static games with incomplete information

part 2

example:
coordination game

1 \ 2	Book launch	Movie
Book launch	$2+t_1$,1	0,0
Movie	0,0	1,$2+t_2$

example:
coordination game - BNE

t_i follows U [0,x]

Bayes-Nash equilibrium:
player 1 plays BL if t_1 above c and M otherwise
player 2 plays M if t_2 above p and BL otherwise

player 1 plays BL with probability (x-c)/x
player 2 plays M with probability (x-p)/x

but $\lim_{x\to 0} (x-c)/x = \lim_{x\to 0} (x-p)/x = 2/3$

dynamic games with incomplete information

part 1

roadmap

perfect Bayesian equilibrium (PBE)

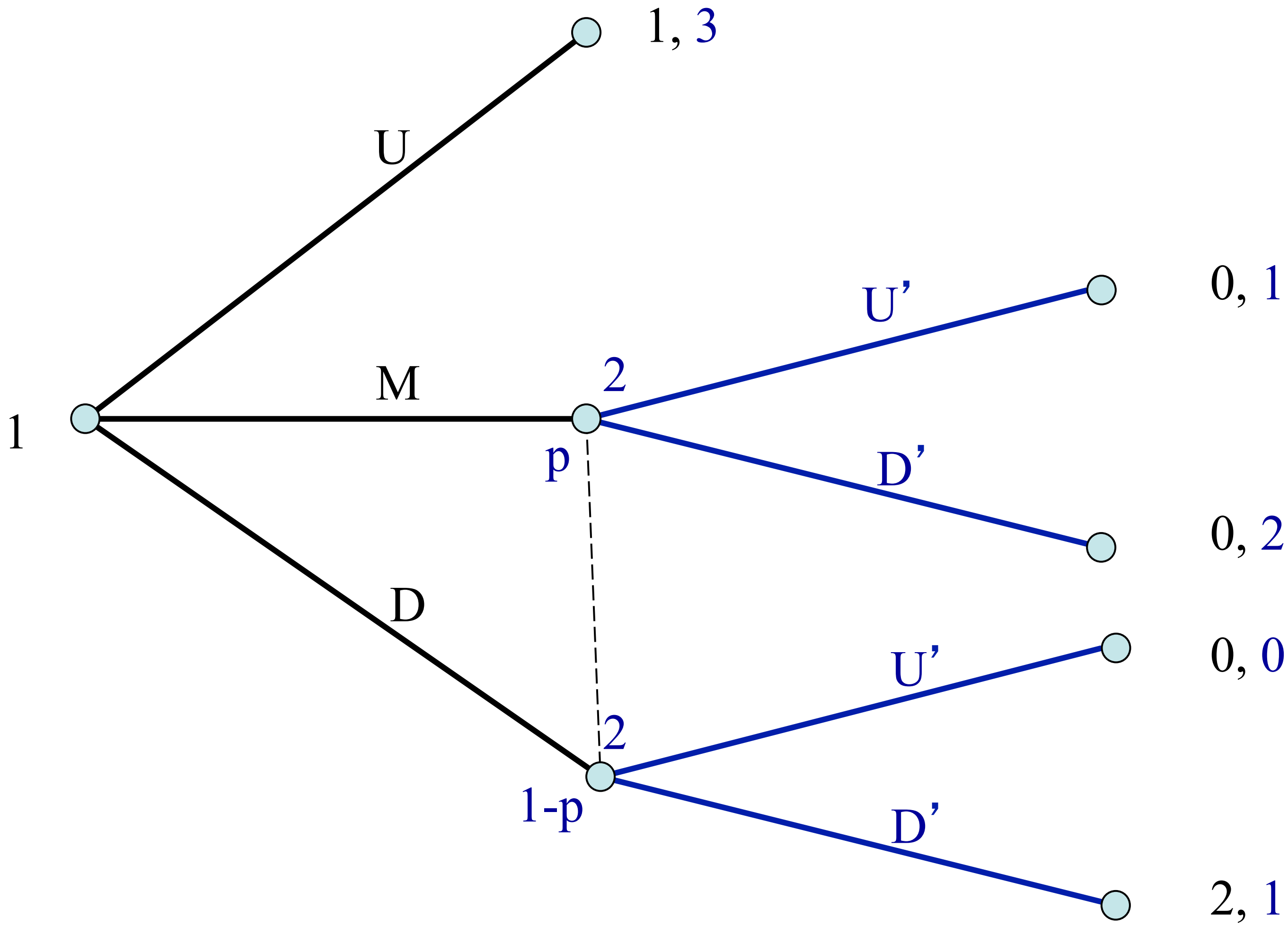
- example
- definition

references

chap. 24 of Dutta

chap. 4 of Gibbons

perfect Bayesian Equilibrium: example



perfect Bayesian Equilibrium: example

1 \ 2	U'	D'
U	1,3	1,3
M	0,1	0,2
D	0,0	2,1

perfect Bayesian Equilibrium: example

subgames: this game has no proper subgames. So any NE is an SPNE. In particular, (U, U') and (D, D') are SPNE

