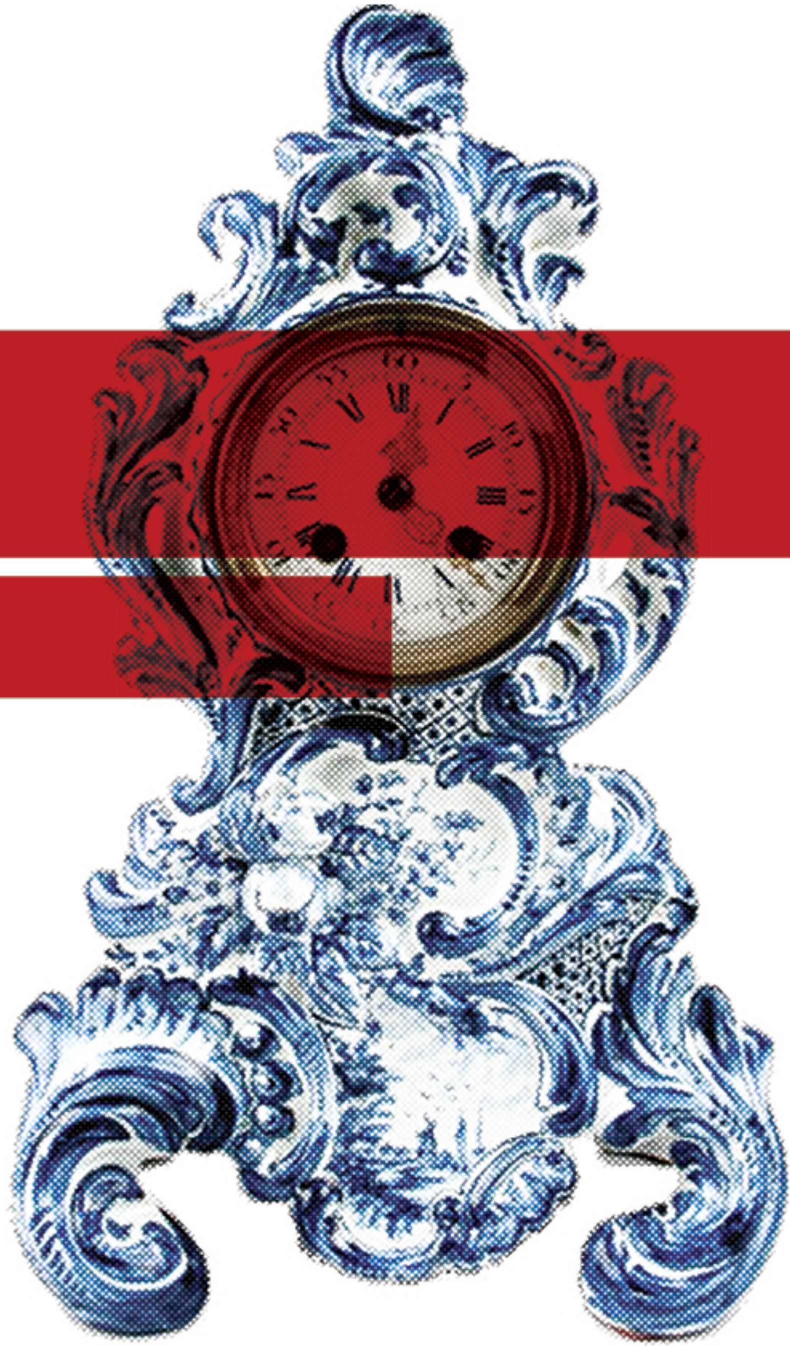
Masters in FINANCE

RISKY DEBT - I
Merton's Model

Corporate Investment Appraisal

Fall 2014



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OUTLINE

The Option Theory approach;

The discrete version of Merton's model;

A possible practical implementation.

The "Option Pricing" approach

Suppose a company has debt with face value D.

Contingent on the value of the firm's assets at maturity of this debt, the payoffs to shareholders and debt-holders will be:

(note: disregarding "bankruptcy costs.")

