SOLUÇÕES DOS EXERCÍCIOS

CAPÍTULO 1

1.

Sem reposição:

a) Ω={(1,2), (1,3), (1,4), (1,5), (2,1), (2,3), (2,4), (2,5), (3,1), (3,2), (3,4), (3,5), (4,1), (4,2), (4,3), (4,5), (5,1), (5,2), (5,3), (5,4)}

b) A_1 = {(1,2), (1,3), (1,4), (1,5), (2,1), (2,3), (2,4), (2,5)}

A_2 = {(2,1), (3,1), (4,1), (5,1), (1,2), (3,2), (4,2), (5,2)}

A_3 = {(1,2), (2,1)} = $A_1 \cap A_2$

$A_4 = A_1 \cup A_2$

$A_5 = A_4 - A_3$

A_6 = {(1,2), (1,3), (1,4), (1,5), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1), (4,2), (5,1)}

Com reposição:

a) Ω={(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)}

b) A_1 = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5)}

A_2 = {(1,1), (2,1), (3,1), (4,1), (5,1), (1,2), (2,2), (3,2), (4,2), (5,2)}

A_3 = {(1,1), (1,2), (2,1), (2,2)} = $A_1 \cap A_2$

$A_4 = A_1 \cup A_2$

$A_5 = A_4 - A_3$

A_6 = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)}

2.

$$\Omega = \{(x, y) : 0 \le x \le 1600 \wedge 0 \le y \le 1600\} \subset \Re^2$$

$$A = \{(x, y) \in \Omega : x \le 1000 \wedge y \le 1000\}$$

$$B = \{(x, y) \in \Omega : (x > 1000 \wedge y \le 1000) \vee (x \le 1000 \wedge y > 1000)\}$$

$$C = \{(x, y) \in \Omega : x \ge 2y \vee y \ge 2x\}$$

$$D = \{(x, y) \in \Omega : x + y < 2000\}$$

3.

a)	b)	c)	d)	e)
$A \cap \overline{B} \cap \overline{C}$	$A \cap \overline{B} \cap C$	$A \cup B \cup C$	$(A \cap B) \cup (A \cap C) \cup (B \cap C)$	$A \cap B \cap C$

f) $\overline{A} \cap \overline{B} \cap \overline{C}$ g) $(\bar{A} \cap \bar{B}) \cup (\bar{A} \cap \bar{C}) \cup (\bar{B} \cap \bar{C})$ h) $\overline{A \cap B \cap C}$

i) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$ j) Ω

4. a) $\overline{A_1}$ b) $A_1 \cup A_2$ c) $\overline{A_1} \cap \overline{A_2}$ d) $\overline{A_1} \cup A_2$ e) $A_1 \cap \overline{A_2}$ f) $\overline{A_1} \cap A_2$ g) $E \cup F$

   b) São

   c) $G = E \cup F$

   d) A realização de F implica a realização de A $(F \subset A)$

5. a) 0.35 b) 0.22 c) 0.65

6. a) 0.08 b) 0.37

7. a) 19/27

8. $1-(1/2)^{10}$, $1-(1/2)^{20}$, $1-(1/2)^{20}$, $1-(1/2)^{10}$, $(1/2)^{10}-(1/2)^{20}$ e $(1/2)^{20}$

9. 2/3

11. $1-\sqrt{3}/2$

12. $B_k = A_k - \bigcup_{i=1}^{k-1} A_i$

13. a) 0.312 b) 2/3

14. 0.2143

15. 0.6778

16. a) 0.9745 b) 0.6274

17. a) 0.2(6) b) 0.8061

18. a) 0.02 b) 0.188

19. a) 0.19 b) 0.80 c) 0.01

20. a) 5/8 b) São...

