normal form games with complete information

part 4

the story so far

normal form (or strategic form) games

definition: players, strategy sets, payoffs

solutions:

- equilibrium in dominant strategies
- elimination of (strictly) dominated strategies
- IE(S)DS
- Nash equilibrium

other concepts: rationality, common knowledge, best-response,...

roadmap

mixed strategies
- definition
- examples
- dominant strategies
- Nash equilibria

references
sec 1.3 of Gibbons
Ch 8 of Dutta

mixed strategies

example: coordination game

1 \ 2	Book launch	Movie
Book launch	2,1	0,0
Movie	0,0	1,2

mixed strategies

example: coordination game

		50%	50%
	1 \ 2	Book launch	Movie
50%	Book launch	2,1	0,0
50%	Movie	0,0	1,2

mixed strategies

example: coordination game

		q	1-q
	1 \ 2	Book launch	Movie
p	Book launch	2,1	0,0
1-p	Movie	0,0	1,2

mixed strategies

definition

suppose a player has M pure strategies $s^1, s^2,\ldots, s^M$;
a mixed strategy for this player is a probability distribution over her pure strategies $(p^1, p^2,\ldots, p^M)$, where $p^k \geq 0$ for all k and $\Sigma_k p^k = 1$

rmk: A pure strategy is also a mixed strategy

the support of a mixed strategy p is the set of pure strategies with positive probability under p

mixed strategies

example: coordination game

		q	1-q
	1 \ 2	Book launch	Movie
p	Book launch	2,1	0,0
1-p	Movie	0,0	1,2

mixed strategies
payoff

the right way to evaluate the uncertainty in a mixed strategy is to take its **expected payoff**

mixed strategies

expected payoff - example

player 1's expected payoff in the coordination game is

$$2.p.q + 0.p.(1-q) + (1-p).q.0 + (1-p).(1-q).1$$

mixed strategies

expected payoff – general case

- each player i has M pure strategies $s_i^1, s_i^2, \ldots, s_i^M$ and plays a mixed strategy $(p_i^1, p_i^2, \ldots, p_i^M)$, where $p_i^k \geq 0$ for all k and $\Sigma_k p_i^k = 1$
- STEP 1: weight the payoff of each pure strategy profile by the probability with which it is played
- STEP 2: add up weighted payoffs

So,

- player i's expected payoff is $\Sigma_j \Sigma_k p_i^j . p_{-i}^k \, u_i(s_i^j, s_{-i}^k)$

mixed strategies

example: best response

1 \ 2	L	M1	M2	R
U	1,0	<u>4</u>,2	<u>2</u>,<u>4</u>	<u>3</u>,1
M	2,<u>4</u>	2,1	<u>2</u>,2	2,1
D	<u>4</u>,2	1,<u>4</u>	<u>2</u>,0	<u>3</u>,1

mixed strategies

best response

the expected payoff of a mixed strategy is simply a weighted average of the payoffs of the pure strategies in the support of this mixed strategy

a mixed strategy $(p^1, p^2, \ldots, p^M)$ is a best response of player i to s_{-i} iff each of the pure strategies in its support is itself a best response to s_{-i}

(in that case, **any** mixed strategy over that support will be a best response)

mixed strategies

example: best response

1 \ 2	L	M1	M2	R
U	1,0	4,2	2,4	3,1
M	2,4	2,1	2,2	2,1
D	4,2	1,4	2,0	3,1

mixed strategies

example: domination and IEDS

	1 \ 2	L (1/3)	M1 (1/3)	M2 (1/3)	R
1/2	U	1,0	4,2	2,4	3,1
	M	2,4	2,1	2,2	2,1
1/2	D	4,2	1,4	2,0	3,1

mixed strategies and domination

a mixed strategy may dominate pure strategies

adding mixed strategies has no impact on dominant strategy equilibria:

- if there is a pure strategy that dominates every other pure strategy, it also dominates every other mixed strategy
- if there is no dominant strategy in pure strategies, there cannot be one in mixed strategies either

adding mixed strategies may generate new IEDS solutions; however, if a IEDS solution in pure strategies exists, that solution will also be the mixed-strategy IEDS solution*

