Capítulo VIII

2401. $\frac{1}{2n-1}$. **2402.** $\frac{1}{2n}$. **2403.** $\frac{1}{2^{n-1}}$. **2404.** $\frac{1}{n^2}$. **2405.** $\frac{n+2}{(n+1)^2}$. **2406.** $\frac{2n}{3n+2}$. **2407.** $\frac{1}{n(n+1)}$. **2408.** $\frac{1\cdot 3\cdot 5 \dots (2n-1)}{1\cdot 4\cdot 7 \dots (3n-2)}$. **2409.** $(-1)^{n+1}$. **2410.** $n^{(n-1)^{n+1}}$. **2416.** Diverge. **2417.** Converge. **2418.** Diverge. **2419.** Diverge. **2420.** Diverge. **2421.** Diverge. **2422.** Diverge. **2423.** Diverge. **2424.** Diverge. **2425.** Converge. **2426.** Converge. **2427.** Converge. **2428.** Converge. **2429.** Converge. **2430.** Converge. **2431.** Converge. **2432.** Converge. **2433.** Converge. **2434.** Diverge. **2435.** Diverge. **2436.** Converge. **2437.** Diverge. **2438.** Converge. **2439.** Converge. **2440.** Converge. **2441.** Diverge. **2442.** Converge. **2443.** Converge. **2444.** Converge. **2445.** Converge. **2446.** Converge. **2447.** Converge. **2448.** Converge. **2449.** Converge. **2450.** Diverge. **2451.** Converge. **2452.** Diverge. **2453.** Converge. **2454.** Diverge. **2455.** Diverge. **2456.** Converge. **2457.** Diverge. **2458.** Converge. **2459.** Diverge. **2460.** Converge. **2461.** Diverge. **2462.** Converge. **2463.** Diverge. **2464.** Converge. **2465.** Converge. **2466.** Converge. **2467.** Diverge. **2468.** Diverge. **Indicação.** $\frac{a_{n+1}}{a_n} > 1$. **2470.** Converge condicionalmente. **2471.** Converge condicionalmente. **2472.** Converge absolutamente. **2473.** Diverge. **2474.** Converge condicionalmente. **2475.** Converge absolutamente. **2476.** Converge absolutamente. **2477.** Converge absolutamente. **2478.** Converge absolutamente. **2479.** Diverge. **2480.** Converge absolutamente. **2481.** Converge condicionalmente. **2482.** Converge absolutamente. **2484.** a) Diverge; b) converge absolutamente; c) diverge; d) converge condicionalmente. **Indicação.** Nos exemplos a) e d) examinar a série $\sum_{k=1}^{\infty}(a_{2k-1}+a_{2k})$, e nos b) e c) investigar separadamente as séries $\sum_{k=1}^{\infty} a_{2k-1}$ e $\sum_{k=1}^{\infty} a_{2k}$. **2485.** Diverge. **2486.** Converge absolutamente. **2487.** Converge absolutamente. **2488.** Converge condicionalmente. **2489.** Diverge. **2490.** Converge absolutamente. **2491.** Converge absolutamente. **2492.** Converge absolutamente. **2493.** Sim. **2494.** Não. **2495.** $\sum_{n=1}^{\infty}\frac{1+(-1)^n}{3^n}$; converge. **2496.** $\sum_{n=1}^{\infty}\frac{1}{2n(2n-1)}$; converge. **2497.** Diverge. **2499.** Converge. **2500.** Converge. **2501.** $|R_4| < \frac{1}{120}$, $|R_5| < \frac{1}{720}$; $R_4 < 0$, $R_5 > 0$. **2502.** $R_n < \frac{a_n}{2n+1} = \frac{1}{2^n(2n+1)n!}$. **Indicação.** O resto da série pode ser avaliado através da soma da progressão geométrica que excede a este resto:

$$R_n = a_n\left[\frac{1}{2}\,\frac{1}{n+1} + \left(\frac{1}{2}\right)^2\frac{1}{(n+1)(n+2)} + \dots\right] < a_n\left[\frac{1}{2}\cdot\frac{1}{n+1} + \left(\frac{1}{2}\right)^2 \times \times \frac{1}{(n+1)^2} + \dots\right].$$

