REAL OPTION VALUE

CHAPTER 7 SEQUENTIAL INVESTMENT OPTIONS

Thus far, it has been assumed that the investment amount is paid instantaneously upon exercise of the real option, that is when commencing the investment. Often, investment opportunities require a sequence of expenditures, so that interim "mini-investments" are necessary over a time line in order to keep the ultimate investment opportunity option alive. This chapter allows for sequential investment options (also termed instalment options), where it is assumed that interim expenditures are completely sunk costs, having no alternative or scrap value.

Assume first the investment program involves required initial expenditures (the real option premium), a second phase of required investment expenditures (D), and a final development phase, when then the project values (V) are realized. The essential aspect of this characterized program is that managers have a choice of whether to pay the interim expenditure, and then the development cost (K). This program constitutes a call option on a further call option. If all costs are considered "sunk costs", the initial expense at t_0 is an irrecoverable premium for a call option to pay D at t_1, which is itself a premium for an option to pay K at t_2, to receive then the project values. Without management flexibility not to pay D or K, perhaps such a program should be valued using present values. With management discretion, real option models are appropriate since future expenditures can be cancelled. The first stage decisions are based on the difference between perceived value (including future options) and cost at or before exercise dates. The transitions between the stages are sequential options.

These models are suitable for any investment program, where there are required interim expenditures for program continuance such as: (a) a telecommunications company contemplating providing intermediate services and looking to maintain or increase line usage, or a mobile operator initially bidding for a 4G license, that requires R&D at a first stage, and then implementation expenditures; (b) an E_Commerce software or a search service provider, which aims to add advertising, and then content in sequences, each requiring R&D and marketing expenditures; and (c) a property developer, who pays an initial price for development land, where there are required interim decontamination expenses, and then final construction costs.

Here are several real option valuation methods, starting with a simple European compound option, extended to a European compound exchange option. Then an approximated American finite sequential exchange option is considered. Finally, an American perpetual, and an American perpetual exchange option are presented, allowing for several stages of investment expenditures (and critical values which justify making those expenditures).

The simplest European sequential model is the Geske (1979) compound call on a call option similar to the previous Chapter, where it is used in the process of approximating an American option. The simple European exchange option is an adapted Margrabe (1978) exchange option, set in a compound option format. This assumes that both the development costs and the ultimate project value are both stochastic, and costs (D) must be spent at t_1 in order to keep alive the option to exchange K for V at $T(=t_2)$.

The sequential finite American exchange option model is a more realistic characterization of sequential investment options than standard American or European option models. The Paxson (2007) approximation of an American sequential exchange option value is presented, where the asset is expected to have a significant current income. Finally, building on Adkins and Paxson (2013), multi-stage sequential American perpetual and American perpetual exchange models are provided.

7.1 SEQUENTIAL EUROPEAN REAL OPTIONS

Geske (1979) developed an analytic framework for a European option, where in order to keep the option alive an interim expenditure is required. Whereas in the previous chapter, the decision was about continuing an option on an income yielding asset, here the decision is about making a required payment in order to maintain the ultimate option. There is a critical value V* which justifies making the interim expenditure. As before, assume that developed values (V) follow a geometric Brownian motion:

$$dV = (\mu_V - \delta_V)Vdt + \sigma_V V dz_v \tag{7.1}$$

where μ_V is the equilibrium expected drift rate, δ_V is the income rate (or payout rate) of V, and σ_V is the volatility. Let the value of a call on a call be the real option value C_c, where D is the interim expenditure required at time $\tau' = .5\,\tau$, and K the investment cost at time τ. The value of a call on a call C_c is given by

$$C_c = Ve^{-\delta\tau}B(a_1, d_1; \rho) - Ke^{-r(\tau-\tau')}B(a_2, d_2; \rho) - De^{-r\tau'}N(a_2) \tag{7.2}$$

where ρ is the correlation coefficient between the overlapping Brownian motion increments, which is defined as $\rho = \sqrt{\tau'/\tau}$, and N(.) and B(.) are the standard cumulative univariate and bivariate normal distributions with parameters:

$$a_1 = \frac{\ln(V/V^*) + (r - \delta + 0.5\sigma^2)\tau'}{\sigma\sqrt{\tau'}},\ a_2 = a_1 - \sigma\sqrt{\tau'}, \tag{7.3}$$

$$d_1 = \frac{\ln(V/K) + (r - \delta + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}},\ d_2 = d_1 - \sigma\sqrt{\tau}.$$

The critical price, V^*, is obtained by solving the value matching condition:

$$V^* e^{-\delta(\tau-\tau')}N(d_1^*) - Ke^{-r(\tau-\tau')}N(d_2^*) = D \tag{7.4}$$

$$d_1^* = \frac{\ln(V^*/K) + (r - \delta + 0.5\sigma^2)(\tau - \tau')}{\sigma\sqrt{\tau - \tau'}},\ d_2^* = d_1^* - \sigma\sqrt{\tau - \tau'} \tag{7.5}$$

Figure 7.1

	A	B	C	D
1	**GESKE EUROPEAN COMPOUND CALL OPTION**			
2				
3	DEVELOPMENT TIME τ'	1	10	
4	INVESTMENT TIME τ	2	20	
5	INTEREST RATE	0.04	0.04	
6	V YIELD	0.04	0.04	
7	VALUE VOLATILITY	0.20	0.20	
8	V	100.00	100.00	
9	D	20.00	20.00	
10	K	80.00	80.00	
11	**REAL OPTION VALUE**	**7.1768**	**10.6141**	B8*EXP(-B6*B4)*B26-B10*EXP(-B5*B4)*B27-B9*EXP(-B5*B3)*B25
12	$\tau-\tau'$	1.00	10.00	B4-B3
13	V*	99.58	94.80	
14	d1	1.19	0.58	(LN(B13/B10)+((B5-B6+0.5*B7^2)*B12))/(B7*SQRT(B12))
15	d2	0.99	-0.05	B14-B7*SQRT(B12)
16	N1	0.88	0.72	NORMSDIST(B14)
17	N2	0.84	0.48	NORMSDIST(B15)
18	EQ 7.4	20.00	20.00	B13*EXP(-B6*B12)*B16-B10*EXP(-B5*B12)*B17
19	EQ 7.4-D	0.00	0.00	B18-B9
20	$\rho(\tau'/\tau)$	0.71	0.71	SQRT(B3/B4)
21	d1,t1	0.12	0.40	(LN(B8/B13)+(B5-B6+0.5*B7^2)*B3)/(B7*SQRT(B3))
22	d1,t2	0.93	0.70	(LN(B8/B10)+(B5-B6+0.5*B7^2)*B4)/(B7*SQRT(B4))
23	d2,t1	-0.08	-0.23	B21-B7*SQRT(B3)
24	d2,t2	0.65	-0.20	B22-B7*SQRT(B4)
25	N2	0.47	0.41	NORMSDIST(B23)
26	B1	0.53	0.59	BiVariateNormalCDF(B21,B22,B20)
27	B2	0.44	0.29	BiVariateNormalCDF(B23,B24,B20)
28				
29	**The first five inputs are the D and K timing estimates, the interest rate, and the**			
30	**value yield and volatility.**			
31	**The next three inputs are V, D and K estimates.**			
32	**Real call option value assumes V* is the value above which D should be paid at τ'.**			
33	**USE TOOLS/SOLVER, SETTING B19=0 BY CHANGING B13.**			

Using standard parameters, the Geske European sequential investment model is shown in Figure 7.1. Use Tools/Solver to solve equation (7.4)-D=0. In column B, V* is almost 100, the current value, for this is a more or less at the money compound call option. If V is 100 at time τ', the payment D=20 should be made in order to keep the ultimate call option alive.

