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**FUNAIR : CASE QUESTIONS:**

1. What are the initial assumptions, advantages and disadvantages of the three equipment renewal models?
2. Should Michael renew the ten old aircraft based on his initial calculations, and why?
3. Should Michael renew the ten old aircraft based on the new development, and why?
4. What are the critical items in this decision that cause Michael many sleepless nights?

(c) This case was prepared for the purpose of class discussion only and not as an illustration of either good or bad business practices. The characters of Michael Funagan and John Leahha are fictitious, and the revenues and costs are invented by a troll in Tullamore.

## **FUNAIR: KEEPING UP TO DATE**

Michael Funagan, CEO and controlling shareholder of FUNAIR, had introduced a new flying concept in Europe, offering no-frills service and very low cost flights between less congested airports such as Manchester and Dublin, Stansted and Oporto, and Liverpool and Santiago. His amazing success had led to a relatively modern fleet of 20 American Boeing 937s, half purchased ten years ago and half purchased five years ago. Each set of planes with an original investment cost of \$100 million, produced an initial annual revenue of \$80 million, with all in operating costs of \$20 million. FUNAIR was registered in the Marshall Islands, and so paid no tax.

Michael kept accurate revenue and cost records over the ten years. He observed that annual revenues declined and operating costs increased over time as shown in Table 1. Currently for the older planes revenues were \$60 million and operating costs \$30 million, still a fairly wide operating margin. Older planes required larger irregular maintenance repairs, and spare parts, not considered normal operating costs, and so deducted from revenue to arrive at net revenue. Also many passengers preferred newer planes, so even for the five year planes, ticket prices and load factors were higher than for the older planes, which also were fuel inefficient. It became obvious that the revenue declines and cost increases quarter by quarter were far from constant, and negatively correlated.

Boeing's chief salesman John Leahha approached Michael with an interesting proposition. Although generally aircraft prices had increased over the last two years, he would exchange the ten older planes for new planes at exactly the same net investment cost, that is \$100 million.

Michael always liked to keep up to date and cost efficient. He was assured that the new planes would achieve exactly the target revenues and costs as the old planes did ten years ago. He wondered "is this a good deal", if riskless interest rates are 7% ?

Table 1

	A	B	C	D	E	F	G	
1	<b>FUNAIR: REVENUE AND COSTS FOR TEN PLANES</b>							
2	Quarter	P	C	RETURN		CUM DRIFT		
3	40	80	20	P	C	$\theta_p$	$\theta_c$	
4	39	79.79	20.11	-0.0027	0.0053			
5	38	80.20	19.90	0.0052	-0.0106			
6	37	79.99	20.00	-0.0027	0.0054			
7	36	74.00	20.22	-0.0778	0.0109			
8	35	79.00	20.05	0.0654	-0.0086			
9	34	76.00	22.00	-0.0387	0.0928			
10	33	75.85	22.09	-0.0020	0.0040			
11	32	76.20	23.00	0.0046	0.0405			
12	31	75.57	20.00	-0.0084	-0.1398			
13	30	75.55	20.01	-0.0003	0.0005			
14	29	70.00	20.14	-0.0763	0.0066			
15	28	68.00	24.00	-0.0290	0.1753			
16	27	68.46	23.68	0.0067	-0.0135			
17	26	67.98	23.00	-0.0070	-0.0291			
18	25	67.95	20.00	-0.0005	-0.1398			
19	24	67.89	20.03	-0.0008	0.0016			
20	23	67.59	20.21	-0.0044	0.0088			
21	22	70.00	20.53	0.0350	0.0158			
22	21	67.00	25.00	-0.0438	0.1969			
23	20	66.53	25.35	-0.0070	0.0138			
24	19	66.18	25.62	-0.0053	0.0106			
25	18	65.67	26.01	-0.0077	0.0152			
26	17	63.00	28.00	-0.0415	0.0737			
27	16	62.83	28.15	-0.0027	0.0053			
28	15	63.16	27.85	0.0052	-0.0106			
29	14	62.99	28.01	-0.0027	0.0054			
30	13	62.64	28.31	-0.0055	0.0109			
31	12	62.91	28.07	0.0043	-0.0086			
32	11	62.52	28.42	-0.0063	0.0124			
33	10	62.39	28.53	-0.0020	0.0040			
34	9	62.68	28.27	0.0046	-0.0093			
35	8	62.16	28.74	-0.0084	0.0165			
36	7	62.15	28.75	-0.0003	0.0005			
37	6	61.94	28.94	-0.0033	0.0066			
38	5	61.64	29.23	-0.0050	0.0099			
39	4	62.05	28.84	0.0067	-0.0135			
40	3	61.62	29.24	-0.0070	0.0138			
41	2	61.62	29.24	0.0000	0.0000			
42	1	60.00	30.00	-0.0266	0.0257	-2.88%	4.05%	
43	ANNUALIZED DRIFT			4*AVERAGE()	4*AVERAGE()			
44	VOLATILITY			STDEV()*SQRT(4)	STDEV()*SQRT(4)			
45	CORRELATION			CORREL( , )				
46	$\theta_p$	LN(B42/\$B\$3)/(((\$A\$3+1-A42)/4)						
47	$\theta_c$	LN(C42/\$C\$3)/(((\$A\$3+1-A42)/4)						