	$V \leq D$	$V > D$
Shareholders	0	V-D
Bondholders	V	D
Value	V	V

Shareholders have a call option, with payoffs at maturity:

VE = max (0,V-D)

Exercise Price = D

Underlying Asset = V

Debtholders wrote a put option, together with having riskless debt:

VD = D + min (0,V-D)

Exercise Price = D

Underlying Asset V

If we intended to apply, for example, the Black-Scholes formula, we would obtain:

$$VE = V \cdot N(d_1) - e^{-R_f T} \cdot D \cdot N(d_2)$$

$$d_1 = \frac{\ln\left(V/D\right) + R_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

We could then determine the value of debt via put-call parity.

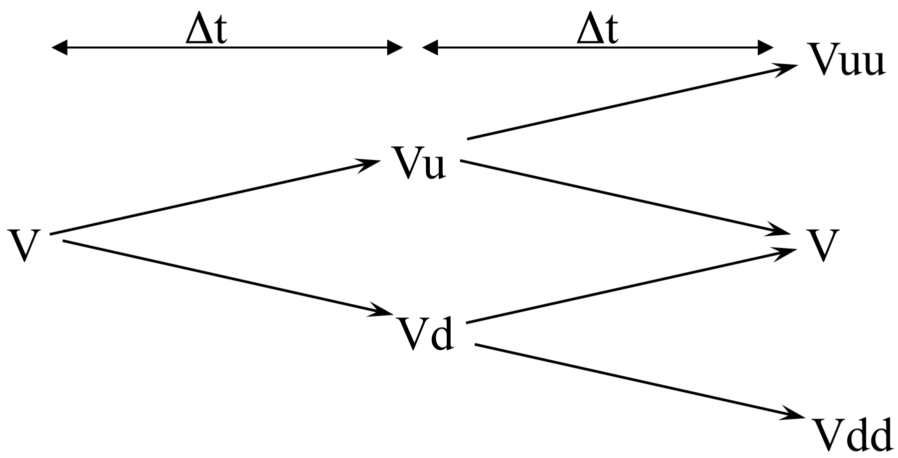
Discrete Version of Merton's Model

- Merton used a formalization in continuous time:

$$\frac{dV}{V} = \mu dt + \sigma dW$$

- Possibly the easiest way to implement this type of analysis is using the binomial tree.

Tree to Value the Assets



$u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$, where σ is the annualized volatility.

$$p = \frac{(1 + r\Delta t) - d}{u - d}$$

u and d incorporate a term of volatility, taking into account the ways in which the price can evolve.

r is the riskless interest rate.

p is a *pseudo-probability* (risk-neutral).

In the limit, for a very small t, this tree converges to a Geometric Brownian Motion.

Example of Valuation (à la Merton)

Value of Firm Equity = 3

Volatility of Equity = 25%

Term Structure of Interest Rates flat at 7%

The firm has a loan

- Maturity within six months;
- Face value 10;
- There are no coupons until maturity.

There will be default if the assets are worth less than the face value of debt.

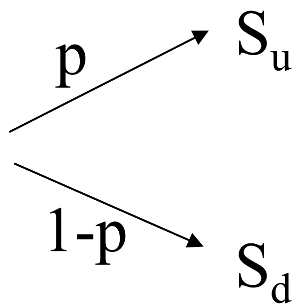
Tree for the Stock Price (Equity)

At debt maturity, the value of the shares = the value of the assets after payments to debt-holders

$$S_{0.5} = \max(V - 10, 0)$$

We use the standard binomial approach under risk neutrality in order to determine prices:

$$\frac{pS_u + (1-p)S_d}{1 + r\Delta t}$$



We need values for V and sigma in order to build the tree for S.

