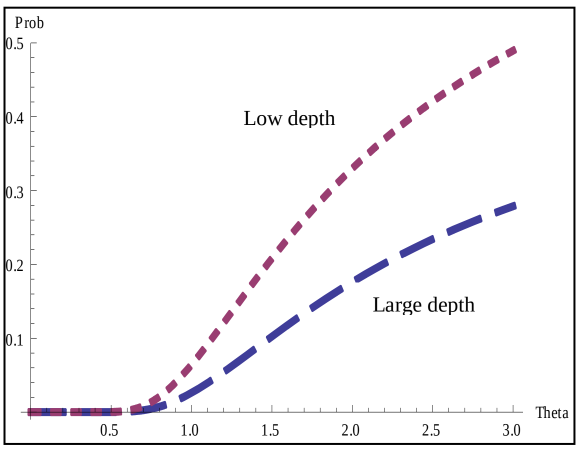
Proposition 2 follows from the optimization problem facing the two firms- a trading firm solves(using equation (18)): $\left[\underbrace{-\beta(\sigma_n \varepsilon_{n,t} - 2\theta_{tr,t}^{U,c})}_{\text{incremental profits from trading}}\right] - \underbrace{\frac{\partial \phi}{\partial \theta_{tr,t}^{U,c}} n(\phi)\left[Y_{tr,t+1}^{U,c} - M_0\right]}_{\text{loss in continuation value}} = 0$ while a joint firm solves $\left[\underbrace{-\beta\left(\sigma_n\varepsilon_{n,t}-2\theta^{U,c}_{jo,t}\right)}_{\substack{incremental\ profits\\ from\ trading}}\right]-\underbrace{\frac{\partial\phi}{\partial\theta_t}g(\varphi)\left[Y^{U,c}_{jo,t+1}-xJ_0\right]}_{incremantal\ loss\ in\ continuation\ value}=0$. Given the same trading opportunities, the trading firm positions are lower when the penalty for trading is higher, i.e., when $\frac{\partial\phi}{\partial\theta^{U,c}_{tr,t}}n(\phi)\left[Y^{U,c}_{tr,t+1}-M_0\right]>\frac{\partial\phi}{\partial\theta^{U,c}_{jo,t}}g(\varphi)\left[Y^{U,c}_{jo,t+1}-xJ_0\right]$.

Figure 1.

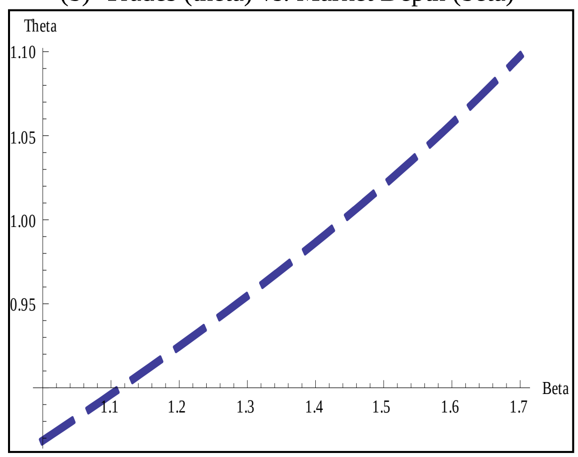
Trading Firm: Impact of Commodity volatility and Market Depth

Figure 1(a) illustrates the effect of trading positions (labeled theta) on the probability of default (Prob) for two levels of market depth (beta), Figure 1(b) graphs the optimal trading position (theta) as a function of market depth (beta). Figure 1 (c) graphs optimal positions (theta) as a function of the commodity volatility (sigma s). We assume that $R = 1.1$, $\sigma_n = 10$, $\sigma_S = 50$, $\beta = 1$, $\varepsilon_{n,t} = -1$ and $M_0 = 100$ where required.

(a) Probability of default vs Trades (theta)



(b) Trades (theta) vs. Market Depth (beta)



(c) Trades (theta) vs. Commodity Volatility (sig s)

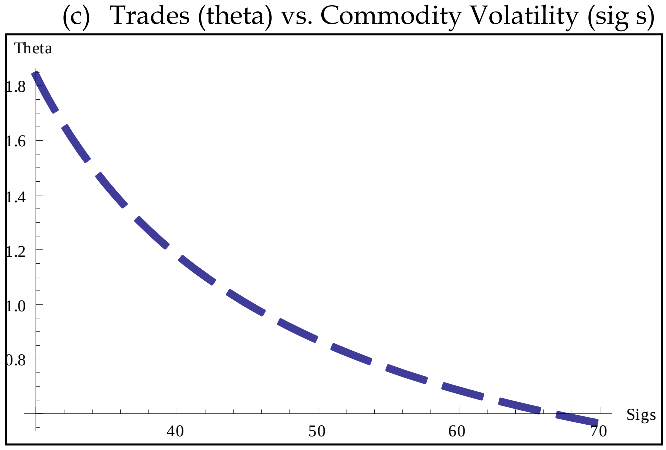


Figure 2.
Trading Positions in Joint and Separate Firms

This figure illustrate the optimal trading positions as a function of the proportion in trading for a joint firm and an independent firm. We assume that $R = 1.1$, $\sigma_n = 10$, $\delta_{pr} = 10$, $\sigma_{pr} = 0.5$, $\sigma_S = 50$, $\varepsilon_{n,t} = -1$, $J_0 = 100$.