remark: (U, U') clearly depends on a non-credible threat; if player 2 gets to move, playing D' dominates U', so player 1 should not be induced to play U by player 2's threat to play U'

perfect Bayesian Equilibrium: definition, part I

requirement 1: at each information set, the player who moves must have a **belief** about which node in the information set has been reached

requirement 2: given their beliefs, the players' strategies must be **sequentially rational**, i.e., the players' actions must be optimal given the player's belief at that information set and the other players' subsequent strategies

perfect Bayesian Equilibrium: example

player 2 must have a belief about whether player 1 has played M or D; this belief is represented by p and 1 – p attached to the relevant nodes

the expected payoff from playing
- U' is $p.1 + (1 - p).0 = p$
- D' is $p.2 + (1 - p) \cdot 1 = 1 + p$

player 2 always chooses D' – we can eliminate (U, U')

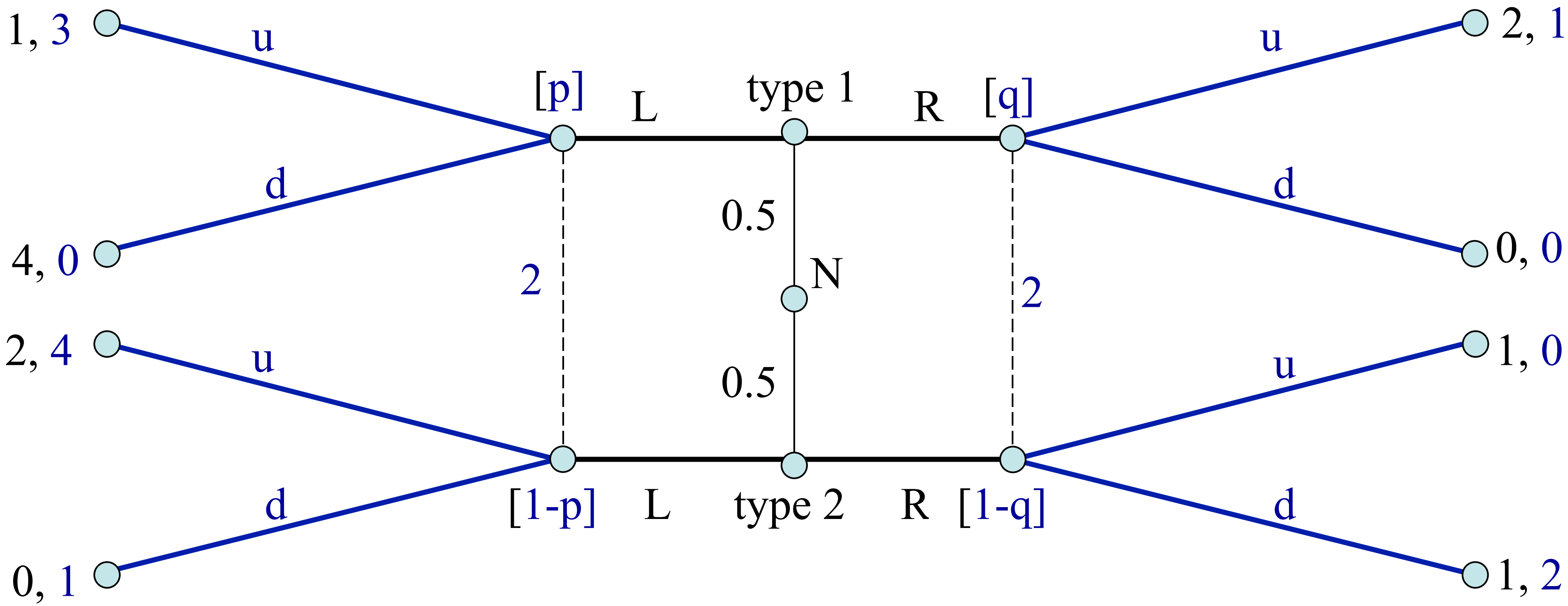
perfect Bayesian Equilibrium: definition, part II