Since the Geske compound option model is European, it is at best a first estimate for long-lived sequential options. As also shown in Figure 7.1 column C, the compound option value does not increase substantially as the time to ultimate exercise increases (with D at the half way time).

It is easy to extend this compound option model to a European sequential exchange real option. Suppose that the D and development costs K follow a diffusion process similar to that for V:

$$dK = (\mu_K - \delta_K)Kdt + \sigma_K K dz_K \tag{7.6}$$

where μ_K is the drift term (the expected cost escalation), δ_K is the payout rate on similar investment cost businesses, σ_K is the volatility of the investment cost, and the correlation between the Wiener processes is ρ. Assuming that the exercise price of the first (compound) option, D, is expressed as a fixed proportion (Q%) of K, i.e., D=QK, Carr (1988) gives the solution for the European compound exchange call option:

$$w_C(V,K,D,\tau,\tau') = Ve^{-\delta_V \tau} B\left(a_1, b_1; \sqrt{\frac{\tau'}{\tau}}\right) - Ke^{-\delta_K \tau} B\left(a_2, b_2; \sqrt{\frac{\tau'}{\tau}}\right) - De^{-\delta_K \tau'} N(a_2) \tag{7.7}$$

where

$$a_1 = \frac{\ln(X/X^*) + (\delta_K - \delta_V + 0.5\sigma^2)\tau'}{\sigma\sqrt{\tau'}},\ a_2 = a_1 - \sigma\sqrt{\tau'}, \tag{7.8}$$

$$b_1 = \frac{\ln(X) + (\delta_K - \delta_V + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}},\ b_2 = b_1 - \sigma\sqrt{\tau}, \tag{7.9}$$

$$X = \frac{V}{K},\ \sigma = \sqrt{\sigma_V^2 - 2\rho\sigma_V\sigma_K + \sigma_K^2}.$$

N(.) and B(, ;) are the standard normal cumulative univariate and bivariate distributions. At time τ’, one would exercise the compound call and obtain the underlying European exchange call option if the critical price ratio is such that $X^* < X_{\tau'}$. The critical price ratio, X^*, above which the compound option should be exercised at time τ’ can be obtained using the solution for the European exchange call option:

$$X^* e^{(\delta_K - \delta_V)(\tau - \tau')} N(b_1^*) - N(b_2^*) = Q \qquad (7.10)$$

where

$$b_1^* = \frac{\ln(X^*) + (\delta_K - \delta_V + 0.5\sigma^2)(\tau - \tau')}{\sigma\sqrt{\tau - \tau'}}, \quad b_2^* = b_1^* - \sigma\sqrt{\tau - \tau'} \qquad (7.11)$$

Figure 7.2

	A	B	C
1	**EUROPEAN COMPOUND EXCHANGE OPTION**		
2			
3	DEVELOPMENT TIME τ'	1	
4	INVESTMENT TIME τ	2	
5	V YIELD	0.04	
6	K YIELD	0.04	
7	VALUE VOLATILITY	0.20	
8	K VOLATILITY	0.20	
9	CORRELATION	0.50	
10	EXCHANGE VOLATILITY	0.20	SQRT(B7^2+B8^2-2*B9*B7*B8)
11	V	100.00	
12	D	20.00	
13	K	80.00	
14	X=V/K	1.25	B11/B13
15	**REAL OPTION VALUE**	**7.1768**	B11*EXP(-B5*B4)*B30-B13*EXP(-B6*B4)*B31-B12*EXP(-B6*B3)*B29
16	τ-τ'	1.00	B4-B3
17	X*	1.24	
18	d1	1.22	(LN(B11/B13)+((B5-B6+0.5*B10^2)*B16))/(B10*SQRT(B16))
19	d2	1.02	B18-B10*SQRT(B16)
20	N1	0.89	NORMSDIST(B18)
21	N2	0.85	NORMSDIST(B19)
22	EQ 7.10	0.25	B17*EXP(-B5*B16)*B20-EXP(-B6*B16)*B21
23	EQ 7.10-D/K	0.00	B22-(B12/B13)
24	ρ(τ'/τ)	0.71	SQRT(B3/B4)
25	d1,t1	0.12	(LN(B14/B17)+(B6-B5+0.5*B10^2)*B3)/(B10*SQRT(B3))
26	d1,t2	0.93	(LN(B14)+(B6-B5+0.5*B10^2)*B4)/(B10*SQRT(B4))
27	d2,t1	-0.08	B25-B7*SQRT(B3)
28	d2,t2	0.65	B26-B7*SQRT(B4)
29	N2	0.47	NORMSDIST(B27)
30	B1	0.53	BiVariateNormalCDF(B25,B26,B24)
31	B2	0.44	BiVariateNormalCDF(B27,B28,B24)
32			
33	**The first five inputs are the D and K timing estimates, the**		
34	**value and cost yields and volatilities, and correlation.**		
35	**After calculating the exchange volatility, the next three inputs are V, D and K estimates.**		
36	**Real call option value assumes X* is the value above which D should be paid at τ'.**		
37	**USE TOOLS/SOLVER, SETTING B23=0 BY CHANGING B17.**		

The European sequential option model assumes that D cannot occur until τ' and K is only paid or exercised at τ. This is mechanical, and does not allow management any flexibility, except to choose whether to make the investment decisions. The input parameters are chosen so that the ROV is the same as in the previous figure.

Different inputs for K yield and volatility, and correlation, will yield different results.

7.2 AMERICAN FINITE SEQUENTIAL EXCHANGE REAL OPTIONS

A finite American sequential exchange option allows for management choice on the timing of the ultimate investment. If investing in an early project stage (such as decontamination) allows further investment opportunities, exercising the first compound option may provide an option to exchange at any time thereafter construction cost for the value of the ultimate property.

This compound exchange option process is simplified by assuming that the first compound option is European, so it can be only exercised at a fixed time, τ'. Then the early exercise premium for an underlying American exchange option is modelled, which provides the right to exchange one asset for another at any time before expiration τ. The value of such an American compound exchange option is simply the sum of the Carr European compound exchange option and the early exercise premium for the underlying American exchange option between τ and τ'.