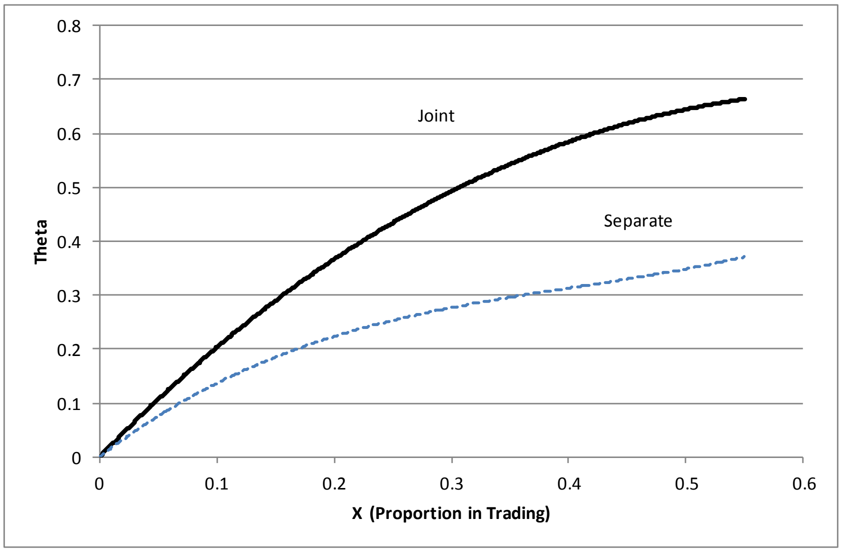
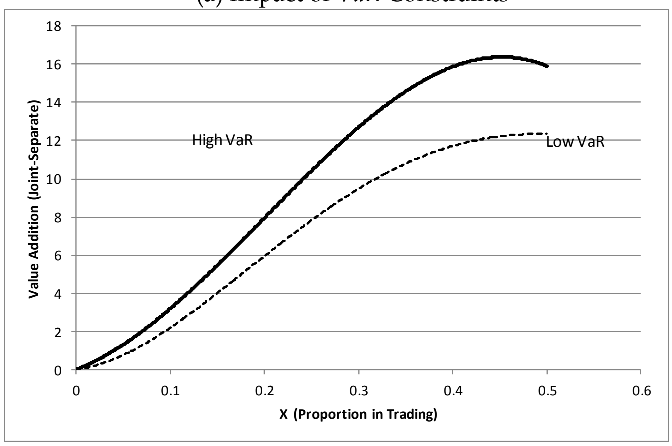


Figure 3.
Incremental Value from Integration

Overall increase in value when a firm integrates its trading and production operations as a function of the proportion invested in trading activities (*x*). We assume that $R = 1.1$, $\sigma_n = 10$, $\delta_{pr} = 10$, $\sigma_{pr} = 1$, $\sigma_S = 50$, $J_0 = 100$.

(a) Impact of *VaR* Constraints



(b) Impact of Liquidity

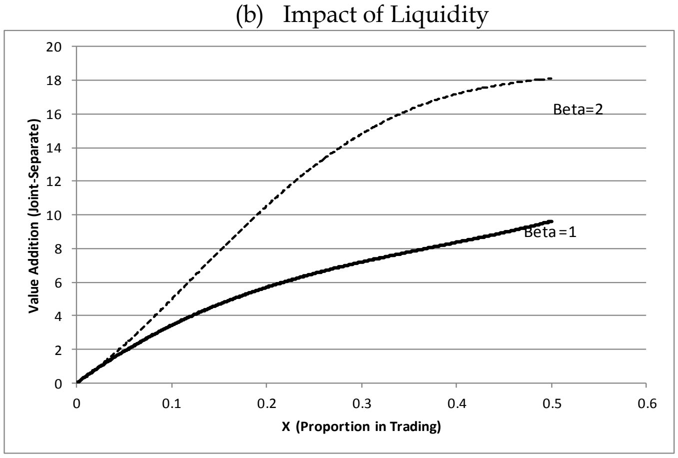


Figure 4.
Value Added to Equity Holders from Additional Funds via Debt

This figure illustrates the increase in the value of equity in a trading firm as a function of incremental leverage. We assume that $R = 1.1$, $\sigma_n = 10$, $\sigma_{pr} = 1$, $\sigma_S = 50$, initial equity=50, and recovery rate $\alpha_{tr} = 0.6$. Newly issued debt is sold at par.