21. a) 0.125 b) 0.124 c) 0.332

22. 0.5

23. a) 25% b) 40%

24. a) 0.125 b) Não (0.76 < 0.80)

25. a) 13/38 b) 0.381, 0.190 e 0.4289

26. 0.7792

27. a) 6.73% b) Verdade (0.577 > 0.423)

28. a) 0.375 b) 1/3

29. a) Sim (0.965 > 0.95); b) alfa3 (3/7> 2/7)

30. 0.2

31. a) 0.9 b) 0.675 c) 0.308

33. a) 0.294 b) 0.01(8)

CAPÍTULO 2

35.

a) $X^{-1}\left([3,5)\right)=\left\{(P_1\cap A_2),(A_1\cap P_2),(P_1\cap V_2),(V_1\cap P_2),(A_1\cap A_2)\right\}$

b) $F(x)=\begin{cases}0, & x<2\\ 2/21, & 2\le x<3\\ 5/21, & 3\le x<4\\ 21/35, & 4\le x<5\\ 4/5, & 5\le x<6\\ 1, & x\ge 6\end{cases}$ c) 0.7024

36.

a) $F(x)=\begin{cases}0, & x<0\\ 0.3, & 0\le x<1\\ 0.6, & 1\le x<2\\ 0.8, & 2\le x<3\\ 0.9, & 3\le x<4\\ 1, & x\ge 4\end{cases}$ b) 2 c) $f(y)=\begin{cases}0.3 & y=0\\ 0.3, & y=1\\ 0.3, & y=2\\ 0.1, & y=3\end{cases}$

37. a) 0.2 e 0.3 b) 0.6

c)

Y	0	1	2
F(y)	0.2	0.2	0.6

38. a) 0.0139 b) P.V.

39.

a) $F(x)=\begin{cases}0, & x<0\\ x^2/8, & 0\le x<2\\ -x^2/8+x-1, & 2\le x<4\\ 1, & x\ge 4\end{cases}$ b) $f(y)=\begin{cases}y/16, & 0<y<4\\ (8-y)/16, & 4<y<8\end{cases}$ c) 3.3675

40.

a) $F(x)=\begin{cases}0, & x<0\\ x^2/2, & 0\le x<1\\ x/2, & 1\le x<2\\ 1, & x\ge 2\end{cases}$ b-i) $f(y)=\begin{cases}(y+2)/16, & -2<y<2\\ 1/8, & 2<y<6\end{cases}$

b-ii) $f(w)=\begin{cases}(w-\sqrt{w})/2w, & 1<w<4\\ \sqrt{w}/4w, & 4<w<9\end{cases}$ c) $F(u)=\begin{cases}0, & u<-1\\ 1/8, & -1\le u<0\\ 3/4, & 0\le u<1\\ 1, & u\ge 1\end{cases}$ (discreta)

41. a) 0.0625 b) 0.136

42. b) 0.875 c) $F(y)=\begin{cases}0, & y<0\\ (y^3+1)/2, & 0\le y<1\\ 1, & y\ge 1\end{cases}$

43.

a) 0.42 b) 27%

c) $f_1(x)=\begin{cases}0.1, & x=0\\ 0.2, & x=1\\ 0.4, & x=2\\ 0.3, & x=3\end{cases}$ $f_2(y)=\begin{cases}0.1, & y=0\\ 0.5, & y=1\\ 0.4, & y=2\end{cases}$ d) $f(z)=\begin{cases}0.01, & z=0\\ 0.07 & z=1\\ 0.18 & z=2\\ 0.31, & z=3\\ 0.31, & z=4\\ 0.12, & z=5\end{cases}$

44. a) e b)

↓y x→	0	1	2	$f_2(y)$
0	0.81	0.126	0.0049	0.9409
1	0.054	0.0042		0.0582
2	0.0009			0.0009
$f_1(x)$	0.8649	0.1302	0.0049	1

Não são independentes

c) 0.062

d)

Z	0	1	2
f(z)	0.81	0.18	0.01

45. 5/36

46. a) 6 b) são independentes

47. a) 2, não são independentes b) 87.5% c) 25% d) 75%

e) $f(y\mid x)=1/x, 0<y<x$; x fixo em $]0,2[$

f) $f(x\mid y)=1/(2-y), y<x<2$; y fixo em $]0,2[$; 1/5

48. a) 6 b) $f_1(x)=3(1-x)^2, 0<x<1$; $f_2(y)=3(1-y)^2, 0<y<1$; Não c) 0.5

49. a) $a=b^2/2$ b) $f_2(y)=y/8, 0<y<4$ c) 9/32

50. a) 50% b) são independentes

51. $f(y_1) = -\ln y_1, 0 < y_1 < 1$

52.a) $f(x,y) = 2(5-x)/125, 0 < x < 5; 0 < y < 5$

$$F(x,y) = \begin{cases} 0, & x < 0 \vee y < 0 \\ (10xy - x^2 y)/125, & 0 \le x < 5, 0 \le y < 5 \\ (50x - 5x^2)/125, & 0 \le x < 5, y \ge 5 \\ y/5, & x \ge 5, 0 \le y < 5 \\ 1, & x \ge 5, y \ge 5 \end{cases}$$

b) 1/3 c) 1/3 d) 0.072

53. $f(u) = \begin{cases} \frac{4}{3} e^{2u}\left(\frac{1}{3} - u\right), & u \le 0 \\ \frac{4}{9} e^{-u}, & u > 0 \end{cases}$ Não, $P(U < 0) = 5/9$