Let w_C denote a European compound call on a European exchange call option as specified in the previous section. When the asset to be received in the exchange pays a sufficiently large yield, there is always a positive probability that the underlying American exchange option should be exercised prior to expiration. Considering the early exercise premium for the American exchange call option with time horizon $[\tau', \tau]$, the value of the European compound option on American exchange call option, W_C, becomes

$$W_C = w_C + W(\tau - \tau') - w_1(\tau - \tau') \qquad (7.12)$$

where w_C is the Carr European compound exchange call option. W is an American exchange call and w_1 is a European exchange call option.

$$W(\tau - \tau') = \max\left[U^* - \frac{\left(U^* - W_2^*\right)^2}{U^* - w_1}, V - K\right] \tag{7.13}$$

Maximizing W_2, a twice-exercisable exchange call option, and employing an upper bound of a perpetual American exchange call option yields the CEA (confined exponential approximation) model.

U* for CEA equals the value of a perpetual American exchange call option:

$$W_\infty = \frac{K}{\beta_1 - 1}\left(\frac{V}{V_\infty^*}\right)^{\beta_1} \tag{7.14}$$

Here, $V_\infty^*(W)$ denotes the optimal exercise price at which the perpetual American exchange call option should be exercised, and its value is equal to

$$V_\infty^*(W) = \frac{\beta_1}{\beta_1 - 1}K \tag{7.15}$$

where

$$\beta_1 = \frac{-(\delta_K - \delta_V - 0.5\sigma^2) + \sqrt{(\delta_K - \delta_V - 0.5\sigma^2)^2 + 2\sigma^2\delta_K}}{\sigma^2} \tag{7.16}$$

Figure 7.3 shows that this real sequential American exchange approximation ($7.86) is higher than the Geske-Margrabe compound exchange value ($7.18), since the option can be exercised anytime after the interim stage. Also this real option value is sensitive to increases in the time horizon, approaching the American perpetual exchange value as τ approaches infinity. The value of the management flexibility to decide on the optimal timing of the ultimate option exercise is worth around 9.5% more than the alternative European setting with the same parameters. This management flexibility would be worth more with higher V, lower K, higher asset yield, and lower correlation of development value and development cost, indicating the practical considerations in using the more complex model.

Figure 7.3

	A	B	C
1	***AMERICAN FINITE SEQUENTIAL EXCHANGE OPTION***		
2			
3	***INPUT***		
4	τ'	1	
5	τ	2	
6	σ1	0.2000	
7	σ2	0.2000	
8	ρ	0.5000	
9	V	100	
10	K	80.000	
11	D	20.000	
12	δ1	0.04	
13	δ2	0.04	
14	***OUTPUT***		
15	D+I	100.000	
16	σ	0.2000	SQRT(B6^2-2*B6*B7*B8+B7^2)
17	Critical Prices X*	1.2332	
18	V*	98.73	
19		0.0000	B17*EXP((B13-B12)*(B5-B4))*B26-B27-B23
20		0.0000	B18-B10-B18*EXP(-0.5*B12*(B5-B4))*B41+B10*EXP(-0.5*B13*(B5-B4))*B42
21	**Carr(1988)**	0.0000	ABS(B19)+ABS(B20)
22	X=P/K	1.25	B9/B10
23	Q=D/K	0.2500	B11/B10
24	b*1(X*t1)	1.1480	(LN(B28)+(B13-B12+0.5*B16^2)*(B5-B4))/(B16*SQRT(B5-B4))
25	b*2(X*t1)	0.9480	B24-B16*SQRT(B5-B4)
26	N(b*1X*t1)	0.8745	NORMSDIST(B24)
27	N(b*2X*t1)	0.8284	NORMSDIST(B25)
28	X*(Critical Price Ratio)	1.2332	
29	a1(X/X*,t1)	0.1677	(LN(B22/B28)+(B13-B12+0.5*B16^2)*B4)/(B16*SQRT(B4))
30	a2(X/X*,t1)	-0.0323	B29-B16*SQRT(B4)
31	b1(X,T)	0.9304	(LN(B22)+(B13-B12+0.5*(B16^2))*B5)/(B16*SQRT(B5))
32	b2(X,T)	0.6475	B31-B16*SQRT(B5)
33	ρ (τ'/τ)	0.7071	SQRT(B4/B5)
34	B[a1,b1;rho]	0.5438	BiVariateNormalCDF(B29,B31,B33)
35	B[a2,b2;rho]	0.4559	BiVariateNormalCDF(B30,B32,B33)
36	N(a2)	0.4871	NORMSDIST(B30)
37	**Carr European Compound = W**	**7.1697**	B9*EXP(-B12*B5)*B34-B10*EXP(-B13*B5)*B35-B23*B10*EXP(-B13*B4)*B36
38	***APPROX. AMERICAN COMPOUND EXCHANGE OPTION***		
39	d1*(0.5Vt1)	1.5581	(LN(B43/B10)+(B13-B12+0.5*B16^2)*0.5*(B5-B4))/(B16*SQRT(0.5*(B5-B4)))
40	d2*(0.5Vt1)	1.4167	B39-B16*SQRT(0.5*(B5-B4))
41	N(d1*Vt1)	0.9404	NORMSDIST(B39)
42	N(d2*Vt1)	0.9217	NORMSDIST(B40)
43	V*(Critical Value)	98.73	B18
44	c1(V/V*,0.5(T-t))	0.1611	(LN(B9/B43)+(B13-B12+0.5*B16^2)*0.5*(B5-B4))/(B16*SQRT(0.5*(B5-B4)))
45	c2(V/V*,0.5(T-t))	0.0197	B44-B16*SQRT(0.5*(B5-B4))
46	d1(V/K,T-t)	1.2157	(LN(B9/B10)+(B13-B12+0.5*B16^2)*(B5-B4))/(B16*SQRT(B5-B4))
47	d2(V/K,T-t)	1.0157	B46-B16*SQRT(B5-B4)
48	ρ	0.7071	SQRT(0.5*(B5-B4)/(B5-B4))
49	B[-c1,d1,-ρ)	0.3335	BiVariateNormalCDF(-B44,B46,-B48)
50	B[-c2,d2,-ρ)	0.3498	BiVariateNormalCDF(-B45,B47,-B48)
51	N(c1)	0.5640	NORMSDIST(B44)
52	N(c2)	0.5079	NORMSDIST(B45)
53	Twice Exercisable = W2(T-t1)	20.6085	B9*EXP(-B12*(B5-B4))*B49-B10*EXP(-B13*(B5-B4))*B50+B9*EXP(-B12*0.5*(B5-B4))*B51-B10*EXP(-B13*0.5*(B5-B4))*B52
54	d1(T-t)	1.2157	(LN(B9/B10)+(B13-B12+0.5*B16^2)*(B5-B4))/(B16*SQRT(B5-B4))
55	d2(T-t)	1.0157	B54-B16*SQRT(B5-B4)
56	N(d1)	0.8880	NORMSDIST(B54)
57	N(d2)	0.8451	NORMSDIST(B55)
58	European Exchange = W1(T-t1)	20.3552	B9*EXP(-B12*(B5-B4))*B56-B10*EXP(-B13*(B5-B4))*B57
59	β	2.0000	(-(B13-B12-0.5*B16^2)+SQRT((B13-B12-0.5*B16^2)^2+2*B16^2*B13))/(B16^2)
60	V*(Critical Value)	200.0000	B59*B15/(B59-1)
61	Perpetual Exchange = W3(D= 0)	20.0000	(B10/(B59-1))*(B9/B60)^B59
62	CEA American (T-t1)	21.0423	B61+(B53-B61)^2/(B58-B61)
63	**Confined Exponential Approximation for the American Compound Exchange Option**		
64	***American Compound Exchange CEA***	***7.8568***	B37+B62-B58
65	The first five inputs are the D & K timing estimates, and the		
66	value and investment cost volatility (and correlation) estimates.		
67	The next three inputs are V, K and D%K estimates.		
68	The last two inputs are the "on-going" yields of V and (K&D).		
69	Use Tools/Solver, setting B19=0 by changing B15:B16.		