2503. $R_n < \frac{n+2}{(n+1)(n+1)}$; $R_{10} < 3\cdot 10^{-8}$. **2504.** $\frac{1}{n+1} < R_n < \frac{1}{n}$. **Solução.** $R_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots > \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)} + \dots = \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \left(\frac{1}{n+2} - \frac{1}{n+3}\right) + \dots = \frac{1}{n-1}$;

$R_n < \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} + \dots = \frac{1}{n}$. **2505.** Para a série dada é fácil achar o valor exato do resto:

$$R_n = \frac{1}{15}\left(n + \frac{16}{15}\right)\left(\frac{1}{4}\right)^{2n}$$

Solução. $R_n = (n+1)\left(\frac{1}{4}\right)^{2n} + (n+2)\left(\frac{1}{4}\right)^{2n+2} + \dots$

Multiplicamos por $\left(\frac{1}{4}\right)^2$:

$$\frac{1}{16}R_n = (n+1)\left(\frac{1}{4}\right)^{2n+2} + (n+2)\left(\frac{1}{4}\right)^{2n+4} + \dots$$

Subtraindo, obtemos:

$$\frac{15}{16}R_n = n\left(\frac{1}{4}\right)^{2n} + \left(\frac{1}{4}\right)^{2n} + \left(\frac{1}{4}\right)^{2n+2} + \left(\frac{1}{4}\right)^{2n+4} + \dots =$$

$$= n\left(\frac{1}{4}\right)^{2n} + \frac{\left(\frac{1}{4}\right)^{2n}}{1-\frac{1}{16}} = \left(n + \frac{16}{15}\right)\left(\frac{1}{4}\right)^{2n}.$$