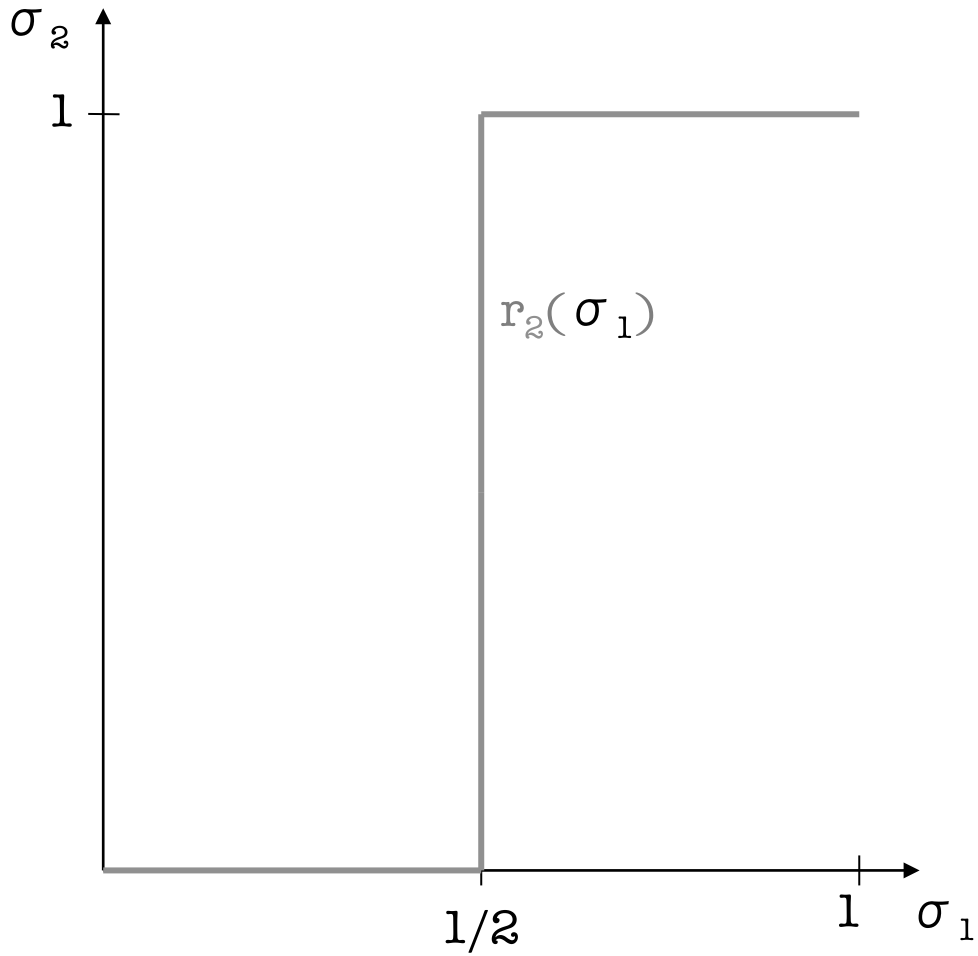
mixed strategies

example: Nash equilibrium

		σ_2	$1-\sigma_2$
	1 \ 2	Heads	Tails
σ_1	Heads	-1,+1	+1,-1
$1-\sigma_1$	Tails	+1,-1	-1,+1