7.3 PERPETUAL AMERICAN MULTI-STAGE SEQUENTIAL REAL OPTION

This section studies a project comprising a multiple sequential investment opportunity. The analytical solution depends on assuming a probability of catastrophic failure at each investment stage that declines in value as the project nears completion, which is a characteristic of many R&D, exploration and infrastructure projects. This real sequential investment opportunity is a set of distinct, ordered investments that have to be made before the project can be completed. The project can then be interpreted as a collection of investment stages, such that no stage investment, except the first, can be started until the preceding stage has been completed. Success at each stage is not guaranteed because of the possibility of a catastrophic failure that reduces the option value to zero. The project value is realized when all the stages have been successfully completed. A typical four-stage opportunity involves: (i) undertaking basic research. (ii) developing a marketable product, (iii) testing its viability and (iv) implementing the infrastructure for launch and delivery. Multiple sequential investment opportunities are common amongst industries as diverse as oil exploration and mining, aircraft manufacture, pharmaceuticals and consumer electronics.

Schwartz and Moon (2000) model a new drug development process which consists of four distinct phases, each with a positive probability of failure, although not necessarily declining over time. Cortazar, Schwartz and Casassus (2003) describe four natural resource exploration stages of a project with technical success probability increasing over each phase, and then a production phase which is subject to commodity price uncertainty. Pennings and Sereno (2011) study the development path of a new medicine over seven phases, with a probability of failure declining over time.

Making an investment at a stage depends on whether the prevailing project value is of sufficient magnitude to economically justify committing the investment cost, or whether it is more desirable to wait for more favorable conditions. After making the stage investment, there is no absolute guarantee that the stage will be successfully completed, because of the presence of irresolvable difficulties in converting intentions into reality owing to technological, technical or market impediments. This means that the stage investment opportunity is subject to a catastrophic failure that causes the option value to be entirely destroyed, and the project as an entity becomes irredeemably lost. There are three sources of uncertainty, the stochastic project value and the investment cost, and the probability of a catastrophic failure, which are considered in a closed-form rule for the investment decision at each of the project stages.

Other authors simplify the multiple investment stage problems for obtaining a meaningful solution. Building on the valuation of sequential exchange opportunities by (Carr 1988), (Lee and Paxson 2001) use an element of European style compound options (and approximation of an American option phase) for formulating a two-stage sequential investment. (Brach and Paxson 2001) examine a two-stage sequential investment opportunity similar to the formulation currently under study but they confine their attention more to valuation. (Childs and Triantis 1999) formulate a multiple sequential investment model with interaction and obtain a solution through using a trinomial lattice. For all of these expositions, the solution is either not analytical or is restricted to only two stages.

Cassimon et al. (2004) study American-type investment options, but provide a solution based on the complex multivariate distribution available in some mathematical programmes. Building on Adkins and Paxson (2011), Adkins and Paxson (2013) suggest an analytic solution for N-stage sequential investments.

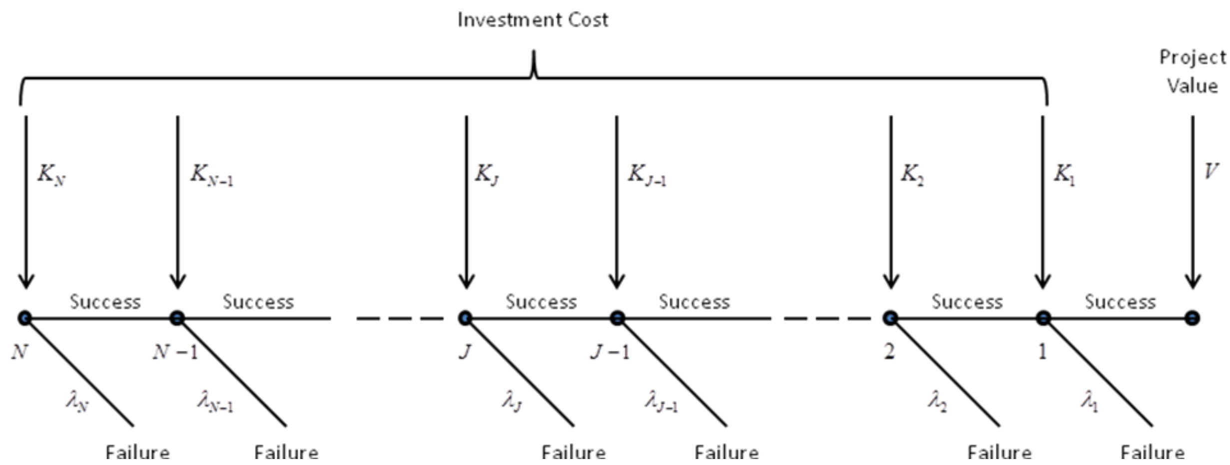
Consider an investment project made up of a discrete number of sequential stages, each involving an individual non-zero investment cost. The project as an entity is not fully implemented and the project value not realized until all of the sequential stages have been successfully completed. Each successive investment stage relies on the successful completion of the investment made at the preceding stage, but the stage timing is not specified. Each investment stage is ordered by the number J of remaining stages, including the current one, until project completion. The decision making position is first examined for the ultimate stage where $J=1$, and then by replication for the preceding stages, incrementally. At the ultimate stage, the decision whether or not to make an investment in a real asset is decided by whether or not the option value at $J=1$ fully compensates the expected net present value of the cash flow stream rendered by the asset. At the penultimate stage $J=2$, whether to make an expenditure to obtain the investment option at $J=1$ depends on whether or not the option value at $J=2$ fully compensates for the net option value at $J=1$. This procedure is then replicated incrementally for stages greater than 2.