Note there are two formulas suggested for calculating annualized P and C drifts: four times the average continuously compounded quarterly returns  $[\ln (P_{t+1}/P_t)]$  and also times  $[\ln (P_{t=1}/P_{t=n})]/(\text{cumulative number of quarters}/4)$ . Annual volatility is the standard deviation of the continuously compounded quarterly returns times the square root of four; the correlation is the Excel function CORREL for the quarterly returns of P and C.

A little rusty on capital budgeting skills, Michael turned to his former university teacher, the great grandson of Faustmann (1849). Use net present values (“deterministic”), said Professor W. Faustmann, who although aged was very wise, because that is a trusted and established method, also used by Professor Marshall at the University of Cambridge and Professor Jevons at the University of Manchester. But an Irish Professor Dubbs argued that Faustmann is out-of-date, since sales are highly variable, as you know, so use the adjusted Dobbs (2004) method (“stochastic P”), approved by British accounting and finance journals. Wait, said an up and coming American finance Professor Riskins. Faustmann and Dubbs are wrong. As you know both aircraft revenues and operating costs are variable, and hardly related, so use the new Adkins and Paxson (2011) approach (“stochastic P and C”). What a palaver, thought Michael, that these so-called experts are so disagreeable, and malign each other’s methods. What difference does it make anyhow?

Here is Professor Riskins’ story.

Asset renewal is relevant for aircraft and other assets. New aircraft tend to command premium prices relative to their incumbent rivals because of novel features, convenience and fuel efficiency. Following the onset of quality deterioration, these differentiating features fade because of the possible emergence of new competing assets. As a result, revenues decline and operating costs increase. Eventually, a threshold performance level is reached justifying replacing the planes. This regenerative process of eroding revenues and escalating operating costs followed by renewal investment is also observed for most service type industries. Examples

include other travel businesses such as cruise liners and vehicle rentals, entertainment businesses such as theaters, stadia and theme parks, and professional sport clubs such as football teams. Indeed most human resources, especially professionals and those with a high degree of specialization, require revitalization through periodic re-education so that ideas and presentational skills are enhanced and cost efficiency restored.