54. a) $F(x,y) = \begin{cases} 0, & x < 0 \vee y < 0 \\ x^2 y^2, & 0 \le x < 1, 0 \le y < 1 \\ x^2, & 0 \le x < 1, y \ge 1 \\ y^2, & x \ge 1, 0 \le y < 1 \\ 1, & x \ge 1, y \ge 1 \end{cases}$ b) $g(u,v) = 1; 0 < u < 1, 0 < v < 1$

c) $f(w) = \begin{cases} 32w^3/3, & 0 < w < 1/2 \\ -32w^3/3 + 16w - 16/3, & 1/2 < w < 1 \end{cases}$

CAPÍTULO 3

55. a) 2.1; 2.09; b) 1.3; 0.41; c) 0.448(3); 0.0869

56. a) 13/12; 35/144; b) 1+0.5ln2; c) 7/3; 55/12; 1/8

57. a) 25%; b)25%; c) 25%

58. a) (i) $b = c$; (ii) $c = 0$ e $a = 1 - 2b$; b = 0 e $a = 1 - 2c$

b)

W	0	2
$f(w)$	$a + 2b$	$2c$

59. a) 0.859; b) são independentes...; c) $x = 2.16, \quad 0 < y < 2$

60. a) Não são independentes; b) 0.25; c) $y = \frac{2}{3}x, \quad 0 < x < 1$

61. a) e b)

↓y x→	0	1	$f_2(y)$
0	0.1	0.3	0.4
1	0.4	0.2	0.6
$f_1(x)$	0.5	0.5	1

c) – 0.408; d) 1/3

62. b) 0.375; c) $E[Y \mid x] = 0.4x, \quad 0 < x < 1$

63. a) 8/3; b) 20/7

64. a) 17.36; b) 0.167

65. 0.688; - 0.1748; 1.599; 1.3; 2; $[2,3]$; 1;

0.455; 0.114; 2.007; 0.42014; 0.793; 1; 0.8438

66. a) 50/32; 295/256; c) – 0.0366

67. 0.91

68. a) 0.5; b) 63/64

69. Não, há mais de 88.8%

70. 38;

71 a) 0.9775; b) 0.9456; c) 0.9838

CAPÍTULO 4

72. a) $f(x)=1/1000,\quad x=0,1,...,999$; $E[X]=499.5;\quad Var(X)=83333.25$

b) $f(y)=1/1000,\quad y=0,5,...,4995$;

$E[Y]=2497.5;\quad Var(X)=2083331.25$

c) € 2497.5

73. $f(y)=\dfrac{1}{b-(a-1)},\quad y=a,a+1,...,b$;

$E[Y]=\dfrac{a+b}{2};\quad Var(Y)=\dfrac{[b-(a-1)]^2-1}{12}$ 74. a) $s=3;\ E[Lucro]=24/5$

b) $s=3;\ E[Lucro]=23/5$

75. 0.376

76. 0.5367

77. a) 2

b) 0.5599; 0.1891

78. a) 1/8

b) 7/8

79. a) 0.0988

b) 0.0754

80. a) 63.28%

b) 1.5

81. a) $f(x)=\left(\dfrac{364}{365}\right)^{x-1}\dfrac{1}{365},\quad x=1,2,3,..$

b) 365; 132860; 364.5

c) 0.3337; 0.5597

82. 0.1024

83. 0.3697

84. (i) 0.4420; (ii) 0.5396; (iii) 0.0184

85. 0.0242

86. a) 3000

b) 0.3679

c) 0.4866

87. 1/4

90. 0.7

92. a) 0.398

b) 0.902

c) 0.496

d) 0.6; 0.68

93. a) 0.01765

b) $\dfrac{\binom{6}{5}\binom{1}{1}\binom{42}{0}}{\binom{49}{6}} \cong 0$

94. a) 0.3416

b) 0.5

c) $f(x) = \dfrac{\binom{12}{x}\binom{12}{5-x}}{\binom{24}{5}}, \quad x = 0,1,2,...,5; \qquad E[X] = 2.5; \qquad Var(X) = 1.0326$