A representation of the sequential investments process for a $J=N$ stage project is illustrated in Figure 7.4. This figure reveals the ordered sequence of stage investments comprising the project. It also shows that after an investment, the possible outcomes are success and failure. If all the stage outcomes are successful, then the entire project is successfully completed and its value can be realized. However, there is a possibility of failure at each stage. Although the investment is committed, the stage may not be successfully completed owing to fundamental irresolvable technical or market impediments, in which case, the option value instantly falls to zero and the project is abandoned without any value. The probability of failure at stage J is denoted by λ_J where $0 \leq \lambda_J < 1 \, \forall J$.

Situations where an investment can produce an innovative breakthrough and generate an unanticipated increase in the project value are ignored. Also, other forms of optionality, such as terminating a project before completion for its abandonment value, are not considered.

Figure 7.4

Sequential Investment Process



The value of the project is defined by V. The investment expenditure made at any stage J is denoted by K_J for all possible values of J. Both the project value and the set of investment expenditures are treated as stochastic. It is assumed that they are individually well described by the geometric Brownian motion process:

$$\mathrm{d}X = \alpha_X X \mathrm{d}t + \sigma_X X \mathrm{d}z_X \tag{7.17}$$

for $X \in \{V, K_J \, \forall J\}$, where α_X represent the respective drift parameters, σ_X the respective instantaneous volatility parameter, and $\mathrm{d}z_X$ the respective increment of a standard Wiener process. Dependence between any two of the factors is represented by the covariance term; so, for example, the covariance between the real asset value and the investment expenditure at stage J is specified by:

$$\mathrm{Cov}[\mathrm{d}V, \mathrm{d}K_J] = \rho_{VK_J} \sigma_V \sigma_{K_J} \mathrm{d}t.$$

Different stages may have different factor volatilities and correlations. The risk-free rate is r, and the investment expenditure at each stage K is assumed to be instantaneous.

<u>One-Stage Model</u>

The stage $J=1$ model represents the investment opportunity for developing a project value V following the investment cost K_1, given that the research effort may fail totally with probability λ_1. The value F_1 of the investment opportunity at stage $J=1$ depends on the project value and the investment cost, so $F_1 = F_1(V, K_1)$. By Ito's lemma, the risk neutral valuation relationship is:

$$\tfrac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 F_1}{\partial V^2} + \tfrac{1}{2}\sigma_1^2 K^2 \frac{\partial^2 F_1}{\partial K_1^2} + \rho_{VK_1}\sigma_V\sigma_{K_1} VK_1 \frac{\partial^2 F_1}{\partial V \partial K_1}$$
$$+ \theta_V V \frac{\partial F_1}{\partial V} + \theta_{K_1} K_1 \frac{\partial F_1}{\partial K_1} - (r + \lambda_1) F_1 = 0, \tag{7.18}$$

where the θ_X for $X \in \{V, K_J \ \forall J\}$ denote the respective risk neutral drift rate parameters. The generic solution is the two-factor power function:

$$F_1 = A_1 V^{\beta_1} K_1^{\eta_{11}}, \tag{7.19}$$

where β_1 and η_{11} denote the generic unknown parameters for the two factors, project value and investment cost, and A_1 denotes a generic unknown coefficient. The power parameter values satisfy the characteristic root function:

$$Q_1(\beta_1, \eta_{11})$$
$$= \tfrac{1}{2}\sigma_V^2\beta_1(\beta_1 - 1) + \tfrac{1}{2}\sigma_{K_1}^2\eta_{11}(\eta_{11} - 1) + \rho_{VK_1}\sigma_V\sigma_{K_1}\beta_1\eta_{11} + \theta_V\beta_1 + \theta_{K_1}\eta_{11} - (r + \lambda_1) = 0. \tag{7.20}$$

The threshold levels for the project value and the investment cost signaling the optimal exercise for the investment option at stage $J=1$ are denoted by $\hat{V}_1$ and $\hat{K}_{11}$, respectively. The value matching relationship describes the conservation equality at optimality that the option value $\hat{F}_1 = F_1(\hat{V}_1, \hat{K}_{11})$ exactly compensates the net asset value $\hat{V}_1 - \hat{K}_{11}$. Then:

$$A_{12}\hat{V}_1^{\beta_{12}}\hat{K}_{11}^{\eta_{112}} = \hat{V}_1 - \hat{K}_{11}. \tag{7.21}$$

There are two associated smooth pasting conditions, one for each factor, which can be expressed as:

$$A_{12}\hat{V}_1^{\beta_{12}}\hat{K}_{11}^{\eta_{112}} = \frac{\hat{V}_1}{\beta_{12}} = -\frac{\hat{K}_{11}}{\eta_{102}}. \tag{7.22}$$

Since the option value is always non-negative, $A_{12} \geq 0$. We conjecture that $\beta_{12} \geq 0$ and $\eta_{112} < 0$, and $\beta_{12} + \eta_{112} = 1$. Replacing η_{112} by $1 - \beta_{12}$ yields:

$$Q_1\left(\beta_{12}, 1-\beta_{12}\right) = \tfrac{1}{2}\sigma_1^2\beta_{12}\left(\beta_{12}-1\right) + \beta_{12}\left(\theta_V - \theta_{K_1}\right) - \left(r + \lambda_1 - \theta_{K_1}\right) = 0, \tag{7.23}$$

where $\sigma_1^2 = \sigma_V^2 + \sigma_{K_1}^2 - 2\rho_{V,K_1}\sigma_V\sigma_{K_1}$. Further, the threshold levels are related by:

$$\hat{V}_1 = \frac{\beta_{12}}{\beta_{12}-1}\hat{K}_{11}, \tag{7.24}$$

with $A_{12} = \beta_{12}^{-\beta_{12}}\left(\beta_{12}-1\right)^{\beta_{12}-1}$. The MEF (markup effective factor) is simply $\frac{\beta_{12}}{\beta_{12}-1} = \hat{V}_1 / \hat{K}_{11}$.

<u>Two-Stage Model</u>

At the preceding stage, $J = 2$, the viability of committing an investment K_2 to acquire the option to invest F_1 is compared to the value of the compound option F_2 with the net benefits $F_1 - K_2$. F_2 depends on the three factors V, K_1 and K_2, so $F_2 = F_2\left(V, K_1, K_2\right)$. By Ito's lemma, the risk neutral valuation relationship for F_2 is:

$$\begin{aligned}
&\tfrac{1}{2}\sigma_V^2V^2\frac{\partial^2 F_2}{\partial V^2} + \tfrac{1}{2}\sigma_{K_1}^2K_1^2\frac{\partial^2 F_2}{\partial K_1^2} + \tfrac{1}{2}\sigma_{K_2}^2K_2^2\frac{\partial^2 F_2}{\partial K_2^2} \\
&+\rho_{V,K_1}\sigma_V\sigma_{K_1}VK_1\frac{\partial^2 F_2}{\partial V\partial K_1} + \rho_{V,K_2}\sigma_V\sigma_{K_2}VK_2\frac{\partial^2 F_2}{\partial V\partial K_2} + \rho_{K_1,K_2}\sigma_{K_1}\sigma_{K_2}K_1K_2\frac{\partial^2 F_2}{\partial K_1\partial K_2} \\
&+\theta_V V\frac{\partial F_2}{\partial V} + \theta_{K_2}K_2\frac{\partial F_2}{\partial K_2} + \theta_{K_1}K_1\frac{\partial F_2}{\partial K_1} - \left(r + \lambda_2\right)F_2 = 0.
\end{aligned} \tag{7.25}$$

The solution to (7.25) is a product power function, with generic form:

$$F_2 = A_2V^{\beta_{24}}K_1^{\eta_{21}}K_2^{\eta_{22}}, \tag{7.26}$$

where β_2, η_{21} and η_{22} denote the generic unknown parameters for the three factors, project value and investment expenditure at stage-one and -two respectively, and A_2 denotes an unknown coefficient.