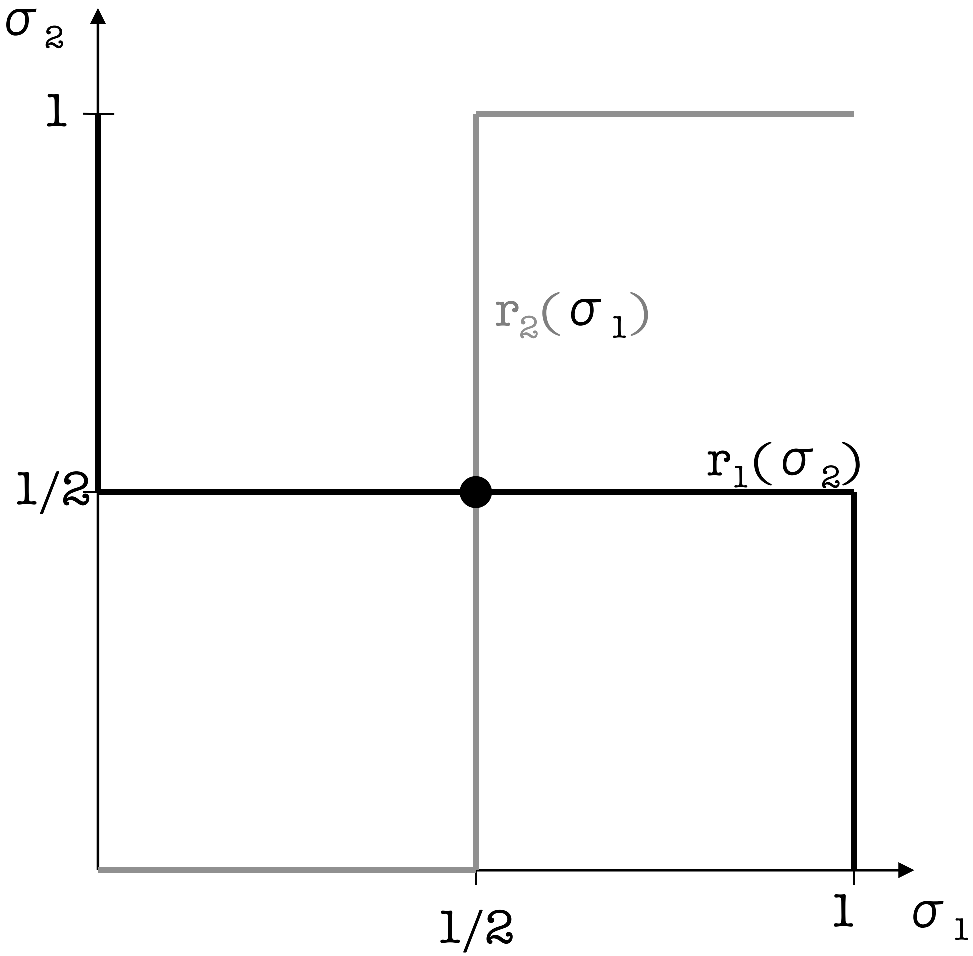
mixed strategies

example: Nash equilibrium



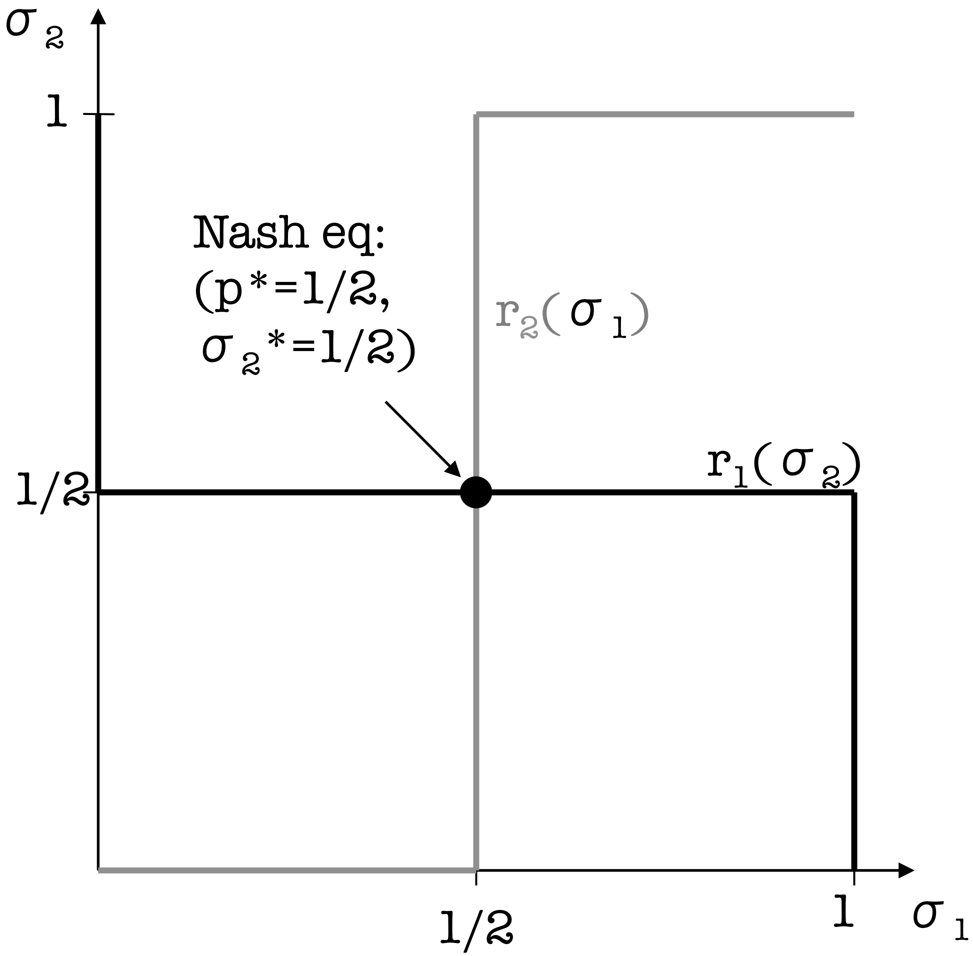
mixed strategies

example: Nash equilibrium



mixed strategies

example: Nash equilibrium



mixed strategies and Nash equilibrium (NE)

in every game there is always a NE in mixed strategies

are mixed strategies reasonable?