Daqui encontramos o valor de R_n dado mais acima. Fazendo $n = 0$, achamos a soma da série $S = \left(\frac{16}{15}\right)^2$. **2506.** 99; 999. **2507.** 2; 3; 5. **2508.** $S = 1$. **Indicação.** $a_n = \frac{1}{n} - \frac{1}{n+1}$. **2509.** $S = 1$ quando $x > 0$; $S = -1$ quando $x < 0$; $S = 0$ quando $x = 0$. **2510.** Quando $x > 1$ é absolutamente convergente, quando $x \leqslant 1$, é divergente. **2511.** Quando $x > 1$ converge absolutamente, quando $0 < x \leqslant 1$ converge não absolutamente, quando $x \leqslant 0$ diverge. **2512.** Quando $x > e$ converge absolutamente, quando $1 < x \leqslant e$ converge não absolutamente, quando $x \leqslant 1$ diverge. **2513.** $-\infty < x < \infty$. **2514.** $-\infty < x < \infty$. **2515.** Converge absolutamente quando $x > 0$, diverge quando $x \leqslant 0$. **Solução.** 1) $|a_n| \leqslant \frac{1}{e^{nx}}$ e quando $x > 0$ a série com o termo geral $\frac{1}{e^{nx}}$ converge; 2) $\frac{1}{e^{nx}} \geqslant 1$ quando $x \leqslant 0$ e o $\cos nx$ não tende a zero quando $n \to \infty$, já que do $\cos nx \to 0$ resultaria que $\cos 2nx \to -1$; desta forma, quando $x \leqslant 0$ não é válido o critério necessário de convergência. **2516.** Converge absolutamente quando $2k\pi < x < (2k+1)\pi (k = 0, \pm 1, \pm 2, \dots)$; nos demais pontos diverge. **2517.** Diverge em toda a parte. **2518.** Converge absolutamente quando $x \neq 0$. **2519.** $x > 1$, $x \leqslant -1$. **2520.** $x > 3$, $x < 1$. **2521.** $x \geqslant 1$, $x \leqslant -1$. **2522.** $x \geqslant 5\frac{1}{3}$, $x < 4\frac{2}{3}$. **2533.** $x > 1$, $x < -1$. **2524.** $-1 < x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$. **Indicação.** Para estes valores de x converge, tanto a série $\sum_{k=1}^{\infty} x^k$, como a série $\sum_{k=1}^{\infty}\frac{1}{2^k x^k}$. Quando $|x| \geqslant 1$ e quando $|x| \leqslant \frac{1}{2}$ o termo geral da série não tende a zero. **2525.** $-1 < x < 0$, $0 < x < 1$. **2526.** $-1 < x < 1$. **2527.** $-2 \leqslant x < 2$. **2528.** $-1 < x < 1$. **2529.** $-\frac{1}{\sqrt{2}} \leqslant x \leqslant \frac{1}{\sqrt{2}}$. **2530.** $-1 < x \leqslant 1$. **2531.** $-1 < x < 1$. **2532.** $-1 < x < 1$. **2533.** $-\infty < x < \infty$. **2534.** $x = 0$. **2535.** $-\infty < x < \infty$. **2536.** $-4 < x < 4$. **2537.** $-\frac{1}{3} < x < \frac{1}{3}$. **2538.** $-2 < x < 2$. **2539.** $-e < x < e$. **2540.** $-3 \leqslant x < 3$. **2541.** $-1 < x < 1$. **2542.** $-1 < x < 1$. **Solução.** A divergência da série quando $|x| \geqslant 1$ é evidente (é interessante assinalar que a divergência da série nos extremos do intervalo de convergência $x = \pm 1$ pode ser comprovada, não só através do critério necessário de convergência, mas também com a ajuda do critério D'Alembert). Quando $|x| < 1$, temos $\lim\limits_{n\to\infty}\left|\dfrac{(n+1)!\,x^{(n+1)!}}{n!\,x^{n!}}\right| = \lim\limits_{n\to\infty}|(n+1)\,x^{n!\,n}| \leqslant \lim\limits_{n\to\infty}(n+1)|x|^n = \lim\limits_{n\to\infty}\dfrac{n+1}{\left|\dfrac{1}{x}\right|^n} = 0$ (a última igualdade pode ser facilmente obtida, aplicando-se a regra de L'Hôspital). **2543.** $-1 \leqslant x \leqslant 1$. **Indicação.** Através do critério de D'Alembert não só se pode achar o intervalo de convergência, mas também investigar a convergência da série dada nos extremos deste intervalo. **2544.** $-1 \leqslant x \leqslant 1$. **Indicação.** Através do critério de Cauchy não só se pode achar o intervalo de convergência, mas investigar também a convergência da série dada nos extremos deste intervalo. **2545.** $2 < x \leqslant 8$. **2546.** $-2 \leqslant x < 8$. **2547.** $-2 < x < 4$. **2548.** $1 \leqslant x \leqslant 3$. **2549.** $-4 \leqslant x \leqslant -2$. **2550.** $x = -3$. **2551.** $-7 < x < -3$. **2552.** $0 \leqslant x < 4$. **2553.** $-\dfrac{5}{4} < x < \dfrac{13}{4}$. **2554.** $-e-3 < x < e-3$. **2555.** $-2 \leqslant x \leqslant 0$. **2556.** $2 < x < 4$. **2557.** $1 < x \leqslant 3$. **2558.** $-3 \leqslant x \leqslant -1$. **2559.** $1 - \dfrac{1}{e} < x < 1 + \dfrac{1}{e}$. **Indicação.** Quando $x = 1 \pm \dfrac{1}{e}$ a série diverge, já que $\lim\limits_{n\to\infty}\dfrac{\left(1+\dfrac{1}{n}\right)^{n^2}}{e^n} = \dfrac{1}{\sqrt{e}} \neq 0$. **2560.** $-2 < x < 0$. **2561.** $1 < x \leqslant 3$. **2562.** $1 \leqslant x < 5$. **2563.** $2 \leqslant x \leqslant 4$. **2564.** $|z| < 1$. **2565.** $|z| < 1$. **2566.** $|z - 2i| < 3$. **2567.** $|z| < \sqrt{2}$. **2568.** $z = 0$. **2569.** $|z| < \infty$. **2570.** $|z| < \dfrac{1}{2}$.