The stage-two threshold levels signaling an optimal exercise are represented by $\hat{V}_2$, $\hat{K}_{21}$ and $\hat{K}_{22}$ for V, K_1 and K_2, respectively. The set $\{\hat{V}_2, \hat{K}_{21}, \hat{K}_{22}\}$ forms the boundary that discriminates between the "exercise" decision and the "wait" decision. The equilibrium amongst the threshold levels is the value matching relation that is expressed as:

$$A_{24}\hat{V}_2^{\beta_{24}}\hat{K}_{12}^{\eta_{214}}\hat{K}_{22}^{\eta_{224}} = A_{12}\hat{V}_2^{\beta_{12}}\hat{K}_{12}^{1-\beta_{12}} - \hat{K}_{22}, \tag{7.27}$$

where A_{12} and β_{12} are known from the evaluation for stage-one. There are three smooth pasting conditions, one for each of the three factors V, K_1 and K_2, respectively, can be expressed as:

$$\beta_{24}A_{24}\hat{V}_2^{\beta_{24}}\hat{K}_{12}^{\eta_{214}}\hat{K}_{22}^{\eta_{224}} = \beta_{12}A_{12}\hat{V}_2^{\beta_{12}}\hat{K}_{12}^{1-\beta_{12}}, \tag{7.28}$$

$$\eta_{214}A_{24}\hat{V}_2^{\beta_{24}}\hat{K}_{12}^{\eta_{214}}\hat{K}_{22}^{\eta_{224}} = \left(1-\beta_{12}\right)A_{12}\hat{V}_2^{\beta_{12}}\hat{K}_{12}^{1-\beta_{12}}, \tag{7.29}$$

$$\eta_{224}A_{24}\hat{V}_2^{\beta_{24}}\hat{K}_{12}^{\eta_{214}}\hat{K}_{22}^{\eta_{224}} = -\hat{K}_{22}. \tag{7.30}$$

As a simplification in calculating the solution values, let $\phi_{24} = \beta_{24}/\beta_{12} \geq 0$, then by using the substitutions $\beta_{24} = \phi_{24}\beta_{12}$, $\eta_{214} = \left(1-\beta_{12}\right)\phi_{24}$ and $\eta_{224} = 1-\phi_{24}$, the quadratic function Q_2 can be expressed as:

$$\begin{aligned} &Q_2\left(\beta_{12}\phi_{24}, \left(1-\beta_{12}\right)\phi_{24}, 1-\phi_{24}\right) \\ &\quad = \tfrac{1}{2}\phi_{24}\left(\phi_{24}-1\right)\sigma_2^2 + \phi_{24}\left\{\tfrac{1}{2}\beta_{12}\left(\beta_{12}-1\right)\sigma_1^2 + \beta_{12}\left(\theta_V - \theta_{K_1}\right) + \left(\theta_{K_1} - \theta_{K_2}\right)\right\} \\ &\qquad\qquad - \left(r + \lambda_2 - \theta_{K_2}\right) = 0, \end{aligned} \tag{7.31}$$

where

$$\begin{aligned} \sigma_2^2 &= \beta_{12}^2\sigma_V^2 + \left(1-\beta_{12}\right)^2\sigma_{K_1}^2 + \sigma_{K_2}^2 \\ &\quad + 2\beta_{12}\left(1-\beta_{12}\right)\rho_{VK_1}\sigma_V\sigma_{K_1} - 2\beta_{12}\rho_{VK_2}\sigma_V\sigma_{K_2} - 2\left(1-\beta_{12}\right)\rho_{K_1K_2}\sigma_{K_1}\sigma_{K_2}. \end{aligned}$$

The value of ϕ_{24} is evaluated as the positive root of $Q_2 = 0$, , where β_{12} is the previously calculated stage-one solution. The values of β_{24}, η_{214} and η_{224} are then obtained from ϕ_{24} and β_{12}.

$$\begin{aligned}\hat{V}_2 &= \frac{\beta_{12}}{\beta_{12}-1}\left\{\frac{\phi_{24}\left(\beta_{12}-1\right)}{\phi_{24}-1}\right\}^{\frac{1}{\beta_{12}}} \hat{K}_{12}^{\frac{\beta_{12}-1}{\beta_{12}}} \hat{K}_{22}^{\frac{1}{\beta_{12}}} \\ &= \frac{\beta_{12}}{\beta_{12}-1}\left\{\frac{\beta_{24}\left(\beta_{12}-1\right)}{\beta_{24}-\beta_{12}}\right\}^{\frac{1}{\beta_{12}}} \hat{K}_{12}^{\frac{\beta_{12}-1}{\beta_{12}}} \hat{K}_{22}^{\frac{1}{\beta_{12}}}.\end{aligned} \tag{7.32}$$

For consistency with the Stage 1 markup factor, the Stage 2 markup effective factor, MEF, is simply $\hat{V}_2 / (\hat{K}_{12} + \hat{K}_{22})$.

The Q_2 function can be expressed as:

$$Q_2 = \tfrac{1}{2}\phi_{24}\left(\phi_{24}-1\right)\sigma_2^2 + \phi_{24}\left(r + \lambda_1 - \theta_{K_2}\right) - \left(r + \lambda_2 - \theta_{K_2}\right) = 0. \tag{7.33}$$

The parameter ϕ_{24}, which is required to be greater than one, is evaluated as the positive root of the quadratic function Q_2 (7.33).

