Master in Actuarial Science, Exam 06/01/2012. **2h30m**
Ratemaking and Experience Rating, 2nd year, 1st semester

1. Consider a motor insurance portfolio where the population is classified into categories A, B and C, respectively, where A is Good drivers, B is Bad drivers and C is *Sports* drivers. The population of drivers is split as follows: 70% is in category A, 25% in B and 5% in C. For each driver in category A, there is a probability of 0.75 of having no claims in a year, a probability of 0.2 of having one claim and a probability of 0.05 of having two or more claims in a year. For each driver in category B these probabilities are 0.25, 0.4 and 0.35, respectively. For each driver in category C these probabilities are 0.3, 0.4 and 0.3, respectively.

   Risk parameter representing the kind of driver is denoted by θ, which is a realization of the random variable Θ. The insurer does not know the value of that parameter. Let X be the (observable) number of claims per year for a risk taken out at random from the whole portfolio. For a given $\Theta = \theta$ yearly observations $X_1, X_2, ...,$ make a random sample from risk X. The insurer finds crucial that the annual premium for a given risk might be adjusted by its claim record.

   (a) Consider a risk X taken out at random from the portfolio.
      i. Calculate the mean and variance of X.
      ii. Compute the probability function of X.

   (b) For a particular risk of the portfolio we observed in the last two years $X_1 = x_1 = 0$ and $X_2 = x_2 = 2$.
      i. For a given $\Theta = \theta$ of risk X observations, $X_1, X_2, \ldots$, are a random sample but X_1 and X_2 are not independent. Comment briefly.
      ii. Compute $Cov[X_1, X_2]$.
      [**Note**: For r.v.'s X, Y and Z, $Cov[X, Y] = E[Cov[X, Y|Z]] + Cov[E[X|Z]; E[Y|Z]]$.]
      iii. Compute the posterior probability function of Θ given $(X_1 = 0, X_2 = 2)$.
      iv. You do not know from which risk category the above sample comes. Carry out appropriate calculations to determine from which category the sample is most likely to have come.
      v. We need to compute a (pure) premium for the next year:
         A. Compute the collective pure premium.
         B. Compute the Bayes premium $E[X_3|\mathbf{X} = (0, 2)] = E(\mu(\Theta)|\mathbf{X} = (0, 2))$.
         C. Compute Bühlmann's credibility premium, say, $\tilde{E}(X_3|\theta)$.
         D. Can we talk here on *Exact Credibility*? Comment appropriately.

2. Retrieve the problem and data of Exercise 1 above. Suppose now that the insurer uses a *Bonus-malus* system based on the claims frequency to rate the risks of that portfolio. The system has simply three classes, numbered 1, 2 and 3 and ranked increasingly from low to higher risk. Transition rules are the following: A policy with no claims in one year goes to the previous lower class in the next year unless it is already Class 1, where it stays. In the case of a claim goes to Class 3, if it is already there no change is made.

   Let $\alpha(\theta)$ be the probability of not having any claim in one year for a policy in with risk parameter θ. Entry class is Class 2 and premia vector is $\mathbf{b} = (70, 100, 150)$.

   (a) Consider a policy with risk parameter θ.
      i. Write the transition rules matrix and compute the one year transition probability.
      ii. Comment on the existence of the stationary distribution.
      iii. Calculate the probability of a policy being ranked in Class 1 two years after entering the system.
      iv. Calculate the probability function of the premium for a type A driver after two years of stay in the portfolio. Compute the average premium.
      v. After some time the insurer's chief actuary concluded that for ratemaking purposes it didn't make much difference to keep categories B and C apart, and merged them into, say, B^*. For a driver in this new class, compute the probability function of the premium after one year of staying in the system (since his entry).

   (b) Stationary distribution for a given θ is given by vector $\left(\alpha(\theta)^2; [1 - \alpha(\theta)]\alpha(\theta); 1 - \alpha(\theta)\right)$. Compute the probability function of the premium for a policy taken out at random from the portfolio. Calculate the average premium.