Professor Faustmann's deterministic model assumes that revenues (P) and costs (C) have constant trends (and no volatility around those trends), so there are just a few simple equations for determining the optimal  $\hat{P}$  and  $\hat{C}$  renewal triggers. The optimal renewal time is a function of the interest rate,  $r$ , revenue decline rate  $\theta_P$  and volatility  $\sigma_P$ , and operating cost increase rate  $\theta_C$  and volatility  $\sigma_C$ .  $P_I$  denotes the revenues produced by new planes,  $C_I$  the new operating costs, and  $K$  the investment cost.

$$\hat{P} \left( \frac{1}{r} + \frac{\theta_P}{r} \times \frac{e^{-r\hat{T}}}{r - \theta_P} \right) - \hat{C} \left( \frac{1}{r} + \frac{\theta_C}{r} \times \frac{e^{-r\hat{T}}}{r - \theta_C} \right) = \frac{P_I}{r - \theta_P} - \frac{C_I}{r - \theta_C} - K \quad (1)$$

The LHS is the current value of the old planes at the optimal renewal triggers. The RHS is the net present value of the replacement investment.

The optimal replacement time  $\hat{T}$  is:

$$\hat{T} = \frac{1}{\theta_C} \ln \left( \frac{\hat{C}}{C_I} \right) = \frac{1}{\theta_P} \ln \left( \frac{\hat{P}}{P_I} \right) \quad (2)$$

and

$$\theta_P \beta + \theta_C \eta - r = 0, \quad (3)$$

$$\left( \frac{P_I}{\hat{P}} \right)^\beta \left( \frac{C_I}{\hat{C}} \right)^\eta - e^{-r\hat{T}} = 0 \quad (4)$$

Converting the Dobbs (2004) one-factor (cost) model to a one-factor model with only uncertain sales, so  $\sigma_C=0$ ,  $\theta_C = 0$ , the sales threshold level is:

$$\frac{\hat{P}}{\beta(r - \theta_p)} \left( \beta - 1 + \left( \frac{P_I}{\hat{P}} \right)^\beta \right) - \frac{P_I}{r - \theta_p} + K = 0, \quad (5)$$

$$\beta = \left( \frac{1}{2} - \frac{\theta_p}{\sigma_p^2} \right) - \sqrt{\left( \frac{1}{2} - \frac{\theta_p}{\sigma_p^2} \right)^2 + \frac{2r}{\sigma_p^2}}. \quad (6)$$

Adkins and Paxson provide a two-factor renewal model for an asset characterized by both uncertain revenues and uncertain operating costs, and develop a quasi-analytical solution.

First of all, there is the solution of the “characteristic equation” for two stochastic factors:

$$Q(\beta, \eta) = \frac{1}{2} \sigma_p^2 \beta(\beta - 1) + \frac{1}{2} \sigma_c^2 \eta(\eta - 1) + \rho \sigma_p \sigma_c \beta \eta + \theta_p \beta + \theta_c \eta - r = 0. \quad (7)$$

Then there is the smooth pasting equation:

$$\frac{\hat{P}}{-\beta(r - \theta_p)} - \frac{\hat{C}}{\eta(r - \theta_c)} = 0. \quad (8)$$

Finally, there is the value matching equation:

$$\frac{\hat{C}}{\eta(r - \theta_c)} \left( 1 - \beta - \eta - \frac{P_I^\beta C_I^\eta}{\hat{C}^{\beta + \eta}} \left( \frac{-\beta(r - \theta_p)}{\eta(r - \theta_c)} \right)^{-\beta} \right) - \frac{P_I}{r - \theta_p} + \frac{C_I}{r - \theta_c} + K = 0. \quad (9)$$

It is easy to implement this solution, as shown in the American Aircraft Renewal Template, Table 4. The results can be compared to the Deterministic Method (see Table 2) by setting the revenue and cost volatility and correlation equal to zero. In order to compare to the equivalent Dobbs Method (see Table 3) which assumes the cost drift and volatility are zero, and the revenue and cost correlation also zero, the reversionary cost should be the same as the current cost, which shows the limitations of the one-factor model.