exercise

NE in the battle of sexes

	opera	prize fight
opera	2, 1	0, 0
prize fight	0, 0	1, 2

dynamic games with complete information

part 1

roadmap

extensive form games

- examples
- definition
- strategy

backwards induction

- backwards induction with perfect information
- vs. Nash equilibria
- vs. IEDS

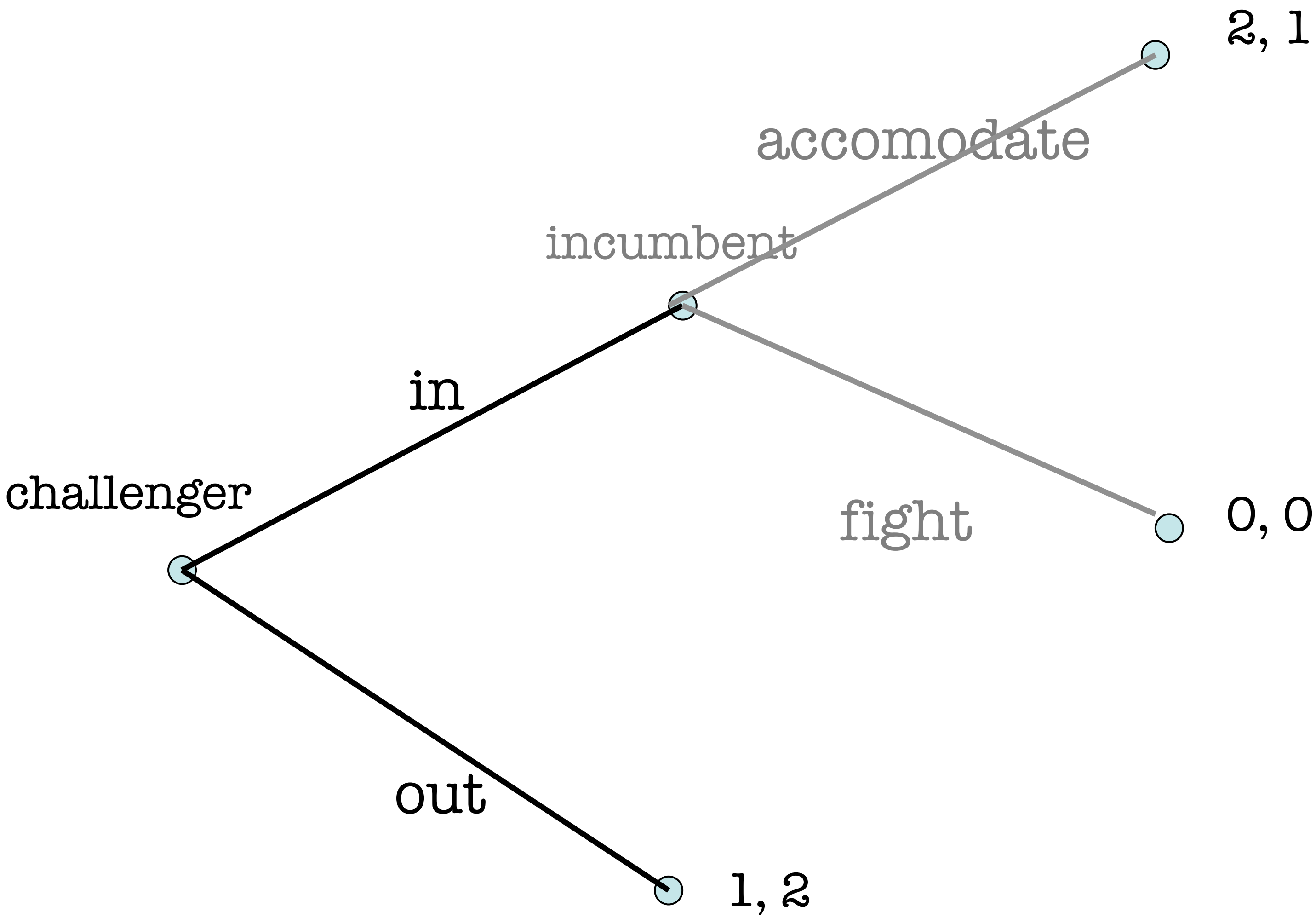
references

sec. 2.1 and 2.4 of Gibbons