2576. $-\ln(1-x)\ (-1 \leqslant x < 1)$. **2577.** $\ln(1+x)\ (-1 < x \leqslant 1)$. **2578.** $\dfrac{1}{2}\ln\dfrac{1+x}{1-x}$ $(|x| < 1)$. **2579.** $\operatorname{arctg} x\,(|x| \leqslant 1)$. **2580.** $\dfrac{1}{(x-1)^2}\ (|x| < 1)$. **2581.** $\dfrac{1-x^2}{(1+x^2)^2}\ (|x| < 1)$. **2582.** $\dfrac{2}{(1-x)^3}\ (|x| < 1)$. **2583.** $\dfrac{x}{(x-1)^2}\ (|x| > 1)$. **2584.** $\dfrac{1}{2}\left(\operatorname{arctg} x - \dfrac{1}{2}\ln\dfrac{1-x}{1+x}\right)$ $|x| < 1$). **2585.** $\dfrac{\pi\sqrt{3}}{6}$. **Indicação.** Examinar a soma da série $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \ldots$ (ver o problema 2579), quando $x = \dfrac{1}{\sqrt{3}}$. **2586.** 3. **2487.** $a^x = 1 + \sum\limits_{n=1}^{\infty}\dfrac{x^n \ln^n a}{n!}$ $(-\infty < x < \infty)$. **2588.** $\operatorname{sen}\left(x + \dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}\left[1 + x - \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \dfrac{x^5}{5!} - \ldots + (-1)^{\frac{n^2-n}{2}}\dfrac{x^n}{n!} + \ldots\right]$ **2589.** $\cos(x + a) = \cos a - x \operatorname{sen} a - \dfrac{x^2}{2!}\cos a + \dfrac{x^3}{3!}\operatorname{sen} a + \dfrac{x^4}{4!}\cos a + \ldots + \dfrac{x^n}{n!}\operatorname{sen}\left[a + \dfrac{(n+1)\pi}{2}\right] + \ldots (-\infty < x < \infty)$. **2590.** $\operatorname{sen}^2 x = \dfrac{2x^2}{2!} - \dfrac{2^3x^4}{4!} + \dfrac{2^5x^6}{6!} - \ldots + (-1)^{n-1}\dfrac{2^{2n-1}x^{2}}{(2n)!} + \ldots (-\infty < x < \infty)$. **2591.** $\ln(2+x) = \ln 2 + \dfrac{x}{2} - \dfrac{x^2}{2\cdot 2^2} + \dfrac{x^3}{3\cdot 2^3} - \ldots + (-1)^{n-1}\dfrac{x^n}{n\cdot 2^n} + \ldots (-2 < x \leqslant 2)$. **Indicação.** Ao investigar o resto, utilize o teorema da integração da série