The templates show hypothetical drift, volatility and correlation figures. For this case, calculate the historical drifts, volatilities and correlation in Table 1, input these

values into the appropriate cells in Table 2 (and appropriate parameter values into Table 3 and Table 4). In Table 2, use Solver, setting B18=0, changing B19:B22, with constraints B23=B24, B15=0, B16:B17=B15. Since only three equations are available to solve four unknowns,  $\hat{C}$ ,  $\hat{P}$ ,  $\beta$  and  $\eta$ , for comparison of models specify  $\hat{C}$  from the deterministic model, and then solve for the other unknowns, seeing whether P is currently above or below the derived  $\hat{P}$ : if below, then aircraft replacement is justified. Note in using these templates, you are first solving equations (1)-(4) for Table 2, and the three equations (7)-(9) for Table 4. The limitations of the one factor model can be viewed by solving equations (5)-(6) for Table 3.

Table 2

	A	B	C	D	E	F	G	H	I	J	K	L
1	<b>American Aircraft Renewal Template</b>											
2	INPUT	<b>Deterministic</b>										
3	P <sub>i</sub>	80.00										
4	C <sub>i</sub>	20.00										
5	K	100.00										
6	C*	29.95										
7	σ <sub>P</sub>	0.00										
8	σ <sub>C</sub>	0.00										
9	ρ	0.00										
10	r	0.07										
11	θ <sub>P</sub>	-0.02										
12	θ <sub>C</sub>	0.04										
13												
14	OUTPUT											
15	Q(β,η)	0.0000										
16	SP	0.0000										
17	VM	0.0000										
18	SUM	0.0000										
19	β	-0.0063										
20	η	1.7469										
21	P*	<b>65.371</b>										
22	C*	29.953										
23	T <sub>C</sub>	10.098										
24	T <sub>P</sub>	10.098										
25	P*-C*	35.418										
26												
27	Deterministic											
28	Q(β,η)	B11*B19+B12*B20-B10				EQ 3						
29	SP	((B3/B21)^B19)*((B4/B22)^B20)-EXP(-B10*B23)				EQ 4						
30	VM	B35-B36-B37										
31	SOLVER	SET B18=0,CHANGING B19:B22,B23=B24										
32	T <sub>C</sub>	(1/B12)*(LN(B22/B4))				EQ 2						
33	T <sub>P</sub>	(1/B11)*(LN(B21/B3))				EQ 2						
34												
35	P* VALUE	831.52				EQ 1						
36	C* VALUE	709.29				EQ 1						
37	Renewal V-K	122.22				EQ 1						
38	NPV=0	0.0000				EQ 1						
39												
40	P* VALUE	B21*((1/B10)+(B11/B10)*(EXP(-B10*B24)/(B10-B11)))										
41	C* VALUE	B22*((1/B10)+(B12/B10)*(EXP(-B10*B24)/(B10-B12)))										
42	Renewal V-K	B3/(B10-B11)-B4/(B10-B12)-B5										
43	NPV=0	B35-B36-B37										
44												
45		ASSET DETERIORATION OVER THE YEARS										
46	YEARS	1	2	3	4	5	6	7	8	9	10	11
47	P	78.42	76.86	75.34	73.85	72.39	70.95	69.55	68.17	66.82	<b>65.50</b>	64.20
48	C	20.82	21.67	22.55	23.47	24.43	25.42	26.46	27.54	28.67	<b>29.84</b>	31.05
49	P-C	57.60	55.20	52.79	50.38	47.96	45.53	43.09	40.63	38.16	<b>35.66</b>	33.15
50	P	=\$B\$3*EXP(\$B\$11*B46)										
51	C	=\$B\$4*EXP(\$B\$12*B46)										

Table 3

	A	B	C	D	E	F
1	<b>American Aircraft Renewal Template</b>					
2	INPUT	<b>Stochastic P</b>				
3	$P_I$	80.00				
4	$C_I$	20.00				
5	K	100.00				
6	$C^*$	20.00				
7	$\sigma_P$	0.30				
8	$\sigma_C$	0.00				
9	$\rho$	0.00				
10	r	0.07				
11	$\theta_P$	-0.02				
12	$\theta_C$	0.00				
13						
14	OUTPUT					
15	$Q(\beta, \eta=0)$	0.0000	EQ 6			
16	VM	0.0000	EQ 5			
17	SUM	0.0000				
18	OUTPUT					
19	$\beta$	-0.7190				
20	$P^*$	<b>51.589</b>				
21	$\eta$	0.0000				
22						
23	Stochastic P					
24	$Q(\beta, \eta=0)$	$(0.5-B11/(B7^2))-\text{SQRT}((0.5-B11/(B7^2))^2+2*B10/(B7^2))-B19$				
25	VM	$(B20/(B19*(B10-B11)))*(B19-1+((B3/B20)^{B19}))-B3/(B10-B11)+B5$				
26						
27	SOLVER	SET B17=0, CHANGING B19:B20.				