3. For tariff modelling purposes, we studied different factors with impact in the claim frequency mean and have found the model showed in the Annex, where

   TVeic – 3 classes, in increasing power order of the vehicle

   TProp – 2 classes (1=Private, 2=Company)

   TUtil – 3 classes (1="male", 2="female", 3="retired")

   Comb – 2 classes (1=petrol, 2=diesel)

   Inexp–3 classes (0=driver's licence aged *five plus*, 1= from 2-5 years, 2=less than 2)

   idSeg – "Dummy" variable valued 1 when policyholder is less than 25 years old

   idveic – "Dummy" variable valued 1 when vehicle is aged more than 10 years

   zona – Usual circulation geographical zone (4 zones in the country)

   Expo – time period of the year the policy was in force (LExpo is the logarithm of Expo).

   All the items below are based on the model presented. You can use an $\alpha = 0.05$ in all tests you make.

   (a) Is it statistically acceptable to set a 30% aggravation for a type 2 vehicle (TVeic=2) when compared to the one of a type 1 (TVeic=1) (*ceteris paribus*)?
   (b) Test if it is acceptable to consider the same rating factor for "female" and "retired".
   (c) Set the situations that make the less and the greater claim frequency mean.
   (d) In the case you have chosen a Poisson model with logarithm link, what would be the effect in the output?
   (e) Why is the variable LExpo considered as "offset"?

Marks (out of 200):

1.*a*)*i*	*ii.*	*b*)*i.*	*ii.*	*iii.*	*iv.*	*v.A*	*B*	*C*	*D*	2.*a*)*i.*	*ii.*	*iii.*	*iv.*	*v.*	*b*)	3.*a*)	*b*)	*c*)	*d*)	*e*)
15	10	10	10	15	10	5	10	15	10	10	10	10	10	10	10	7.5	10	7.5	2.5	2.5
(15	25	35	45	60	70	75	85	100	110	120	130	140	150	160	170	177.5	187.5	195	197.5	200)

ANNEX

```
Call:
glm(formula = NS ~ TVeic + TProp + TUtil + Comb + Exper + Idade +
    IdVeic + Zona, family = quasi(link = "log", variance = "mu"),
    data = dados, offset = LExpo)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.8408  -0.3391  -0.2583  -0.1619   8.8934

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     -2.35449    0.03144 -74.883  < 2e-16 ***
TVeic2           0.28325    0.04803   5.897 3.71e-09 ***
TVeic3           0.37898    0.06028   6.287 3.25e-10 ***
TProp2           0.26397    0.07143   3.695 0.000220 ***
TUtil2          -0.11734    0.04332  -2.708 0.006760 **
TUtil3          -0.16520    0.06957  -2.375 0.017569 *
Comb2            0.24900    0.03093   8.051 8.28e-16 ***
Inexper1         0.23479    0.07510   3.126 0.001770 **
Inexper2         0.68794    0.09663   7.120 1.09e-12 ***
idSeg            0.35912    0.07348   4.888 1.02e-06 ***
idVeic          -0.39583    0.11899  -3.327 0.000879 ***
Zona2           -0.10860    0.03660  -2.967 0.003007 **
Zona3           -0.30775    0.03669  -8.387  < 2e-16 ***
Zona4           -0.62366    0.05496 -11.348  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for quasi family taken to be 1.786155)

    Null deviance: 64625  on 239735  degrees of freedom
Residual deviance: 63681  on 239722  degrees of freedom
AIC: NA
Number of Fisher Scoring iterations: 6
```

Estimate of variance/covariance matrix of estimadors

	(Intercept)	TVeic2	TVeic3	TProp2
(Intercept)	9.886221e-04	-5.388603e-05	-8.361113e-05	-1.799297e-05
TVeic2	-5.388603e-05	2.307227e-03	5.598489e-04	2.957897e-05
TVeic3	-8.361113e-05	5.598489e-04	3.634144e-03	-1.086117e-04
TProp2	-1.799297e-05	2.957897e-05	-1.086117e-04	5.102511e-03
TUtil2	-2.867092e-04	1.464166e-04	1.651684e-04	4.179612e-05
TUtil3	-2.827942e-04	1.002087e-04	1.682527e-04	1.455274e-04
Comb2	-3.113933e-04	-5.232310e-04	-5.208346e-04	-1.607743e-04
Inexper1	-1.772845e-04	5.114019e-05	7.293580e-05	7.907560e-05
Inexper2	-1.792848e-04	1.509841e-05	3.207420e-05	8.365561e-05
idSeg	-1.105421e-04	1.349316e-04	1.330956e-04	3.512015e-05
idVeic	-1.701638e-04	-1.508968e-04	-3.577608e-04	1.798026e-04
Zona2	-7.900101e-04	-1.252341e-05	7.144301e-06	-2.329660e-04
Zona3	-7.752905e-04	-6.746288e-05	-2.593509e-05	-1.201231e-04
Zona4	-7.703890e-04	-1.655381e-04	-5.602541e-05	-9.972035e-05