de potências. **2592.** $\dfrac{2x-3}{(x-1)^2} = -\sum\limits_{n=0}^{\infty}(n+3)\,x^n(|x| < 1)$. **2593.** $\dfrac{3x-5}{x^2-4x+3} = -\sum\limits_{n=0}^{\infty}\left(1 + \dfrac{2}{3^{n+1}}\right)x^n\ (|x| < 1)$. **2594.** $xe^{-2x} = x + \sum\limits_{n=2}^{\infty}\dfrac{(-1)^{n-1}2^{n-1}x^n}{(n-1)!}$ $(-\infty < x < \infty)$. **2595.** $e^{x^2} = 1 + \sum\limits_{n=1}^{\infty}\dfrac{x^{2n}}{n!}$ $(-\infty < x < \infty)$. **2596.** $\sum\limits_{n=0}^{\infty}\dfrac{x^{2n+1}}{(2n+1)!}$ $(-\infty < x < \infty)$. **2597.** $1 + \sum\limits_{n=1}^{\infty}(-1)^n\dfrac{2^{2n}x^{2n}}{(2n)!}$. **2598.** $1 + \dfrac{1}{2}\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n(2x)^{2n}}{(2n)!}$ $(-\infty < x < \infty)$. **2599.** $2\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{(n+2)\,3^{2n}\cdot x^{2n+1}}{(2n+1)!}$ $(-\infty < x < \infty)$. **2600.** $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{9^{n+1}}$ $(-3 < x < 3)$. **2601.** $\dfrac{1}{2} + \dfrac{1}{2}\cdot\dfrac{x^2}{2^3} + \dfrac{1\cdot 3}{2\cdot 4}\dfrac{x^4}{2^5} + \dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\dfrac{x^6}{2^7} + \ldots + \dfrac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\dfrac{x^{2n}}{2^{2n+1}} + \ldots (-2 < x < 2)$. **2602.** $2\sum\limits_{n=0}^{\infty}\dfrac{x^{2n+1}}{2n+1}$ $(|x| < 1)$. **2603.** $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n+1}2^n - 1}{n}x^n\left(-\dfrac{1}{2} < x \leqslant \dfrac{1}{2}\right)$. **2604.** $x + \sum\limits_{n=2}^{\infty}(-1)^n\dfrac{x^n}{(n-1)\,n}$ $(|x| \leqslant 1)$. **2605.** $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{2n+1}$ $(|x| \leqslant 1)$. **2606.** $x + \dfrac{1}{2}\cdot\dfrac{x^3}{3} + \dfrac{1\cdot 3}{2\cdot 4}\dfrac{x^5}{5} + \ldots + \dfrac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\dfrac{x^{2n+1}}{2n+1} + \ldots (|x| \leqslant 1)$. **2607.** $x - \dfrac{1}{2}\cdot\dfrac{x^3}{3} + \dfrac{1\cdot 3}{2\cdot 4}\dfrac{x^5}{5} - \ldots + (-1)^n\dfrac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\dfrac{x^{2n+1}}{2n+1} + \ldots (|x| \leqslant 1)$. **2608.** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{2^{4n-3}x^{2n}}{(2n)!}$ $(-\infty < x < \infty)$. **2609.** $1 + \sum\limits_{n=2}^{\infty}(-1)^{n-1}\dfrac{n-1}{n!}x^n(-\infty < x < \infty)$. **2610.** $8 + 3\sum\limits_{n=1}^{\infty}\dfrac{1 + 2^n + 3^{n-1}}{n!}x^n(-\infty < x < \infty)$. **2611.** $2 + \dfrac{x}{2^2\cdot 3\cdot 1!} - \dfrac{2\cdot x^2}{2^5\cdot 3^3\cdot 2!} + \dfrac{2\cdot 5x^3}{2^8\cdot 3^3\cdot 3!} + \ldots + (-1)^{n-1}\dfrac{2\cdot 5\cdot 8\ldots(3n-4)\,x^n}{2^{3n-1}\cdot 3^n\cdot n!} + \ldots (-\infty < x < \infty)$. **2612.** $\dfrac{1}{6} - \sum\limits_{n=0}^{\infty}\left(\dfrac{1}{2^{n+1}} + \dfrac{1}{3^{n+1}}\right)x^n\ (-2 < x < 2)$. **2613.** $1 + \dfrac{3}{4}\sum\limits_{n=1}^{\infty}\dfrac{(1 + 3^{2n-1})\,x^{2n}}{(2n)!}$ $(|x| < \infty)$. **2614.** $\sum\limits_{n=0}^{\infty}\dfrac{x^{4n}}{4^{n+1}}$ $(|x| < \sqrt{2})$. **2615.** $\ln 2 + \sum\limits_{n=1}^{\infty}(-1)^{n-1}(1 + 2^{-n})\dfrac{x^n}{n}$ $(-1 < x \leqslant 1)$. **2616.** $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{(2n+1)\,(2n+1)!}$ $(-\infty < x < \infty)$. **2617.** $x + \sum\limits_{n=1}^{\infty}(-1)^n\dfrac{x^{2n+1}}{(2n+1)\,n!}$ $(|x| < \infty)$. **2618.** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{x^n}{n^2}$ $(|x| \leqslant 1)$. **2619.** $x + \dfrac{1}{2\cdot 5}x^5 + \dfrac{1\cdot 3}{2^2\cdot 9\cdot 2!}x^9 + \ldots + \dfrac{1\cdot 3\cdot 5\ldots(2n-1)}{2^n(4n+1)\,n!}x^{4n+1} + \ldots (|x| < 1)$. **2620.** $x + \dfrac{x^3}{3} + \dfrac{2x^5}{15} + \ldots$ **2621.** $x - \dfrac{x^3}{3} + \dfrac{2x^5}{15} - \ldots$ **2622.** $e\left(1 - \dfrac{x^2}{2} + \dfrac{x^4}{6} - \ldots\right)$. **2623.** $1 + \dfrac{x^2}{2} + \dfrac{5x^4}{24} + \ldots$. **2624.** $-\left(\dfrac{x^2}{2} + \dfrac{x^4}{12} + \dfrac{x^6}{45} + \ldots\right)$. **2625.** $x + x^2 + \dfrac{1}{3}x^3 + \ldots$.

