LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

Corporate Investment Appraisal

Masters in Finance

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Problem Set Nº 1: Guideline to Solution

1.

- There are no Nash equilibria (NE) in pure strategies. Why? For instance, let's start with (Heads,Heads): If player "Column" believes that player "Row" plays Heads, player "Column" would prefer to choose "Tails"; hence, he wouldn't choose Heads. Therefore (Heads,Heads) cannot be a NE. The same logic applies to the 3 other cases – there is always someone with an incentive to deviate given what (s)he believes the other player is doing.
- However, there is an equilibrium in mixed strategies. Suppose that player "Row" believes that player "Column" plays Heads with probability Ph and plays Tails with probability (1-Ph). Row's expected payoff is:
  - If she plays Heads: 1*Ph-1*(1-Ph)
  - If she plays Tails: -1*Ph+1*(1-Ph)
  - She is indifferent between the two strategies if: Ph=1/2.
  - Conclusion: if Row believes that Column will play Heads with probability ½, then Row is indifferent between playing Heads or Tails.
- Likewise, if player Column has the belief that player Row chooses Heads with probability Qh, Column's expected payoff will be:
  - If he plays Heads: -1*Qh+1*(1-Qh)

    - If he plays Tails: 1*Qh-1*(1-Qh)
    - Column will be indifferent between the two strategies if: Qh=1/2.
- In equilibrium the expectations of all players cannot be violated. Thus, in equilibrium, player Row is indifferent between the two strategies and may well choose Ph=1/2. If Row indeed plays Ph=1/2 and Column forms that belief (in equilibrium he gets it right), Column is indifferent between playing Heads or Tails. In equilibrium, Column chooses Qh=1/2, which would make Row indeed indifferent between Heads or Tails and possibly choosing Ph=1/2, which shows the consistency of this mixed strategy equilibrium, with (Ph=1/2,Qh=1/2).

2.

This is a possible sequence to determine a DS equilibrium:

- From player A's perspective, since 3≥3, 1≥0, and 0≥0, strategy M dominates C regardless of the other player's action). Thus, we eliminate row "Top".
- Since 2≥0 and 0≥0, for player B strategy Center dominates Left. We can eliminate "Left".
- Because 1≥1 and 2≥0, for Player A's strategy Bottom dominates Middle. So, we eliminate "Middle".
- Finally, as 1≥0, for Player B strategy Right dominates Center. We can eliminate "Center".
- We are left with (Bottom,Right), which is the only equilibrium in dominated strategies (DSE).

3.

(a) NE in Pure strategies: (B,L) and (T,R). Explain...

(b) NE in mixed strategies: Bill chooses Top with probability 1/2 and Ted chooses Left with probability 1/2.

(c) When the solution is (B,R) both players have strictly positive payoffs.

If they play the mixed strategy equilibrium, the probability of (B,R) happening is 1/2*1/2 =1/4.

In the case of pure strategies, the outcome (B,R) would not take place.

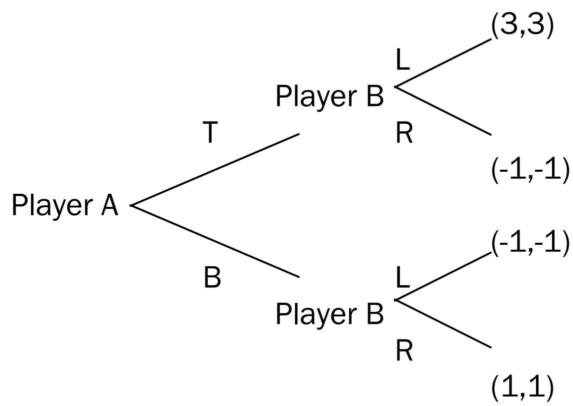
If we meant non-strictly positive payoffs, then the probability 1/4 would be revised to 1-Probability(T,L) = 1-1/2*1/2=3/4.

4.

(a) NE in pure strategies: (T,L), (B,R).

(b) No strategy dominates any other.

(c) and (d)



The sub-game perfect equilibrium of this game is (T,L). Why?

A plays first.

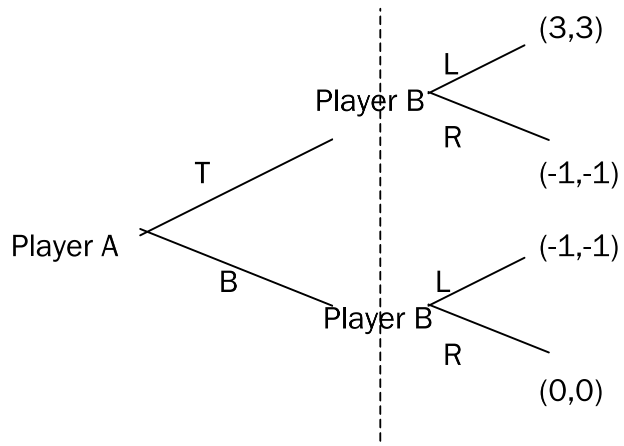
If A plays B, player B will choose R (1>-1). Hence, A would get 1.

If A plays T, then player B will choose L (because 3>-1). Hence A would get 3.

Therefore, player A chooses T, then player B chooses L, and the SPE is (T,L), with payoffs (3,3).

5.

(a) Extensive Form (assuming that player A plays first):



(b) Analysis of equilibrium:

(i) If player A believes that player B plays L with probability q and plays R with probability (1-q), player A knows that:

- If she plays T her expected payoff is 3q-1(1-q)=4q-1
- If she plays B her expected payoff is –q+(1-q)=1-2q

(ii) In equilibrium player A should choose (let's say p is the probability of player A choosing T):

- p=1 if 4q-1> 1-2 q
- p in [0,1] if q=1/3
- p=0 if q<1/3

(iii) If player B believes that player A chooses T with probability p, then he knows that:

- If he plays L his expected payoff is: 3p-1(1-p)=4p-1
- If he plays R his expected is –p+(1-p)=1-2p

(iv) Hence, Player B should choose according to (where q is the probability with which he plays L):

- q=1 if p>1/3
- q in [0,1] if p=1/3
- q=0 if p < 1/3

(v) Finally what will characterize na equilibrium:

- Start with the case in which A chooses $p>1/3$. If B guesses this right, B chooses $q=1$. But if $q=1$, and A guesses this right, then A would choose $p=1$, which is compatible with the initial conjecture of $p>1/3$. We found an equilibrium in which ($p=1$, $q=1$).
- If Player A chooses $p=1/3$, and B guesses this right, B is indifferent between L and R. He may choose any q in the interval [0,1]. In case B chooses $q=1/3$, that would be compatible with A "replying" $p=1/3$, since A would be indifferent. We found another PBE with ($p=1/3$,$q=1/3$).
- Finally, if A chooses $p<1/3$, and player B guesses this correctly, player B chooses $q=0$. But if B chooses $q=0$, and player A guesses this correctly, then player A should respond with $p=0$ (which is compatible with the conjecture that $p<1/3$). We found the third PBE of this game, with ($p=0$,$q=0$).