Parameters for the value of the Assets

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We try to guess sigma=25%,

$$u = e^{0.25\sqrt{0.25}} = 1.1331$$

$$d = e^{-0.25\sqrt{0.25}} = 0.8825$$

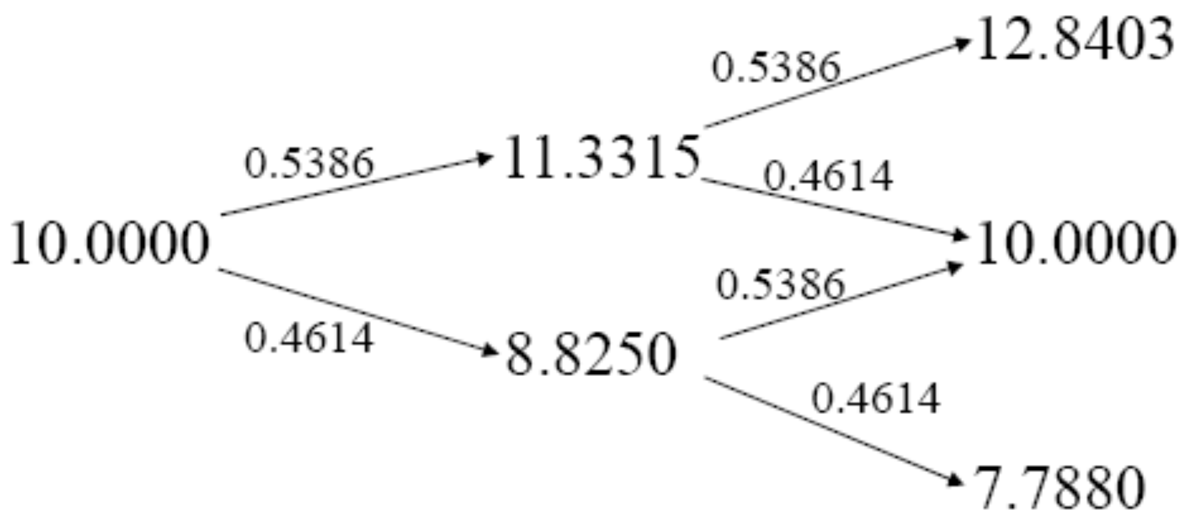
$$p = \frac{\left(1 + 0.07/4\right) - d}{u - d} = 0.5386$$

We try to guess value of assets equal to 10;

We use these values to build the tree.

Asset Value Tree

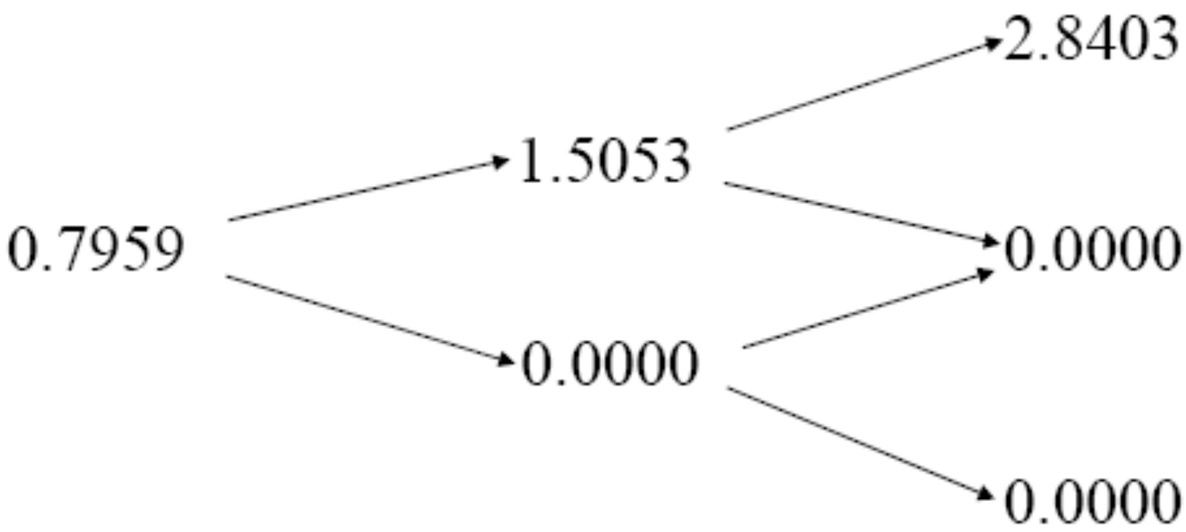
Using the parameters suggested before:



We work recursively from here…

Equity Tree:

V=10, sigma=25%



With the parameters we guessed, equity would be worth 0.7959;

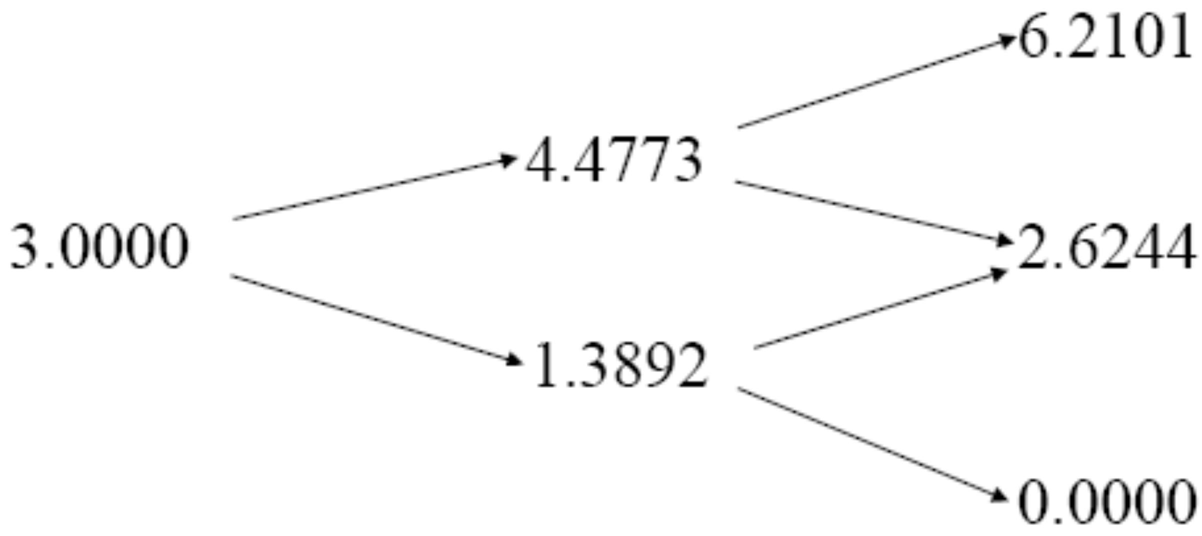
Not exactly what we were looking for...

To reach compatible values for V and S, we can keep sigma constant and change V until we get S=3.

Goalseek in Excel can be used for this.

Equity Tree:

V=12.6244, sigma=25%



Seems better: stock price resembles the real world's.

Is this sufficiently good to compute the value of Debt?

Compute σ_S

We know from Itô's Lemma that: $$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma$$

We expect the volatility of the shares to be higher or lower than the volatility of the assets?

We should check if we have sensible values for σ_S.

A standard way of computing $\frac{\partial S}{\partial V}$ based on the binomial tree is by looking at S and V in the first branches:

3.0000 → 4.4773
3.0000 → 1.3892

12.6244 → 14.3053
12.6244 → 11.1410

Compute σ_S

Approximately $$\frac{\partial S}{\partial V} = \frac{4.4773 - 1.3892}{14.3053 - 11.1410} = 0.9759$$

Our current parameters produce:

$$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma = 0.9759 \times \frac{12.6244}{3} \times 0.25 = 1.0267$$

The volatility of the shares when using these parameters is almost 103%.

Not exactly ideal yet...

We may keep V=12.6244 constant and alter σ until σ_S=25%.