Ch 2.2 and 11 of Dutta

extensive form games

example: entry game



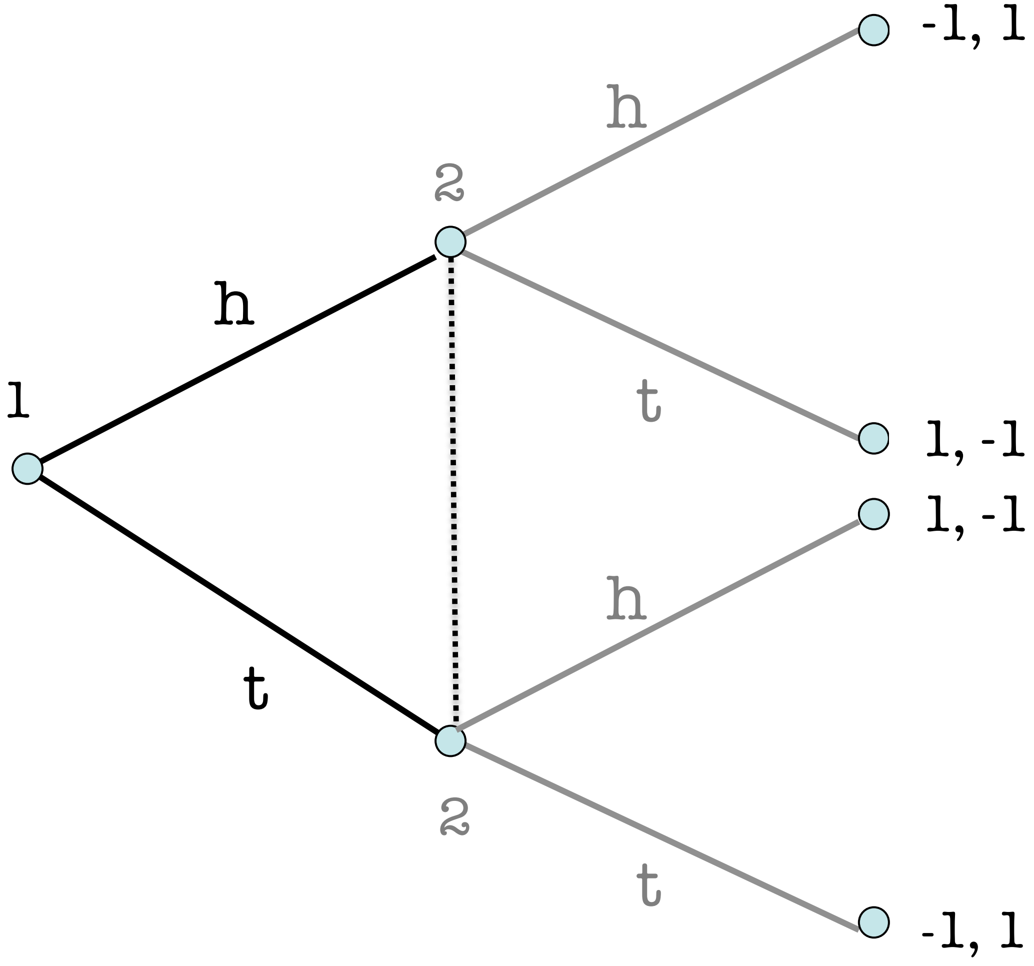
extensive form games

elements

- nodes
    - root (initial node)
    - decision nodes (where one player decides)
    - terminal nodes (with payoffs, game ends)
- branches (correspond to choices)
- information sets (sets of nodes that are indistinguishable by the player)