2626. Indicação. Partindo das equações paramétricas da elipse $x = a\cos t$, $y = b \operatorname{sen} t$, calcular a extensão da elipse e desenvolver a expressão obtida em série de potências ε. **2628.** $x^3 - 2x^2 - 5x - 2 = -78 + 59(x+4) - 14(x+4)^2 + (x+4)^3$ $(-\infty < x < \infty)$. **2629.** $f(x+h) = 5x^3 - 4x^2 - 3x + 2 + (15x^2 - 8x - 3)h + (15x - 4)h^2 + 5h^3$ $(-\infty < x < \infty;\ -\infty < h < \infty)$. **2630.** $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{(x-1)^n}{n}$ $(0 < x \leqslant 2)$.

2631. $\sum\limits_{n=0}^{\infty} (-1)^n (x-1)^n$ $(0 < x < 2)$. **2632.** $\sum\limits_{n=0}^{\infty} (n+1)(x+1)^n$ $(-2 < x < 0)$.

2633. $\sum\limits_{n=0}^{\infty} (2^{-n-1} - 3^{-n-1})(x+4)^n$ $(-6 < x < -2)$. **2634.** $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{(x+2)^{2n}}{3^{n+1}}$ $(-2 - \sqrt{3} < x < -2 + \sqrt{3})$. **2635.** $e^{-2}\left[1 + \sum\limits_{n=1}^{\infty} \dfrac{(x+2)^n}{n!}\right]$ $(|x| < \infty)$. **2636.** $2 + \dfrac{x-4}{2^2} - \dfrac{1}{4} \times \dfrac{(x-4)^2}{2^4} + \dfrac{1\cdot 3}{4\cdot 6} \cdot \dfrac{(x-4)^3}{2^6} - \dfrac{1\cdot 3\cdot 5}{4\cdot 6\cdot 8} \cdot \dfrac{(x-4)^4}{2^8} + \ldots + (-1)^{n-1} \dfrac{1\cdot 3\cdot 5 \ldots (2n-3)}{4\cdot 6\cdot 8 \ldots 2n} \times \dfrac{(x-4)^n}{2^{2n}} + \ldots$ $(0 \leqslant x \leqslant 8)$. **2637.** $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{\left(x - \frac{\pi}{2}\right)^{2n-1}}{(2n-1)!}$ $(|x| < \infty)$. **2648.** $\dfrac{1}{2} + \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{4^{n-1}\left(x - \frac{\pi}{4}\right)^{2n-1}}{(2n-1)!}$ $(|x| < \infty)$. **2639.** $-2 \sum\limits_{n=0}^{\infty} \dfrac{1}{2n+1} \left(\dfrac{1-x}{1+x}\right)^{2n+1}$ $(0 < x < \infty)$. **Indicação.** Fazer a substituição $\dfrac{1-x}{1+x} = t$ e desenvolver $\ln x$ em série de potências de t. **2640.** $\dfrac{x}{1+x} + \dfrac{1}{2}\left(\dfrac{x}{1+x}\right)^2 + \dfrac{1\cdot 3}{2\cdot 4}\left(\dfrac{x}{1+x}\right)^3 + \ldots + \dfrac{1\cdot 3\cdot 5 \ldots (2n-3)}{2\cdot 4\cdot 6 \ldots (2n-2)} \left(\dfrac{x}{1+x}\right)^n + \ldots$ $\left(-\dfrac{1}{2} \leqslant x < \infty\right)$. **2641.** $|R| < \dfrac{e}{5!} < \dfrac{1}{40}$.