	TUtil2	TUtil3	Comb2	inexper1
(Intercept)	-2.867092e-04	-2.827942e-04	-3.113933e-04	-1.772845e-04
TVeic2	1.464166e-04	1.002087e-04	-5.232310e-04	5.114019e-05
TVeic3	1.651684e-04	1.682527e-04	-5.208346e-04	7.293580e-05
TProp2	4.179612e-05	1.455274e-04	-1.607743e-04	7.907560e-05
TUtil2	1.877048e-03	2.488226e-04	8.601928e-05	-1.230231e-04
TUtil3	2.488226e-04	4.839936e-03	1.159281e-04	1.304524e-04
Comb2	8.601928e-05	1.159281e-04	9.566395e-04	8.149603e-05
Inexper1	-1.230231e-04	1.304524e-04	8.149603e-05	5.639870e-03
Inexper2	-2.026008e-04	1.028410e-04	1.469452e-04	2.006136e-03
idSeg	7.733099e-06	1.291575e-04	-3.994985e-05	-2.730365e-03
idVeic	9.129981e-05	-4.307946e-04	1.642761e-04	8.513266e-05
Zona2	-1.485289e-05	-6.347523e-05	8.967463e-06	-5.298394e-06
Zona3	2.592926e-05	-1.239841e-05	-2.576541e-05	-1.785598e-05
Zona4	5.632733e-05	3.061035e-07	-1.964603e-05	-5.482391e-05

	inexper2	idade	IdVeic	Zona2
(Intercept)	-1.792848e-04	-1.105421e-04	-1.701638e-04	-7.900101e-04
TVeic2	1.509841e-05	1.349316e-04	-1.508968e-04	-1.252341e-05
TVeic3	3.207420e-05	1.330956e-04	-3.577608e-04	7.144301e-06
TProp2	8.365561e-05	3.512015e-05	1.798026e-04	-2.329660e-04
TUtil2	-2.026008e-04	7.733099e-06	9.129981e-05	-1.485289e-05
TUtil3	1.028410e-04	1.291575e-04	-4.307946e-04	-6.347523e-05
Comb2	1.469452e-04	-3.994985e-05	1.642761e-04	8.967463e-06
Inexper1	2.006136e-03	-2.730365e-03	8.513266e-05	-5.298394e-06
Inexper2	9.336699e-03	-3.458548e-03	1.052468e-04	-1.367375e-05
idSeg	-3.458548e-03	5.398878e-03	4.835702e-05	1.520852e-05
idVeic	1.052468e-04	4.835702e-05	1.415830e-02	-2.325554e-05
Zona2	-1.367375e-05	1.520852e-05	-2.325554e-05	1.339684e-03
Zona3	-4.331029e-06	-2.415734e-05	-1.122816e-04	7.998054e-04
Zona4	-2.817194e-05	-1.504866e-05	-1.319008e-04	7.989468e-04

	Zona3	Zona4
(Intercept)	-7.752905e-04	-7.703890e-04
TVeic2	-6.746288e-05	-1.655381e-04
TVeic3	-2.593509e-05	-5.602541e-05
TProp2	-1.201231e-04	-9.972035e-05
TUtil2	2.592926e-05	5.632733e-05
TUtil3	-1.239841e-05	3.061035e-07
Comb2	-2.576541e-05	-1.964603e-05
Inexper1	-1.785598e-05	-5.482391e-05
Inexper2	-4.331029e-06	-2.817194e-05
idSeg	-2.415734e-05	-1.504866e-05
idVeic	-1.122816e-04	-1.319008e-04
Zona2	7.998054e-04	7.989468e-04
Zona3	1.346383e-03	8.078880e-04
Zona4	8.078880e-04	3.020230e-03