Three-Stage Model

The extension of the sequential investment model to the $J = 3$ stage is achieved by replication. The value of the option to invest at the $J = 3$ stage F_3 depends on the project value V, and the investment costs at the $J = 1$, $J = 2$ and $J = 3$ stages, K_1, K_2 and K_3, respectively, so $F_3 = F_3\left(V, K_1, K_2, K_3\right)$. Using Ito's lemma, it can be shown that the risk neutral valuation relationship for F_3 is:

$$F_3 = A_3 V^{\beta_3} K_1^{\eta_{13}} K_2^{\eta_{23}} K_3^{\eta_{33}}, \tag{7.34}$$

with a simplified characteristic root equation (7.36).

$$\hat{V}_3 = \left\{ \frac{\phi_3}{\phi_3 - 1} \frac{\phi_2^{\phi_2}}{(\phi_2 - 1)^{(\phi_2 - 1)}} \left[\frac{\phi_1^{\phi_1}}{(\phi_1 - 1)^{\phi_1 - 1}} \right]^{\phi_2} \right\}^{1/\phi_1\phi_2} \hat{K}_{13}^{(\phi_1-1)/\phi_1} \hat{K}_{23}^{(\phi_2-1)/\phi_1\phi_2} \hat{K}_{33}^{1/\phi_1\phi_2} \quad (7.35)$$

The Stage 3 markup effective factor, MEF, is simply $\hat{V}_3 / (\hat{K}_{13} + \hat{K}_{23} + \hat{K}_{33})$.

where:

$$\begin{aligned} \tfrac{1}{2}\sigma_3^2 &= \tfrac{1}{2}\sigma_V^2\phi_2^2\phi_1^2 + \tfrac{1}{2}\sigma_{K_1}^2\phi_2^2(1-\phi_1)^2 + \tfrac{1}{2}\sigma_{K_2}^2(1-\phi_2)^2 + \tfrac{1}{2}\sigma_{K_3}^2 \\ &+ \rho_{VK_1}\sigma_V\sigma_{K_1}\phi_1(1-\phi_1)\phi_2^2 + \rho_{VK_2}\sigma_V\sigma_{K_2}\phi_1\phi_2(1-\phi_2)\beta_3\eta_{23} - \rho_{VK_3}\sigma_V\sigma_{K_3}\phi_1\phi_2 \\ &+ \rho_{K_1K_2}\sigma_{K_1}\sigma_{K_2}(1-\phi_1)\phi_2(1-\phi_2) - \rho_{K_1K_3}\sigma_{K_1}\sigma_{K_3}(1-\phi_1)\phi_2 - \rho_{K_2K_3}\sigma_{K_2}\sigma_{K_3}(1-\phi_2). \end{aligned}$$

$$Q_3 = \tfrac{1}{2}\sigma_3^2\phi_3(\phi_3 - 1) + \phi_3\left(r + \lambda_2 - \theta_{K_3}\right) - \left(r + \lambda_3 - \theta_{K_3}\right) = 0. \quad (7.36)$$

Numerical Illustrations

Figure 7.5 is a spreadsheet evaluation on an illustration involving a 3-stage sequential investment project. The set of probabilities of catastrophic failure at the stages adheres to the condition $\lambda_1 < \lambda_2 < \lambda_3$. Initially, the variances for the investment costs at the three stages have been set to be equal, the covariance terms between the four factors to equal zero, and the K thresholds are all assumed to be the same as the current value, so the threshold justifying investment at each stage is the ratio of $\hat{V}_{N..3,2.1}$ to the nominal investment costs remaining.

Figure 7.5

	A	B	C	D	E	F	G	H	I	J
1	**SEQUENTIAL MATRIX**	**3 STAGES**		**STAGE**	**VOLATILITY**	**ϕ**	**MEF**	**V^**	**V^-ΣK$_N$**	**ROV**
2	Project value V	100		1	0.2236	1.5177	2.9315	97.7142	64.3812	64.3812
3	θ theta_V	0.02		2	0.3238	1.4045	2.1572	143.8096	77.1436	37.9850
4	sigma_V	20%		3	0.4457	1.2255	2.0810	208.1001	108.1011	21.7932
5	Stage 1									
6	theta_K1	0								
7	sigma_K1	10%		Input the correlations:						
8	Failure probability: lambda	0%			V	K1	K2	K3		Volatility
9	Stage 2			V	100%	0%	0%	0%		20%
10	theta_K2	0		K1	0%	100%	0%	0%		10%
11	sigma_K2	10%		K2	0%	0%	100%	0%		10%
12	Failure probability: lambda	5%		K3	0%	0%	0%	100%		10%
13	Stage 3			Assumes all V, K correlations are the same as in D10, Ks are not correlated.						
14	theta_K3	0								
15	sigma_K3	10%								
16	Risk-free rate	5%								
17	Failure probability: lambda	10%								
18	**Threshold Levels**									
19	K1^	33.333								
20	K2^	33.333								
21	K3^	33.333								
22	**Calculations**									
23	Volatility	20%	10%	10%	10%					
24	Variance-Covariance Matrix									
25	V	0.0400	0.0000	0.0000	0.0000					
26	K1	0.0000	0.0100	0.0000	0.0000					
27	K2	0.0000	0.0000	0.0100	0.0000					
28	K3	0.0000	0.0000	0.0000	0.0100					
29	**Analysis**									
30	**Stage 1**									
31	w	1	column vector with 2 elements							
32		-1								
33	wT (w transpose)	1	-1	TRANSPOSE(B31:B32)						
34	Var-Covar*w	0.0400	MMULT(B25:C26,B31:B32)							
35		-0.0100								
36	sigma^2_1=wT*Var-Covar*w	0.0500	MMULT(B33:C33,B34:B35)							
37	vol_1	0.2236	SQRT(B36)							
38	theta_V - theta_K1	0.02	B3-B6							
39	r + Lamda_1 - theta_K1	5%	B16+B8-B6							
40	T1	0.1000	0.5-B38/B36							
41	T2	2.0100	B40^2+2*B39/B36							
42	phi_1	1.5177	B40+SQRT(B41)							
43	phi_1/(phi_1 - 1)	2.9315	B42/(B42-1)							
44	beta_1	1.5177	B42							
45	eta_11	-0.5177	1-B44							
46	V1^	97.7142	B43*B19							
47	A1	0.3775	((B42-1)^(B42-1))/(B42^B42)							
48	Mark-up Factor MEF	2.9315	B42/(B42-1)							
49	ROV 1	64.3812	IF(B2<B46,B47*(B2^B44)*(B19^B45),MAX(B46-B19,0))							
50	**Stage 2**									
51	w	1.5177	column vector with 3 elements							
52		-0.5177								
53		-1.0000								
54	wT	1.5177	-0.5177	-1.0000						
55	Var-Covar*w	0.0607								
56		-0.0052								
57		-0.0100								
58	sigma^2_2=wT*Var-Covar*w	0.1048	MMULT(B54:D54,B55:B57)							
59	vol_2	0.3238	SQRT(B58)							
60	r + lambda_1 - theta_K2	5%	B16+B8-B10							
61	r + lambda_2 - theta_K2	10%	B16+B12-B10							
62	T1	0.0230	0.5-B60/B58							
63	T2	1.9085	B62^2+2*B61/B58							
64	phi_2	1.4045	B62+SQRT(B63)							
65	B2	0.4303	((B64-1)^(B64-1))/(B64^B64)							
66	beta_2	2.1317	B64*B44							
67	eta_21	-0.7272	B64*B45							
68	eta_22	-0.4045	1-B64							
69	V2^	143.8096	((B64*(B19^(B42-1))*B20)/(B47*(B64-1)))^(1/B42)							
70	A2	0.1096	B65*B47^B64							
71	ROV 2	37.9850	IF(B2<B69,B70*(B2^B66)*(B19^B67)*(B20^B68),MAX(B69-B19-B20,0))							
72	Mark-up Factor_2	4.3143	(B64/(B47*(B64-1)))^(1/B44)							
73	V2^/(K1^+K2^) MEF	2.1572	B69/(B19+B20)							
74	Check: V2^/K2^	4.3143	B69/B20							
75	**Stage 3**									
76	w	2.1317	column vector with 4 elements							
77		-0.7272								
78		-0.4045								
79		-1.0000								
80	wT	2.1317	-0.7272	-0.4045	-1.0000					
81	Var-Covar*w	0.0853								
82		-0.0073								
83		-0.0040								
84		-0.0100								
85	sigma^2_3=wT*Var-Covar*w	0.1987	MMULT(B80:E80,B81:B84)							
86	vol_3	0.4457	SQRT(B85)							
87	r + lambda_2 - theta_K3	10%	B16+B12-B14							
88	r + lambda_3 - theta_K3	15%	B16+B17-B14							
89	T1	-0.0033	0.5-B87/B85							
90	T2	1.5100	B89^2+2*B88/B85							
91	phi_3	1.2255	B89+SQRT(B90)							
92	B3	0.5571	((B91-1)^(B91-1))/(B91^B91)							
93	beta_3	2.6123	B91*B66							
94	eta_31	-0.8911	B91*B67							
95	eta_32	-0.4957	B91*B68							
96	eta_33	-0.2255	1-B91							
97	V3^	208.1001	((B91*(B19^(-B67))*(B20^(-B68))*B21)/(B70*(B91-1)))^(1/B66)							
98	A3	0.0371	B92*B70^B91							
99	ROV 3	21.7932	IF(B2<B97,B98*(B2^B93)*(B19^B94)*(B20^B95)*(B21^B96),MAX(B97-B19-B20-B21,0))							
100	Mark-up Factor_3	6.2431	(B91/(B70*(B91-1)))^(1/B66)							
101	V2^/(K1^+K2^+K3^) MEF	2.0810	B97/(B19+B20+B21)							
102	Check: V2^/K2^	6.2431	B97/B21							

