Master in Actuarial Science, Exam 24/01/2012. **2h30m**
Ratemaking and Experience Rating, 2nd year, 1st semester

1. Consider a motor insurance portfolio where drivers are of different types identified by the parameter θ, and labeled as 1, 2 and 3. Per year each risk in the portfolio can produce 0, 1, or 2 claims. Probabilities are shown in the table below. Suppose that a risk type is chosen at random (types are equally likely), and two risks are chosen with replacement from that class. Suppose that a total of 2 claims was observed from those two risks drawn. Two more risks are then drawn with replacement from the same type, and it is of interest to predict the total on these next two. Let X be the total of claims from the two risks drawn and $f_X(.)$ its probability function.

Type	0	1	2
1	0.40	0.35	0.25
2	0.25	0.10	0.65
3	0.50	0.15	0.35

   (a) Determine the probability function $\pi(\theta)$ of the random variable Θ.
   (b) Determine the conditional probability function $f_{X|\theta}(x|\theta = i)$ for the totals of claims for the two risks drawn when $i = 1, 2, 3$.
   (c) Calculate the probability of the total claims of the first two risks drawn equals 2: $\Pr(X_1 = 2)$
   (d) Calculate the posterior distribution $\pi_{\Theta|X_1}(\theta|2)$.
   (e) Determine the conditional distribution $f_{X_2|X_1}(x_2|2)$ of the total X_2 of the next double draw given that $X_1 = 2$ was observed in the previous double draw
   (f) Determine the Bayesian premium $E(X_2|X_1 = 2)$.
   (g) Compute the structural parameters $\mu = E(\mu(\Theta))$, $\upsilon = E(\upsilon(\Theta))$ and $a = V(\mu(\Theta))$.
   (h) Compute Bühlmann's credibility premium.
   (i) On what condition(s) can we talk on *exact credibility model*? Comment appropriately.

2. Retrieve Problem 1. The insurer uses a *bonus* system based on claim frequency with three classes (Class 1, 2, and 3) in increasing order of riskiness. Transition rules are as follows: following a year without a claim, a policy moves to the next lower risk class, or remains in Class1. Following a year with one or more claims, a policy moves to, or remains in, Class 3.

   Let $\lambda(\theta)$ be the probability of a policy with risk parmeter θ getting a claim (one or more). Class 2 is the entry class and *premia* vector (in €) is given by $\mathbf{b} = (150, 225, 300)$.

   (a) Consider a policy with risk parameter θ.
      i. Set the transition rules matrix and determine the 1-step transition probabilities matrix (make calculations as function of θ);
      ii. Determine the limiting distribution;
      iii. For a policy entering in the system, what is the probability of belonging to Class 2 after two years;
      iv. A certain policyholder who considers himself to be a Type 1 driver is placed is *Bonus* Class 1, in a given year. In that year he had an accident with a corresponding claim amount x. He can report the claim to the insurer, however that would imply a higher premium for the following years. Discuss the situations making appropriate, but simple, calculations.
      v. For a driver with risk parameter $\theta = 2$ calculate the probability function of the premium payment one year after having entered the system. Compute also the average premium.

   (b) Suppose that the stationary distribution for a given θ is given by vector $\left([1-\lambda(\theta)]^2 ; [1-\lambda(\theta)]\lambda(\theta) ; \lambda(\theta)\right)$. Calculate the distribution for a risk taken at random from the portfolio. Compute also the average premium.

3. Suppose you work as an actuary for an insurance company and you need to define a new tariff for the third party liability in motor insurance to put in place in the beginning of 2012.

   (a) Consider the *pros* and *cons* of using the available data of 2011 or that of 2010.

(b) As usual, we model separately the claim frequency and the expected cost per unit. For the expected cost we decided to consider 5 rating factors:

- Driver's age (1=until 20 years of age, 2= from 21-25, 3 = from 26-35, 4= from 36-50 and 5=more than 50)
- Experience (1=2 or less license years , 2=more than 2)
- Region (1=Lisbon urban area, 2=Oporto urban area, 3=South, 4=Centre and 5=North)
- Capital insured (1 = minimum legal, 2 = medium, 3 = Insurer's maximum).
- Power of vehicle (1=till 75 HP, 2 = 75-120 HP and 3= more than 120HP)

Consider the model shown in the Annex.

i. Argue the use of the model and refer possible changes to be made. Use existing information when available.
ii. What is the size of the portfolio on which the model is based?
iii. What is the ratio between the larger and the smaller expected claim cost?
iv. Suppose now a policy from the Centre Region, vehicle with 90HP, Driver with 53 years of age and 30 years of Experience, CI minimum, and expected frequency of 0.1. What will be the corresponding pure premium?

Marks (out of 200):

1.*a*)	*b*)	*c*)	*d*)	*e*)	*f*)	*g*)	*h*)	*i*)	2.*a*)*i*.	*ii*.	*iii*.	*iv*.	*v*.	*b*)	3.*a*)	*b*)*i*.	*ii*.	*iii*.	*iv*.
2.5	15	7.5	10	20	10	15	15	15	10	10	10	10	10	10	5	7.5	2.5	7.5	7.5
(2.5	17.5	25	35	55	65	80	95	110	120	130	140	150	160	170	175	182.5	185	192.5	200)

Annex

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Call:
glm(formula = cost ~ Age + Experience + Region + CILevel +
    Power, family = Gamma(link = "log"), data = dados)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.0741  -0.4282  -0.0868   0.2428   4.8993

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   7.287927   0.018583 392.181  < 2e-16 ***
Age2         -0.072685   0.015306  -4.749 2.07e-06 ***
Age3         -0.045600   0.016796  -2.715  0.00664 **
Age4         -0.234246   0.018794 -12.464  < 2e-16 ***
Age5         -0.254591   0.022173 -11.482  < 2e-16 ***
Experience2   0.087757   0.013683   6.414 1.46e-10 ***
Region2       0.063669   0.015509   4.105 4.06e-05 ***
Region3       0.030505   0.017394   1.754  0.07949 .
Region4       0.128912   0.019063   6.763 1.40e-11 ***
Region5       0.104366   0.021465   4.862 1.17e-06 ***
CILevel2      0.003539   0.009874   0.358  0.72006
CILevel3      0.006322   0.011188   0.565  0.57205
Power2        0.050938   0.010551   4.828 1.39e-06 ***
Power3        0.148721   0.013310  11.174  < 2e-16 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.302994)

    Null deviance: 4590.4  on 16353  degrees of freedom
Residual deviance: 4422.0  on 16340  degrees of freedom
AIC: 260880

Number of Fisher Scoring iterations: 5
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