2642. $|R| < \dfrac{1}{11}$. **2643.** $\dfrac{\pi}{6} \approx \dfrac{1}{2} + \dfrac{1}{2}\,\dfrac{\left(\frac{1}{2}\right)^3}{3} + \dfrac{1\cdot 3}{2\cdot 4}\,\dfrac{\left(\frac{1}{2}\right)^5}{5} \approx 0{,}523$. **Indicação.** Para demonstrar que o erro não excede a 0,001 é preciso avaliar o resto da série através da progressão geométrica que excede a este resto. **2644.** Dois termos, isto é, $1 - \dfrac{x^2}{2}$. **2645.** Dois termos, isto é, $x - \dfrac{x^3}{6}$. **2646.** Oito termos, isto é, $1 + \sum\limits_{n=1}^{7} \dfrac{1}{n!}$. **2647.** 99; 999. **2648.** 1,92. **2649.** $|R| < 0{,}0003$. **2650.** 2,087.

2651. $|x| < 0{,}69$; $|x| < 0{,}39$; $|x| < 0{,}22$. **2652.** $|x| < 0{,}39$; $|x| < 0{,}18$. **2653.** $\dfrac{1}{2} - \dfrac{1}{2^3\cdot 3\cdot 3!} \approx 0{,}4931$. **2654.** 0,7468. **2655.** 0,608. **2656.** 0,621. **2657.** 0,2505.

2658. 0,026. **2659.** $1 + \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{(x-y)^{2n}}{(2n)!}$ $(-\infty < x < \infty;\ -\infty < y < \infty)$.

2660. $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{(x-y)^{2n} - (x+y)^{2n}}{2\cdot(2n)!}$ $(-\infty < x < \infty;\ -\infty < y < \infty)$.

2661. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{(x^2+y^2)^{2n-1}}{(2n-1)!}$ $(-\infty < x < \infty;\ -\infty < y < \infty)$. **2662.** $1 + 2\sum\limits_{n=1}^{\infty} (y-x)^n$; $|x - y| < y$. **Indicação.** $\dfrac{1-x+y}{1+x-y} = -1 + \dfrac{2}{1-(y-x)}$. Utilizar a progressão geométrica. **2663.** $-\sum\limits_{n=1}^{\infty} \dfrac{x^n + y^n}{n}$ $(-1 \leqslant x < 1;\ -1 \leqslant y < 1)$. **Indicação.** $1 - x - y + xy = (1-x)(1-y)$. **2664.** $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1} + y^{2n+1}}{2n+1}$ $(-1 \leqslant x \leqslant 1;\ -1 \leqslant y \leqslant 1)$. **Indicação.** $\operatorname{arctg} \dfrac{x+y}{1-xy} = \operatorname{arctg} x + \operatorname{arctg} y$ (quando $|x| \leqslant 1$, $|y| \leqslant 1$). **2665.** $f(x+h, y+k) = ax^2 + 2bxy + cy^2 + 2(ax+by)h + 2(bx+cy)k + ah^2 + 2bhk + ck^2$. **2666.** $f(1+h, 2+k) - f(1,2) = 9h - 21k + 3h^2 + 3hk - 12k^2 + h^3 - 2k^3$.

