Master in Actuarial Science, Exam 30/01/2013. **2h30m**
Ratemaking and Experience Rating, 2nd year, 1st semester

1. Consider a portfolio of independent risks, where each risk has associated a risk parameter, say θ. For a given risk X of that portfolio with parameter θ, we have n annual observations $\mathbf{X} = (X_1, ..., X_n)$. θ is realization of a random variable Θ such that $\Theta \frown \pi(\theta)$. Given $\Theta = \theta$, $\mathbf{X}$ is a random sample, where X_i is the claims number in year i. Furthermore, given θ, $X_i \frown \text{Poisson}(\theta)$. Prior distribution $\pi(\theta)$ is an Exponencial(β) with mean $1/\beta$.

   (a) Comment briefly on the meaning of credibility hypothesis summarized above (state those hypothesis clearly).

   (b) By computing the probabilitites $\Pr\{X_2 = 0\}$ and $\Pr\{X_2 = 0|X_1 = 0\}$, show that the components of $\mathbf{X}$ are not independent.

   (c) For the following four items only, consider: In the last year it has been observed for the risk that $X_1 = 0$. Also, consider $\beta = 5/3$.

      i. Compute the posterior distribution $\pi_{\Theta|X_1}(\theta|0)$.
      ii. Determine the Bayesian premium $E(X_2|X_1 = 0)$.
      iii. Compute the structural parameters $\mu = E(\mu(\Theta))$, $\upsilon = E(\upsilon(\Theta))$ and $a = V(\mu(\Theta))$.
      iv. Compute Bühlmann's credibility premium. Compare with Bayesian premium and comment apropriately.

   (d) The portfolio is composed by 350 risks that generated 209 claims in the last year according to the following table:

No. of Claims	0	1	2	3
No. of policies	210	81	49	10

   Consider Bühlmann's credibility model and a given risk that has produced two claims in the last year.

      i. Calculate proper estimates for the structural parameters $\mu = E[\mu(\theta)]$, $\upsilon = E[\upsilon(\theta)]$, $a = V[\mu(\theta)$.
      ii. Compute the *(empirical)* Bühlmann's credibility premium for that risk.

2. Consider a *bonus* system based on the claims frequency to rate the risk of some given motor insurance homogeneous portfolio. The system has simply three classes, numbered 1, 2, and 3 and ranked increasingly from low to higher risk. Transition rules are the following: A policy with no claims in one year goes to the previous lower class in the next year unless it is already Class 1, where it stays. In the case of a claim goes up Class 3, if it is already there no change is made. Entry class is Class 2 and premia vector is given by $\mathbf{b} = (150, 225, 300)$.

   Let θ be the probability of not having any claim in one year for risk in the portfolio.

   (a) Write the transition rules matrix and compute the one year transition probability matrix;

   (b) Discuss the existence of the stationary distribution and, considering that it exists, compute it. Compute the average premium.

   (c) For a policy entering in the system:

      i. What is the probability of belonging to Class 3 after two years?
      ii. How much in total will the policyholder expect to pay as premia in the two years?

   (d) For the following two items only: Suppose that the probability of a given policy have one or more claims is 0.1.

      i. Compute the probability distribution of the premia one year after entering in the system. Calculate the average premium.
      ii. Calculate the similar quantitites in stationary conditions. Discuss brief and appropriately.

   (e) Now, consider that θ is an outcome of a random variable following an Uniform distribution: $\Theta \frown \text{U}(0;1)$. Compute the stationary distribution and the average premium. Comment briefly.

3. For tariff modelling purposes, we studied different factors with impact in the single claim severity mean. We have first selected 3 factors labeled as F1 to F3. In F1 five levels were set, three levels in F2 and five levels in F3. For each claim the amount of the claim as well as the level of each factor per policy are available. For decisions consider a significance level of 5%. See Annex.

   Consider the first estimated model in the Annex as main reference.

   (a) Show how could you infer about the relevance of the factors (jointly) considered.

   (b) Test the relevance of level 3 of factor F1 (F13).

   (c) Is it statiscally acceptable to eliminate factor F3?

   (d) Test if it is acceptable an aggravation of 4% to a policy at level 2 of F2 when compared to a policy at level 1 of the same factor.

   (e) What is your estimate for the expected value of a claim of a policy at level 2 in each of the factors (F1-F3).

Marks (out of 200):

1.*a)*	*b)*	*c)i.*	*ii.*	*iii.*	*iv.*	*d)i.*	*ii.*	2.*a)*	*b)*	*c)i.*	*ii.*	*d)i.*	*ii.*	*e)*	3.*a)*	*b)*	*c)*	*d)*	*e)*
10	15	25	10	10	15	15	10	10	10	7.5	7.5	7.5	7.5	15	5	5	5	5	5
(10	25	50	60	70	85	100	110	120	130	137.5	145	152.5	160	175	180	185	190	195	200)

Model 1

```
Call:
glm(formula = cost ~ FF1 + FF2 + FF3, family = Gamma(link = "log"),
    data = dados)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.10522  -0.42749  -0.08666   0.24440   4.94412

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.29139    0.01793 406.722  < 2e-16 ***
FF12        -0.06924    0.01536  -4.508 6.60e-06 ***
FF13        -0.02757    0.01665  -1.656 0.097695 .
FF14        -0.19258    0.01774 -10.857  < 2e-16 ***
FF15        -0.18173    0.01917  -9.479  < 2e-16 ***
FF22         0.05854    0.01052   5.564 2.68e-08 ***
FF23         0.16710    0.01304  12.817  < 2e-16 ***
FF32         0.05952    0.01555   3.827 0.000130 ***
FF33         0.02090    0.01739   1.202 0.229354
FF34         0.11383    0.01897   5.999 2.02e-09 ***
FF35         0.08263    0.02125   3.888 0.000101 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.3055630)

    Null deviance: 4590.4  on 16353  degrees of freedom
Residual deviance: 4434.6  on 16343  degrees of freedom
AIC: 260923

Number of Fisher Scoring iterations: 5
```

Model 2

```
Call:
glm(formula = cost ~ 1, family = Gamma(link = "log"), data = dados)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.16769  -0.43538  -0.08663   0.25163   4.87415

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 7.327974   0.004345    1687   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.3087079)

    Null deviance: 4590.4  on 16353  degrees of freedom
Residual deviance: 4590.4  on 16353  degrees of freedom
AIC: 261493

Number of Fisher Scoring iterations: 5
```

Model 3

```
Call:
glm(formula = cost ~ FF1 + FF2, family = Gamma(link = "log"),
    data = dados)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.12497  -0.42891  -0.08752   0.24524   5.06415

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.33012    0.01434 511.171  < 2e-16 ***
FF12        -0.06457    0.01535  -4.206 2.61e-05 ***
FF13        -0.01958    0.01657  -1.182    0.237
FF14        -0.18099    0.01758 -10.295  < 2e-16 ***
FF15        -0.16911    0.01890  -8.947  < 2e-16 ***
FF22         0.07164    0.01000   7.161 8.34e-13 ***
FF23         0.19183    0.01141  16.814  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.3076068)

    Null deviance: 4590.4  on 16353  degrees of freedom
Residual deviance: 4452.4  on 16347  degrees of freedom
AIC: 260983

Number of Fisher Scoring iterations: 5
```