95. $f(x) = \dfrac{\binom{3}{x}\binom{4}{5-x}}{\binom{7}{5}}, \quad x = 1,2,3; \qquad E[X] = 15/7; \qquad Var(X) = 20/49$

96. a) X_1 : v.a. que representa o número de alunos que só andam a pé, nos 50

....

X_5 : v.a. que representa o número de alunos noutras situações, nos 50

$$f(x) = \frac{50!}{x_1!x_2!x_3!x_4!x_5!} 0.1^{x_1} 0.4^{x_2} 0.2^{x_3} 0.2^{x_4} 0.1^{x_5}, \quad x_i \text{ inteiro} \geq 0, \sum_{i=1}^{5} x_i = 50$$

b) 5,20,10,10,5 e $\begin{bmatrix} 4.5 & -2 & -1 & -1 & -0.5 \\ -2 & 12 & -4 & -4 & -2 \\ -1 & -4 & 8 & -2 & -1 \\ -1 & -4 & -2 & 8 & -1 \\ -0.5 & -2 & -1 & -1 & 4.5 \end{bmatrix}$

c) 5.3144×10^{-7}

97. a) 0.4405

b) 0.0067

c) 0.0062

98. 0.0144

99. a) 0.0803

b) 0.1246

c) 0.5580

100. a) 0.5488

b) 0.9927

101. 0.25

103. a) 0.3849

b) 0.5403

c) 0.0603

d) 0.0013

e) 0.9902

f) -1.282

104. a) 0.9398

b) 0.32

c) 600; 320

d) 109.7; 130.3

105. a) 43.7538

b) 0.4364

106. a) 0.0062

b) 0.9996

107. a) 9.375; 25%; 45%

b) 0.5799

108. 78.88%

109. a) 0.45

b) 0.2709

c) Não, a probabilidade de cumprir é inferior a 0.5

110. 0.1532

111. a) 0.62

b) 0.9987

112. $a = 6.26 \quad b = 27.49; \quad a = 7.261 \quad b \to \infty$

113. 23.5893

114. 8

115 a) 0.05

b) 56.25; 1265.625 (10^3 €)

116. a) -1.812

b) 0.6

c) -1.372

d) 2.228

117. a) -1.782

b) 0.99

c) -1.782

d) 1.782

e) $a = -2.179 \quad b = 2.179; \quad a \to -\infty \quad b = 1.782$

118. a) 20

b) 0.495

119. a) 0.11

b) 0.04

c) 0.03541

120. a) 4.75

b) $1.63(7) \times 10^{-4}$

121. a) 0.3175

b) 0.18484

122. a) 0.05

b) 0.975

c) 0.95

123. $\cong 0.95$

124. 81; 144; 0.5987

125. a) $N\left(86.4; \sqrt{40.96}\right)$

b) 0.4192

CAPÍTULO 5

127. $X \sim B(1,0)$

129. a) 2000

b) 1000

130. 250

131. $n \geq 3$ ($n \geq 10$, pela desigualdade de Chebychev)...

132. Não: $0.091 \notin V_{0.016}(0.05)$

[Com o teorema de Bernoulli, já se aceita, pois $0.091 \in V_{0.043}(0.05)$]

133. a) 74.86%

b) 76744.78

134. 2598

135. 0.0008

136. a) 13

b) $\cong 16500$

137. a) 0.4938

b) 0.03054

138. é insuficiente (0.2177 é uma probabilidade pouco tranquilizadora)

139. a (i) 0.1359; (ii) 0.1697; (iii) 0.1698

b (i) 0; (ii) 0.1052; (iii) 0.1063

140. (i) 0.4004, se não houver o cuidado de escrever previamente cada uma das probabilidades na forma $P(a < X \leq b)$; se houver esse cuidado, obter-se-ão os resultados 0.4004; 0.3686; 0.3133; 0.4557

(ii) 0.4116; 0.4309; 0.3261; 0.4731

(iii) 0.4226; 0.4026; 0.380; 0.4872