2667. $1 + \sum\limits_{n=1}^{\infty} \dfrac{[(x-2) + (y+2)]^n}{n!}$. **2668.** $1 + \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{\left[x + \left(y - \frac{\pi}{2}\right)\right]^{2n}}{(2n)!}$.

2669. $1 + x + \dfrac{x^2 - y^2}{2!} + \dfrac{x^3 - 3xy^2}{3!} + \ldots$. **2670.** $1 + x + xy + \dfrac{1}{2}x^2y + \ldots$.

2671. $\dfrac{c_1 + c_2}{2} - \dfrac{2(c_1 - c_2)}{\pi} \sum\limits_{n=0}^{\infty} \dfrac{\operatorname{sen}(2n+1)x}{2n+1}$; $S(0) = \dfrac{c_1 + c_2}{2}$; $S(\pm\pi) = \dfrac{c_1 + c_2}{2}$.

2672. $\dfrac{b-a}{4}\pi - \dfrac{2(b-a)}{\pi} \sum\limits_{n=0}^{\infty} \dfrac{\cos(2n+1)x}{(2n+1)^2} + (a+b) \sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{\operatorname{sen} nx}{n}$; $S(\pm\pi) = \dfrac{b-a}{2}\pi$. **2673.** $\dfrac{\pi^2}{3} + 4\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{\cos nx}{n^2}$; $S(\pm\pi) = \pi^2$. **2674.** $\dfrac{2}{\pi} \operatorname{sen\,h} a\pi \left[\dfrac{1}{2a} + \sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{a^2 + n^2} (a \cos nx - n \operatorname{sen} nx)\right]$; $S(\pm\pi) = \cos\operatorname{h} a\pi$. **2675.** $\dfrac{2 \operatorname{sen} a\pi}{\pi} \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n \operatorname{sen} nx}{a^2 - n^2}$ se a não é número inteiro; $\operatorname{sen} ax$, se a é um número inteiro. $S(\pm\pi) = 0$.

2676. $\dfrac{2 \operatorname{sen} a\pi}{\pi} \left[\dfrac{1}{2a} + \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{a \cos nx}{a^2 - n^2}\right]$, se a não é número inteiro; $\cos ax$ se a é número inteiro; $S(\pm\pi) = \cos a\pi$. **2677.** $\dfrac{2 \operatorname{sen\,h} a\pi}{\pi} \sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{n \operatorname{sen} nx}{a^2 + n^2}$; $S(\pm\pi) = 0$.

2678. $\dfrac{2 \operatorname{sen\,h} a\pi}{\pi} \left[\dfrac{1}{2a} + \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{a \cos nx}{a^2 + n^2}\right]$; $S(\pm\pi) = \cos\operatorname{h} a\pi$. **2679.** $\sum\limits_{n=1}^{\infty} \dfrac{\operatorname{sen} nx}{n}$.

2680. $\sum\limits_{n=1}^{\infty} \dfrac{\operatorname{sen}(2a-1)x}{2n-1}$; a) $\dfrac{\pi}{4}$, b) $\dfrac{\pi}{3}$; c) $\dfrac{\pi}{2\sqrt{3}}$. **2681.** a) $2\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{\operatorname{sen} nx}{n}$; b) $\dfrac{\pi}{2} - \dfrac{4}{\pi} \sum\limits_{n=1}^{\infty} \dfrac{\cos(2n-1)x}{(2n-1)^2}$; $\dfrac{\pi^2}{8}$. **2682.** a) $\sum\limits_{n=1}^{\infty} b_n \operatorname{sen} nx$, onde $b_{2k-1} = \dfrac{2\pi}{2k-1} - \dfrac{8}{\pi(2k-1)^3}$ e $b_{2k} = -\dfrac{\pi}{k}$; b) $\dfrac{\pi^2}{3} + 4\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{\cos nx}{n^2}$; 1) $\dfrac{\pi^2}{6}$; 2) $\dfrac{\pi^2}{12}$.