LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

Corporate Investment Appraisal

Masters in Finance

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Problem Set 11: Solutions

Risky Debt

1. Present an estimate of the value of debt of a company with the following features:
   - Stock Price: 15
   - Stock Volatility: 60%
   - Debt Par: 360
   - Maturity of Debt: 6 months
   - Risk free rate: 3%
   - Bankruptcy cost rate: 0%

Market Parameters

Stock Price:	15	r:	3%
Stock Vol:	60%	B/Ruptcy Cost:	0%
Debt Par:	360		

Asset Parameters

Asset Value:	369,6299
Asset Vol:	2,46%

Computation Parameters

dS/dV:	0,991499
Implied Stock Vol:	60,00%
Impl-Actual Vol:	0,00%
Impl-Actual Pr:	0,0000

Tree Parameters

Dt:	0,083333
u:	1,007114
d:	0,992936
p:	0,674553

Asset Value Tree

369,6299	372,2596	374,9080	377,5752	380,2614	382,9667	385,6912
	367,0189	369,6299	372,2596	374,9080	377,5752	380,2614
		364,4262	367,0189	369,6299	372,2596	374,9080
			361,8519	364,4262	367,0189	369,6299
				359,2957	361,8519	364,4262
					356,7577	359,2957
						354,2375

Stock Value Tree

15,0000	16,7286	18,4856	20,2617	22,0546	23,8644	25,6912
	11,5324	13,2154	14,9462	16,7012	18,4729	20,2614
		8,1327	9,7295	11,4232	13,1574	14,9080
			4,8854	6,2937	7,9166	9,6299
				2,0040	2,9783	4,4262
					0,0000	0,0000
						0,0000

Debt Value Tree

354,6299	355,5310	356,4224	357,3134	358,2067	359,1022	360,0000
	355,4865	356,4146	357,3134	358,2067	359,1022	360,0000
		356,2935	357,2893	358,2067	359,1022	360,0000
			356,9665	358,1325	359,1022	360,0000
				357,2918	358,8736	360,0000
					356,7577	359,2957
						354,2375

2. Consider the following application of the debt valuation model of Anderson and Sundaresan (1996), with just two periods (to simplify, consider 2 years). Company LM uses a technology such that the present value of its asset (at t=0) is V_0=90. This value evolves annually according to a binomial process with u=1.25 and d=1/u. The project generates annual cash flows (f_t) proportional to its present value, i.e., f_t = 0.2V_t. The risk free interest rate is 4% in both years of analysis (from t=0 to t=1, and from t=1 to t=2). There is a fixed cost of liquidation of the company, estimated as K=55. Suppose that the company issued debt at t=0, and that this debt contract requires an annual debt service in t=1 and t=2 of CS_1=CS_2=15. In t=1 and t=2 the owner/manager chooses the effective debt service to the creditor. If the debt service lies below the contracted amount, the creditor may accept it (and the game continues) or he may liquidate the firm.

Data:

$$CS_1 = CS_2 = 15$$
$$V_0 = 90$$
$$R_f = 4\%$$
$$f_t = 0.2$$
$$\text{Bankruptcy Cost} = K = 55$$

Tree Parameters:

$$D_t = 1$$
$$u = 1.25$$
$$d = 0.8$$
$$p = \frac{1.04(1-0.2)-0.8}{1.25-0.8} = 0.071$$

Asset Value Tree

t=0	t=1	t=2
90	112.5	140.625
	72.0	90
		57.6

Cash Flow Tree

t=0	t=1	t=2
18	22.5	28.125
	14.4	18
		11.52

(a) What is the debt service that the owner-manager of LM should offer at t=1 and at t=2? Explain.

- T=2

The manager will propose to pay:

$$S_2 = \min\left(CS_2, \max\left(V_2 - K, 0\right), f_2\right)$$

$$S_2^{++} = \min\left(15, \max\left(140.625 - 55, 0\right), 28.125\right) = 15$$

$$S_2^{+-} = S_2^{-+} = \min\left(15, \max\left(90 - 55, 0\right), 18\right) = 15$$

$$S_2^{--} = \min\left(15, \max\left(57.6 - 55, 0\right), 11.52\right) = 2.6$$

Note 2: In the final period, the value of debt corresponds to ST, unless there is forced liquidation (which is not the case in this example). Thus:

$$B\left(V_2^{++}\right) = 15$$

$$B\left(V_2^{+-}\right) = B\left(V_2^{-+}\right) = 15$$

$$B\left(V_2^{--}\right) = 2.6$$

- t=1

The manager will propose to pay the following debt services:

$$S_1 = \min\left(CS_1, \max\left(0, \max\left(V_1 - K, 0\right) - \frac{pB\left(uV_1\right) + \left(1-p\right)B\left(dV_1\right)}{1 + R_f}\right), f_1\right)$$

$$S_1^{+} = \min\left(15, \max\left(0, \max\left(112.5 - 55, 0\right) - \frac{0.071*15 + \left(1 - 0.187\right)*15}{1.04}\right), 22.5\right) = 15$$

$$S_1^{-} = \min\left(15, \max\left(0, \max\left(72 - 55, 0\right) - \frac{0.071*15 + \left(1 - 0.071\right)*2.6)}{1.04}\right), 14.4\right) = 13.65213675$$

Note: There is strategic default in state -.

(b) If I told you that the amount of money borrowed at t=0 was 20, would that seem credible to you? Explain why.

The present value of this debt (taking into account the debt services chosen in part (a) is inferior to the amount of the loan.

Let's see:

$$B\left(V_1^+\right) = 15 + \frac{0.071*15 + \left(1 - 0.071\right)*15}{1.04} = 29.4231$$

$$B\left(V_1^-\right) = 13.65213675 + \frac{0.071*15 + \left(1 - 0.071\right)*2.6}{1.04} = 17$$

$$B\left(V_0\right) = \frac{0.071*29.4231 + \left(1 - 0.071\right)*17}{1.04} = 17.1956$$