Table 4

	A	B	C	D	E
1	<b>American Aircraft Renewal Template</b>				
2	INPUT	<b>Deterministic</b>	<b>Stochastic P</b>	<b>Stochastic P &amp; C</b>	
3	P <sub>I</sub>	80.00	80.00	80.00	
4	C <sub>I</sub>	20.00	20.00	20.00	
5	K	100.00	100.00	100.00	
6	C*	29.95	20.00	29.95	
7	σ <sub>P</sub>	0.00	0.30	0.30	
8	σ <sub>C</sub>	0.00	0.00	0.30	
9	ρ	0.00	0.00	0.00	
10	r	0.07	0.07	0.07	
11	θ <sub>P</sub>	-0.02	-0.02	-0.02	
12	θ <sub>C</sub>	0.04	0.00	0.04	
13					
14	OUTPUT				
15	Q(β,η)	0.0000	0.0000	0.0000	EQ 7
16	SP	0.0000	0.0000	0.0000	EQ 8
17	VM	0.0000	0.0000	0.0000	EQ 9
18	PART 1			1241.82	
19	PART 2			0.0984	
20	PART 3			-122.22	
21	SUM	0.0000	0.0000	0.0000	
22					
23	β	-0.0451	-0.7190	-0.5107	
24	η	1.7274	0.0000	0.8040	
25	P*	<b>65.371</b>	<b>51.589</b>	<b>57.076</b>	
26	T*	10.098			
27	Deterministic				
28	EQ 3	B11*B23+B12*B24-B10			
29	EQ 4	((B3/B25)^B23)*((B4/B6)^B24)-EXP(-B10*B26)			
30	T*	(1/B12)*LN(B6/B4)			
31	Stochastic P				
32	EQ 6	(0.5-C11/(C7^2))-SQRT((0.5-C11/(C7^2))^2+2*C10/(C7^2))			
33	EQ 5	(C25/(C23*(C10-C11)))*(C23-1+((C3/C25)^C23))-C3/(C10-C11)+C5			
34	Stochastic P & C				
35	EQ 7	0.5*(D7^2)*D23*(D23-1)+0.5*(D8^2)*D24*(D24-1)+D9*D7*D8*D23*D24+D11*D23+D12*D24-D10			
36	EQ 8	D25*D24*(D10-D12)-D6*D23*(D10-D11)			
37	EQ 9	D18*D19+D20			
38	PART 1	D25/(-D23*(D10-D11))			
39	PART 2	(-D23-D24-(1-(((D3^D23)*(D4^D24))/((D25^D23)*(D6^D24))))))			
40	PART 3	-D3/(D10-D11)+D4/(D10-D12)+D5			
41					
42	SOLVER	SET D21=0,CHANGING D23:D25.			
43		ILLUSTRATIVE			
44		CURRENT	P	C	
45	ANNUALIZED DRIFT		-0.020	0.040	
46	VOLATILITY		0.300	0.300	
47	CORRELATION		0.000		

## References

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## **New Developments:**

After making the initial calculations, Michael was not convinced that immediate replacements of even the older set of planes is warranted. He phoned Leahha with his decision.

Wait, cried Leahha. A new development has just occurred. Because of a special arrangement from the new Federal Aircraft Transportation Support (FATS), in these trying times, the US government has enabled us to reduce permanently the net investment cost (in exchange for the older 937s) to \$90 million. What now?