repeated games

part 1

roadmap

repeated games
- finitely repeated games
  - examples
  - equilibria
- infinitely repeated games
  - examples
  - equilibria

references
Chap. 14, 15 of Dutta

repeated game
what's different

- The fact that a game is repeated in time may make a difference since the prospect of reward or punishment may lead to different behavior
- But reward/punishment is only credible if it is part of a SPE!

repeated game

notions

- A repeated game is a dynamic game of complete information in which a component (simultaneous-move) game is repeated at least twice and the previous plays are observed before the next play

- The component game is sometimes called the *stage game*

- Repetition may be finite or infinite

finitely repeated game

example: once-repeated PD

- Two prisoners make a simultaneous choice between cooperate (C) or defect (D)
- Their choices are revealed to them
- They play the same stage game again, i.e., make the same choices
- The payoffs of the game are the sums of the payoffs in each stage

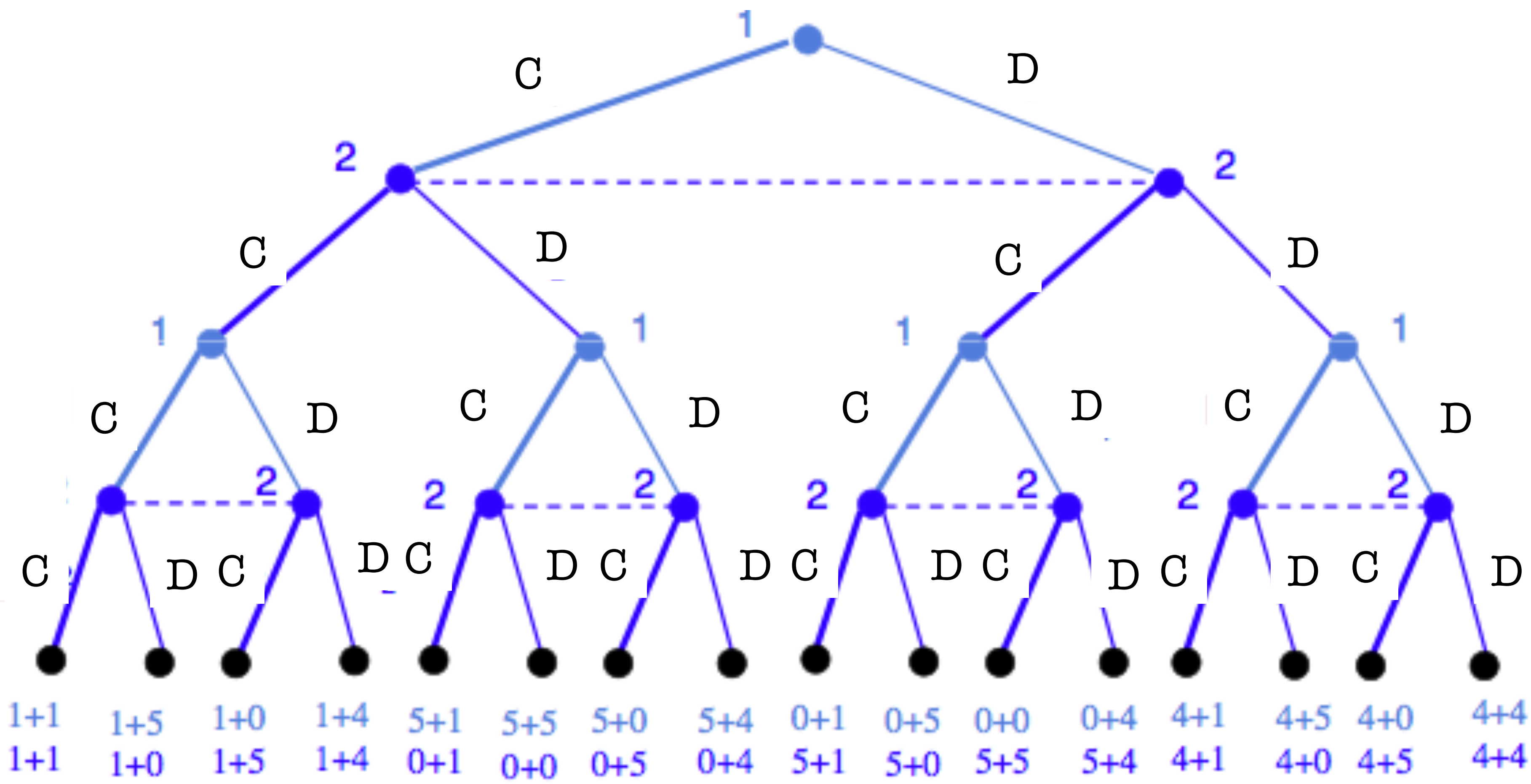
once-repeated PD

stage game

	C	D
C	1, 1	5, 0
D	0,5	4, 4

once-repeated PD

game tree



finitely repeated game
SPE

- The analysis of SPE in a finitely repeated game proceeds by backwards induction

finitely repeated game
example: SPE

- In all proper subgames, the unique NE is (C,C)