Figure 7.5 shows the results, using the backwardation principle so the $J=1$ stage is enumerated first, then the $J=2$ stage, and so on. The volatilities at each of the 3 stages, σ_1, σ_2, and σ_3 are evaluated, as are the parameters ϕ_J for $J=1$ and the mark-up factors for each of the 3 stages. Figure 7.5 illustrates that the volatilities at each stage increase in magnitude as the stage in question becomes more distant from completion. As expected, the parameter values ϕ_J are all greater than one. Note that with these parameter values, $\hat{V}$ increases with the distance of the stage from completion, and with the stage volatility, as does the excess of the $\hat{V}$ over the assumed investment cost over each stage. The real option value (ROV), which is the option to continue the next stages if $V<\hat{V}$, and otherwise V less the remaining investment costs (or zero), decreases with the distance from the final state. According to our results, the MEFs decrease in magnitude as the stage becomes more distant from completion, given these parameter values.

There are many other alternative combinations of changes in value volatility, investment cost volatility at each stage, and probability of failure at each stage that could be simulated, to illustrate the power and surprises of viewing sequential investment opportunities (and eventually investment requirements over stages) using this model.

SUMMARY

Sequential investment options are appropriate when an investment program involves several stages, such as required initial expenditures (equivalent to a real option premium), a second phase of required investment expenditures (D), and a final development phase, when then the project values (V) are realized. The essential aspect of this characterized program is that managers have a choice about whether to pay the interim expenditure, and then the development cost (K).

This chapter presents several real option valuation methods, starting with a simple European compound option, extended to a European compound exchange option. Then an approximated American finite sequential exchange option is considered. Finally, an American perpetual, and an American perpetual exchange option are presented, allowing for several stages of investment expenditures (and critical values which justify making those expenditures).

EXERCISES

EXERCISE 7.1 A bungalow in Putney has a restrictive covenant requiring the permission of the adjacent house owner in order to convert the bungalow into a modern house. That house owner has required extensive design and planning expenditures by the end of the next year prior to the construction of the new house. These expenditures and demolition costs are expected to be £150,000. Provisionally, a house of 3,000 square feet is envisioned (depends on design), which currently would be worth £300 per square foot, and costs £273 per square foot to build. The volatility of Putney houses is 20% and interest rates are 4%. The redevelopment must occur at the end of five years. What is the value of this bungalow site? At what house value should the construction start?

EXERCISE 7.2 A bungalow in Putney has a restrictive covenant requiring the permission of the adjacent house owner in order to convert the bungalow into a modern house. That house owner has required extensive design and planning expenditures by the end of the next year prior to the construction of the new house. These expenditures and demolition costs are expected to be £150,000, and along with construction costs are 50% correlated with housing prices. Provisionally, a house of 3,000 square feet is envisioned (depends on design), which currently would be worth £300 per square foot, and costs £273 per square foot to build. The volatility of Putney houses is 20%, the same as the construction costs, the “yield” on renting such a house is 4%, construction cost escalate by 4%, and interest rates are 4%. The provision planning permission to redevelop at any time after the planning expenditure will expire at the end of five years. What is the value of this bungalow site? At what house value should the construction start?

EXERCISE 7.3 Willard Wang wants to enjoy the fruits of his research involving two expenditures (both equal to 50) K_1 at the end of the first year and K_2 at end of the second year. The current research price is 15, continuous cost is 10, the interest rate is 4% and the research yield is 4%. The research volatility is 20%. What's today's value of WW's research, and at what research price should he make the first and second investment expenditures?

PROBLEMS

PROBLEM 7.4 Susie Wong wants to enjoy the fruits of her research involving two expenditures (both equal to 50) K_1 at the end of the first year and K_2 anytime until the end of the second year. The current research price is 15, continuous cost is 10, the interest rate is 4% and the research yield is 4%. The research volatility is 20%. What's today's value of Susie's research, and at what research price should she make the first and second investment expenditures?

PROBLEM 7.5 Pixit & Dindyck are planning a superior real options product PROD that will indicate optimal timing for perpetual multi-stage projects. They estimate that the current value of PROD is 81, costs 90 to make in three stages of equal investment amounts, has a volatility of 20%, interest rates are only 5%, while the yield on the PROD is expected to be 2%. The failure rate of the initial stage is 10%, the second stage 5%, and there is no failure expected for the final stage. Advise P&D on this adventure.

PROBLEM 7.6 Pixit & Dindyck are planning a superior real options product PROD that will indicate optimal timing for perpetual multi-stage projects. This time they estimate that the current value of PROD is 87, costs 90 to make in three stages of equal investment amounts, has a volatility of 20%, cost volatility is 34%, with a -9%

correlation of PROD value and cost. The yield on the PROD is expected to be 2%, with no yield for the investment cost. The failure rate of the initial stage is 10%, the second stage 5%, and there is no failure expected for the final stage. Advise P&D on this venture.

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