extensive form games

example: (sequential-move) matching pennies



extensive form games

the extensive form representation specifies

1. the players in the game
2. when each player has the move
3. what each player can do at each of her opportunities to move
4. what each player knows at each of her opportunities to move
5. the payoff received by each player for each combination of moves that could be chosen

extensive form games

perfect information

all previous moves are observed before the next move is chosen

(a player knows who has moved and how before she makes a decision)

def: no information set has multiple nodes

example: entry game

extensive form games

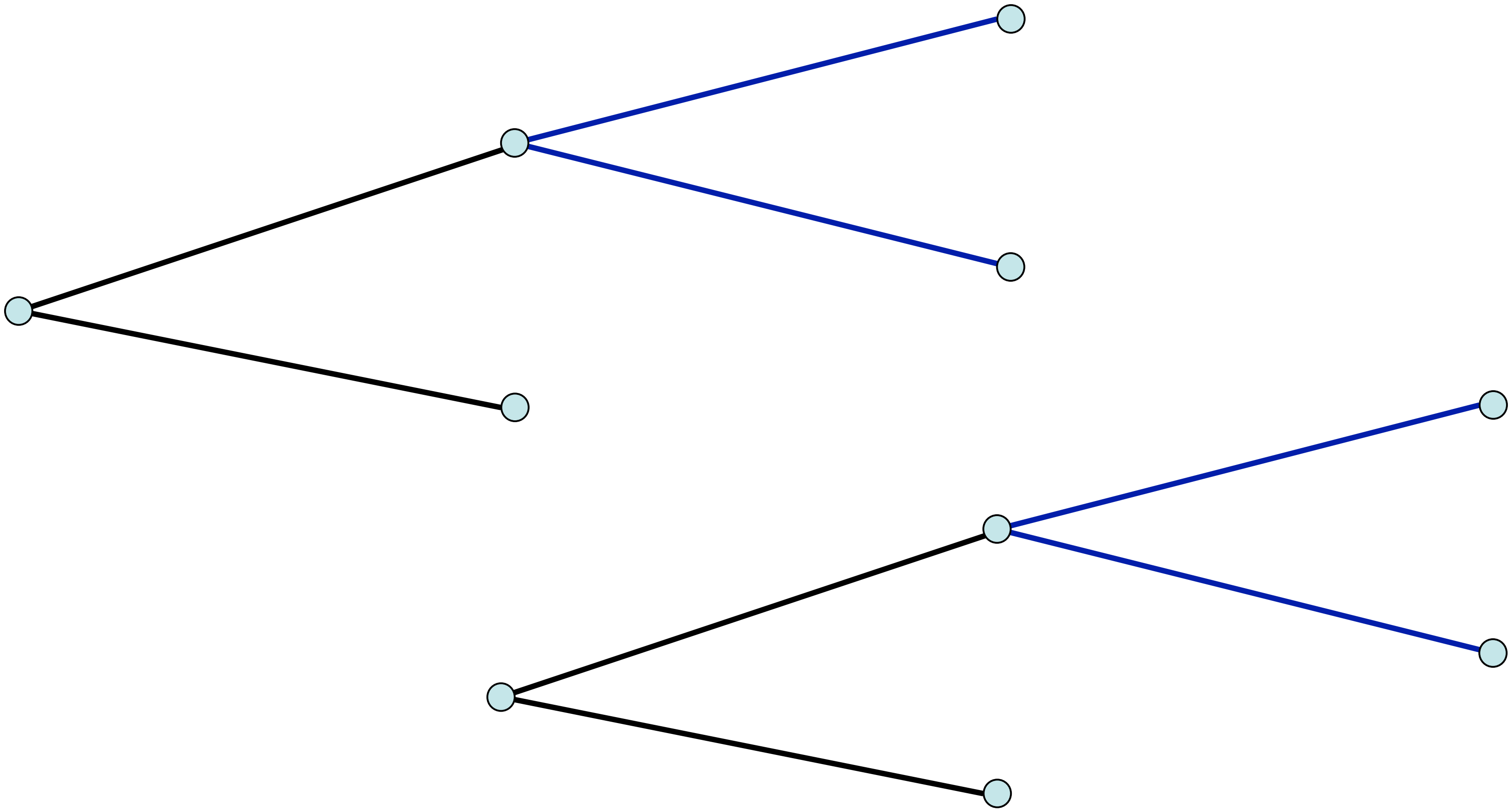
definition of a tree

a game tree is composed of nodes and branches with

1. a single starting point (root)
2. no cycles
3. one way to proceed

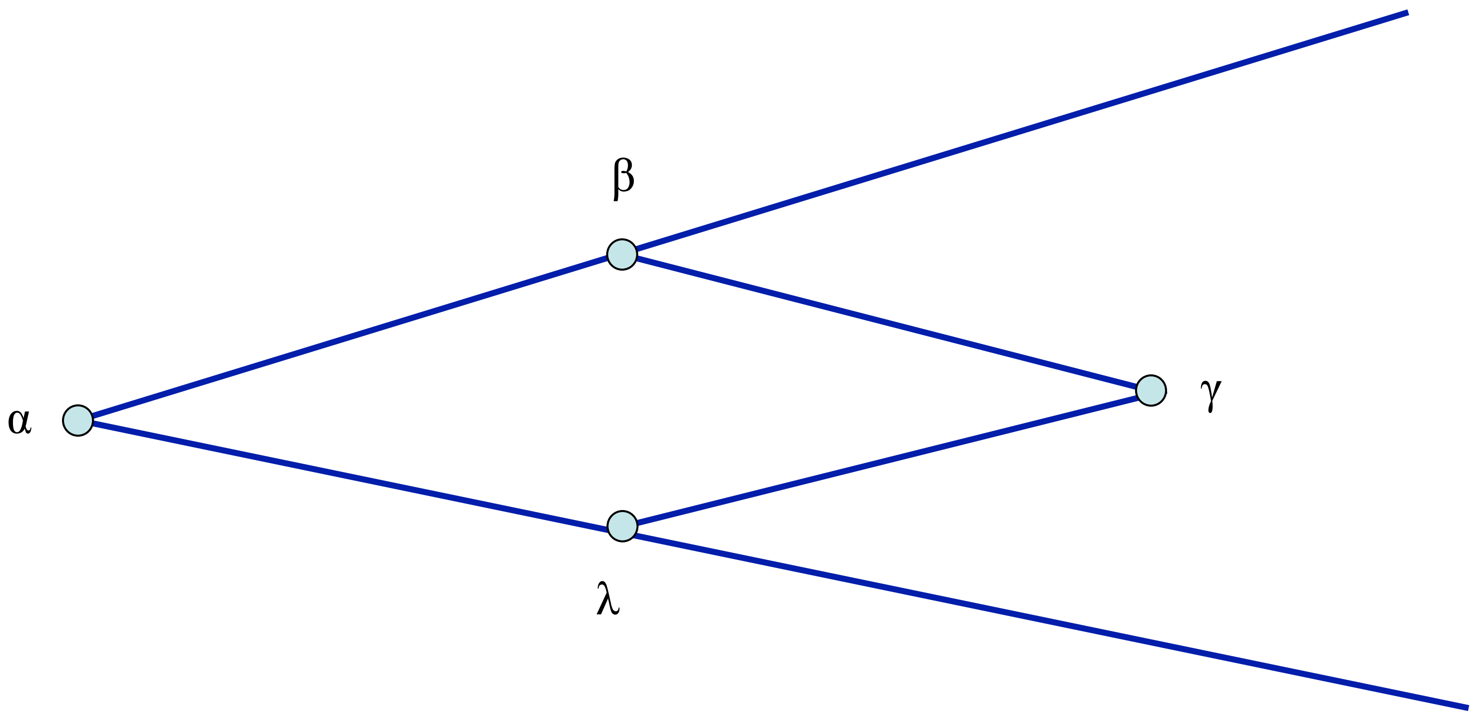
extensive form games

tree: counterexample



extensive form games

tree: counterexample



extensive form games

definition of a tree

predecessor of a node:

1. a node cannot be a predecessor of itself
2. a predecessor's predecessor is also a predecessor
3. predecessors can always be ranked: if β and γ are both predecessors of α, then either β is a predecessor of γ or vice-versa
4. there must be a common predecessor: for two nodes β and γ, neither of which precedes the other, there must be a node α that precedes them both
5. every node other than the root has a unique immediate predecessor
6. a terminal node is a node that has no successors

extensive form games

definition of a tree

a **path** is a sequence of distinct but adjacent nodes

the **length of a path** is the number of edges (or branches) contained in the path

a path from the root to a terminal node represents a complete sequence of moves thay determines the players' payoffs (1 + 2 + 3 above imply such a path is unique)

extensive form games

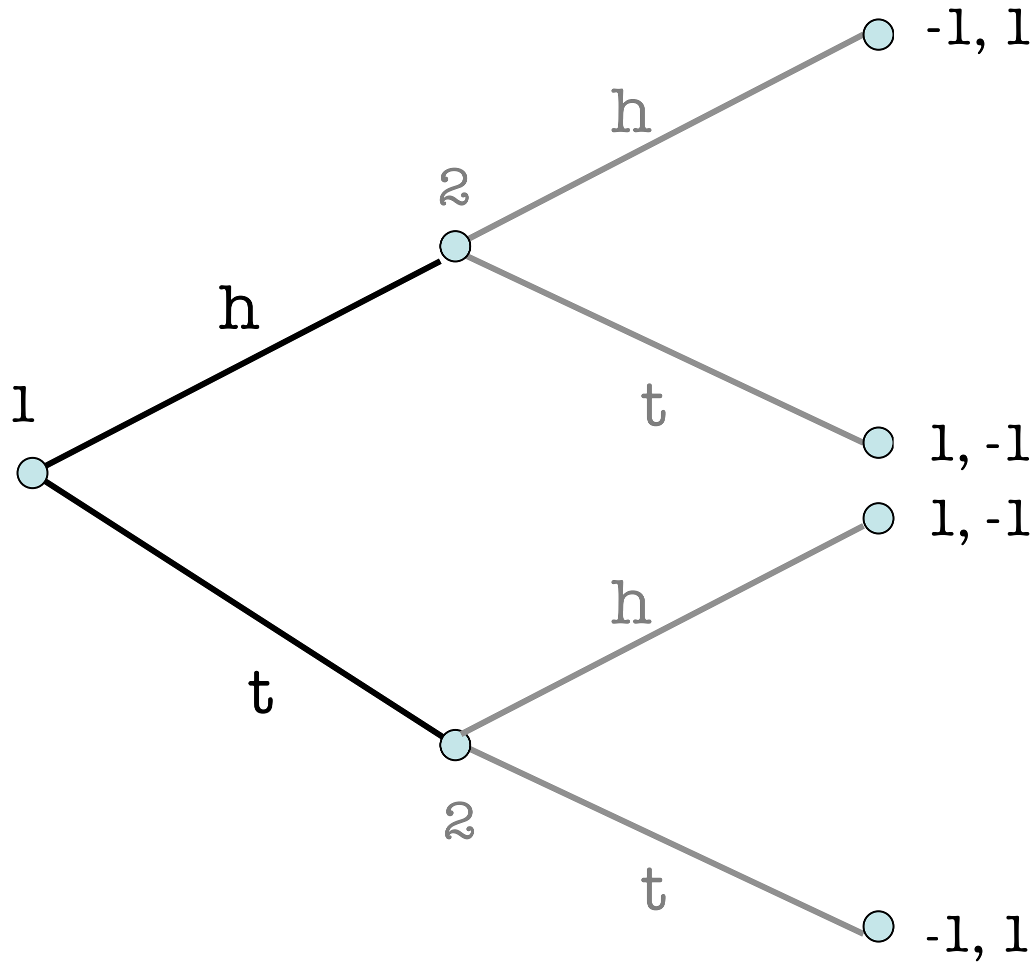
definition of a strategy

a **strategy** is a complete, conditional plan of actions

- **conditional** because it tells each player which branch to follow if arriving at an information set
- **complete** because it tells her what to choose at every information set

extensive form games

example: sequential-move matching pennies



extensive form games

strategies in sequential matching pennies

2 strategies for player 1: {h, t}

2^2 strategies for player 2: {hh, ht, th, tt}