- Folding back to the first round, the only NE is again (C,C). So, the unique SPE is

  ((C,C,C,C,C), (C,C,C,C,C))

- Here, the players play each stage of the game as if they were playing it by itself!

finitely repeated game
SPE

- Proposition: Consider a finitely repeated game (G,T) with G = (I, S, u). Suppose the stage game has one NE, say $(s^*_1, \ldots, s^*_n)$. Then, the repeated game has exactly one SPE, where each player i plays s^*_i at every one of the T stages, regardless of the history of the game

finitely repeated game
example 2

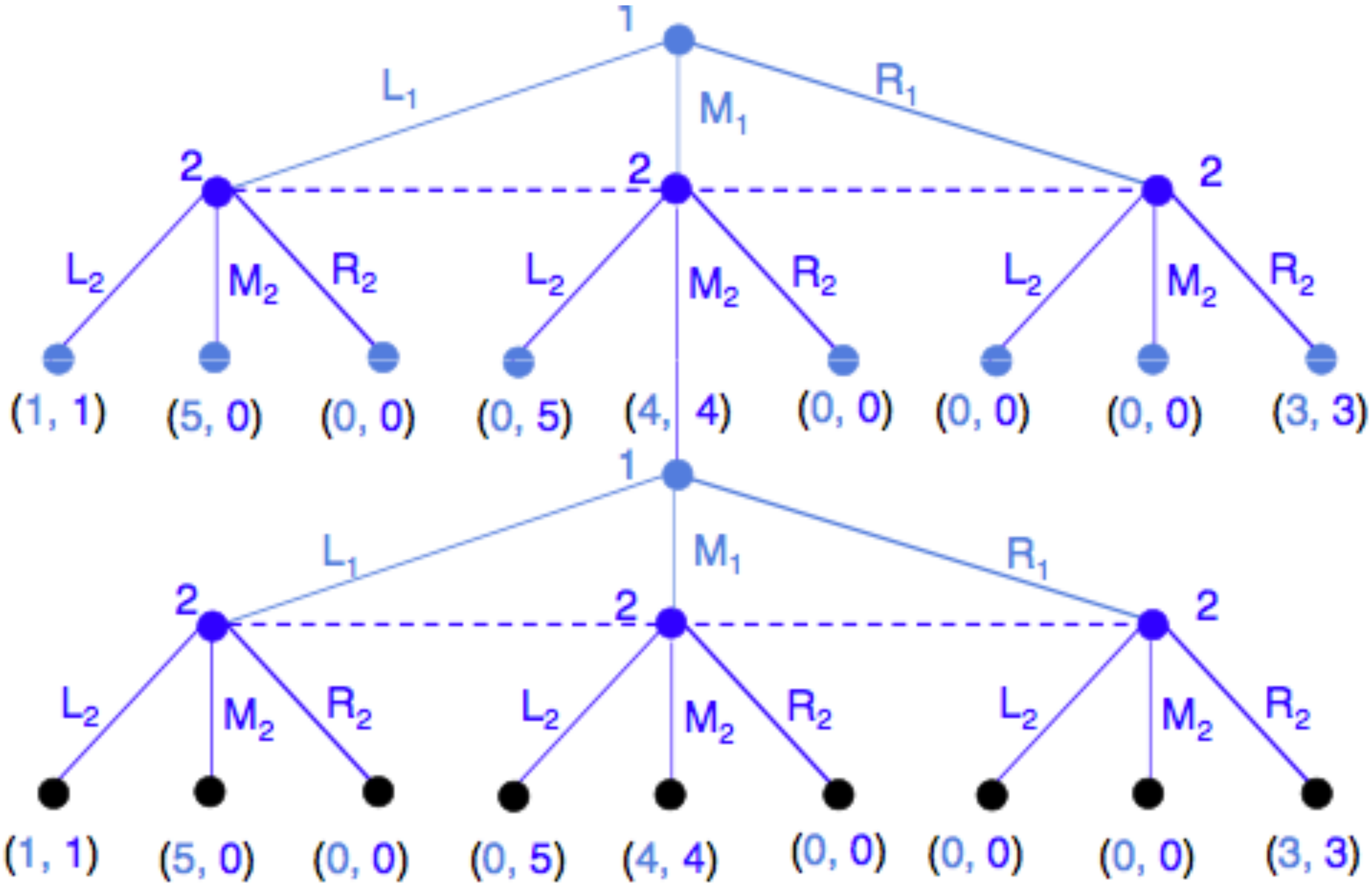
The following stage game is played twice:

	L	M	R
L	1, 1	5, 0	0, 0
M	0, 5	4, 4	0, 0
R	0, 0	0, 0	3, 3

Can we find a SPE where (M,M) is played?

finitely repeated game

example 2: *informal* game tree



finitely repeated game
example 2: SPE

Player 1's strategy:
plays M at stage 1 and at stage 2 plays R if the first stage outcome is (M,M) or L otherwise

Player 2's strategy:
plays M at stage 1 and at stage 2 plays R if the first stage outcome is (M,M) or L otherwise

finitely repeated game
example 2: SPE

Adding payoffs of stage 2 to stage 1:

	L	M	R
L	2, 2	6, 1	1, 1
M	1, 6	7, 7	1, 1
R	1, 1	1, 1	4, 4

finitely repeated game

example 2: SPE

But there are other SPE!

For example, playing L always… or R always…

And others…

finitely repeated game
SPE

If the stage game has multiple NE, there are many SPE of the finitely repeated game.

The nonmyopic behavior is sustained by the expectation of reciprocity: a player may be willing to sacrifice short-term gains (by deviating to L in the example) if he anticipates that she will be rewarded in the future for having made such a sacrifice.

repeated games

part 2

roadmap

repeated games

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Chap. 15 of Dutta

infinitely repeated game
discounting

- 1 Euro in a year's time is different than 1 Euro now!

- 1 Euro could be invested and earn interest; if the interest rate is 5%,

  1 Euro today × 1.05 = 1.05 Euro in a year's time

- So, 1 Euro in a year's time is worth 0.952 today!

  0.952 today × 1.05 = 1 Euro in a year's time

infinitely repeated game

discounting

- $1/1.05 = 0.952$ is the discount factor d

- So, if an agent receives K every year, in the present this amounts to:

$$K + Kd + Kd^2 + \ldots + Kd^t + \ldots$$

$$= K(1 + d + d^2 + \ldots + d^t + \ldots)$$

$$= K\,[1/(1-d)]$$

infinitely repeated game

example: infinitely repeated PD

- The PD is repeated an infinite number of times

- Each player's payoff is the discounted sum of payoffs in the stage game

infinitely-repeated PD

stage game

	C	D
C	1, 1	5, 0
D	0,5	4, 4

infinitely repeated PD

SPE

- Consider the following strategy:

  A player starts by playing D and continues playing D as long as (D,D) is the outcome of all previous stages; if there is one deviation, then C forever!

- This is a *grim trigger strategy*: a deviation from the desired behavior (D,D) triggers punishment forever

infinitely repeated PD

SPE

- Is the pair of grim trigger strategies a SPE?

- Note that there is an infinite number of subgames (no matter how players have played, each time they repeat play, a new subgame begins)

- But, for the grim trigger strategy, there are only two kinds of subgames: those in which no deviation occured (and players chose D,D always) and the others (those in which someone deviated at some point)

infinitely repeated PD

SPE

- Assuming the column player chooses the grim trigger strategy, does the row player have incentives to deviate?

- For the subgames of the second type, the strategy prescribes choosing C, which, since the column player is also choosing C, is the best reply (he cannot change the expected pattern of play therefater!)

infinitely repeated PD

SPE

- For the subgames of the first type (with history (D,D)), does the row player want to chose C at some point?

- Choosing C (against D) gives immediate payoff of 5, but 1 therefater

- Continuing with D gives 4 forever

infinitely repeated PD

SPE

The row player will not deviate iff:

$$4 + 4d + 4d^2 + 4d^3 + \ldots \geq 5 + d + d^2 + d^3 + \ldots$$

$$\Leftrightarrow \frac{4}{1-d} \geq 5 + d\frac{1}{1-d} \Leftrightarrow$$

$$\Leftrightarrow d \geq \frac{1}{4}$$

i.e., playing the grim trigger strategies (ensuring D,Devery period) is a SPE iff the future is important enough!