for a given equilibrium, an information set is on the equilibrium path if it will be reached with positive probability if the game is played according to the equilibrium strategies; it is off the equilibrium path if it is certain not to be reached

requirement 3: at information sets on the equilibrium paths, beliefs are determined according to Bayes' rule and the players' equilibrium strategies

ex: In the SPNE (D, D'), player 2's belief must be $p = 0$

Perfect Bayesian Equilibrium: example



Perfect Bayesian Equilibrium: example

There are four possible pure-strategy perfect Bayesian equilibria in this game:

(1) pooling on L (i.e, both t_1 and t_2 play L)

(2) pooling on R (i.e, both t_1 and t_2 play R)

(3) separation with t_1 playing L and t_2 playing R

(4) separation with t_1 playing R and t_2 playing L

Perfect Bayesian Equilibrium: example

Pooling on L

Suppose 1's strategy is (L, L).

Then 2's information set after L is on the equilibrium path, and 2's belief (p, 1-p) is determined by Bayes' rule and 1's strategy. Clearly, we must have p = 0.5 (due to pooling).

Given this belief, 2's best response is to play u, so that types t_1 and t_2 earn 1 and 2, respectively.

Is 1 willing to choose (L, L)? If 2's response to R is u, the payoff of t_1 is 2 > 1 (deviation incentive). If it is d, the payoffs for t_1 and t_2 are 0 < 1 and 1 < 2.

Under what conditions is d the optimal choice of 2?

2's expected payoff from d is larger than from u iff

$$q \cdot 0 + (1 - q) \cdot 2 \geq q \cdot 1 + (1 - q) \cdot 0 \rightarrow q \leq 2/3$$

So [(L, L), (u, d), p = 0.5, q] is a pooling PBE for q ≤ 2/3.

Perfect Bayesian Equilibrium: example

Pooling on R

Suppose 1 adopts strategy (R, R).

Clearly, we must have q = 0.5 (due to pooling). Given this belief, 2's best response is to play d, so that types t_1 and t_2 earn 0 and 1, respectively. But t_1 can earn 1 by playing L, since 2's best response to L is u for any value of p.

So there cannot be an equilibrium where 1 plays (R, R).

Perfect Bayesian Equilibrium: example

Separating, with t_1 playing L

Suppose 1 adopts strategy (L, R).

Then both of 2's information sets are on the equilibrium path, so both beliefs are determined using Bayes' rule and the eq. strategy: $p = 1$, $q = 0$.

2's best responses to these beliefs are u and d, respectively, and both types earn 1.

Is (L, R) optimal given 2's strategy (u, d)? No: if type t_2 deviates by playing L rather than R, 2 responds with u, earning t_2 a payoff of $2 > 1$ (deviation incentive).

So there cannot be an equilibrium where 1 plays (L, R).

Perfect Bayesian Equilibrium: example

Separating, with t_1 playing R

Suppose 1 adopts strategy (R, L).

2's beliefs are reversed: $p = 0$, $q = 1$. 2's best response is (u, u) and both types earn payoffs of 2.

If t_1 were to deviate by playing L rather than R, 2 would react with u, and t_1's payoff would be $1 < 2$. So there is no incentive to deviate for t_1.

If t_2 were to deviate by playing R rather than L, 2 would react with u, and t_2's payoff would be $1 < 2$. So there is no incentive to deviate for t_2.

So there is a separating PBE [(R, L), (u, u), $p = 0$, $q = 1$].