Trees for V and S:
V=12.6244, σ=5.87%

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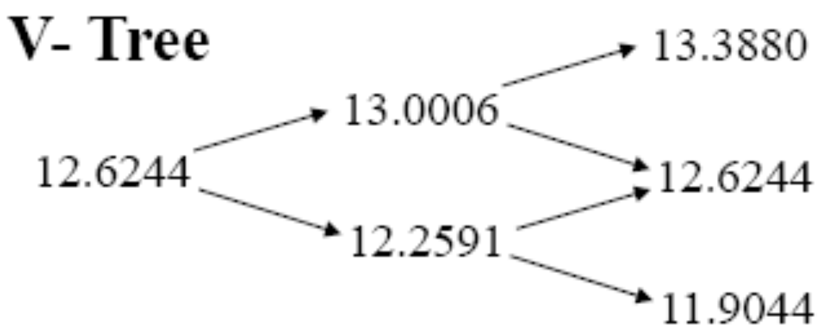
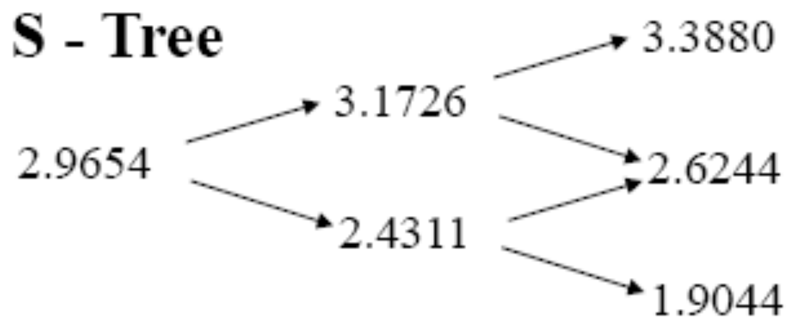




With these trees,

$$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma = \frac{3.1726 - 2.4311}{13.0006 - 12.2591} \times \frac{12.6244}{2.2591} \times 0.0587 = 25\%$$

S is now 2.9654, but we're getting there...

Determining the Parameters for V

Repeatedly we can iterate until being satisfied with both parameters.

This can be done with repeated use of Excel Goalseek, or using Excel Solver once.

In this case, the “correct” parameters are:

$$V = 12.6590$$

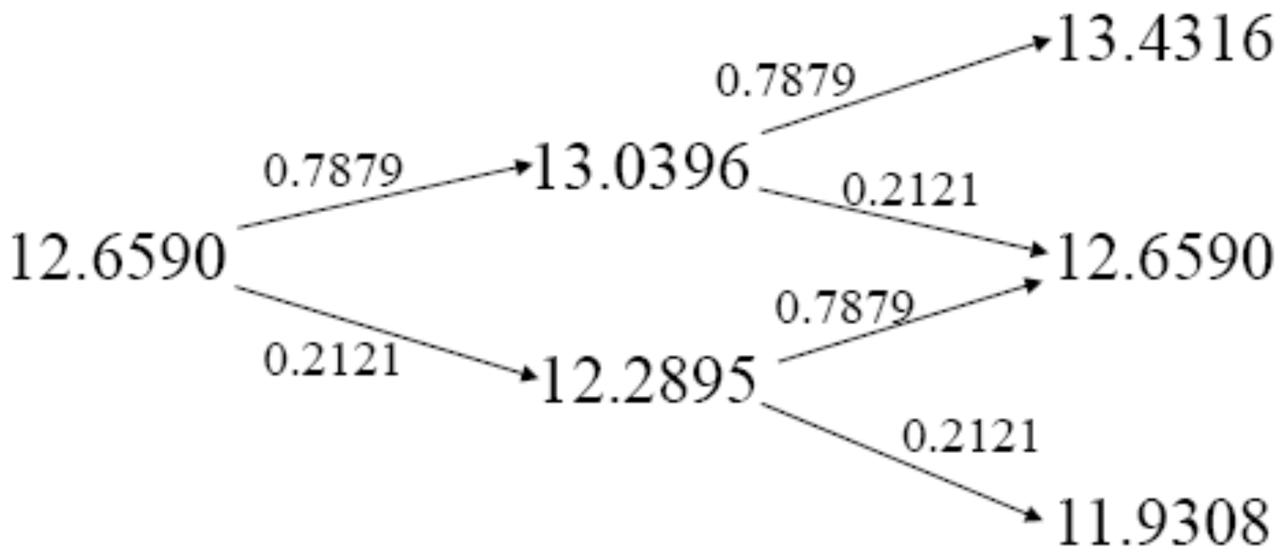
$$\sigma = 5.92\%$$

$$u = 1.0301$$

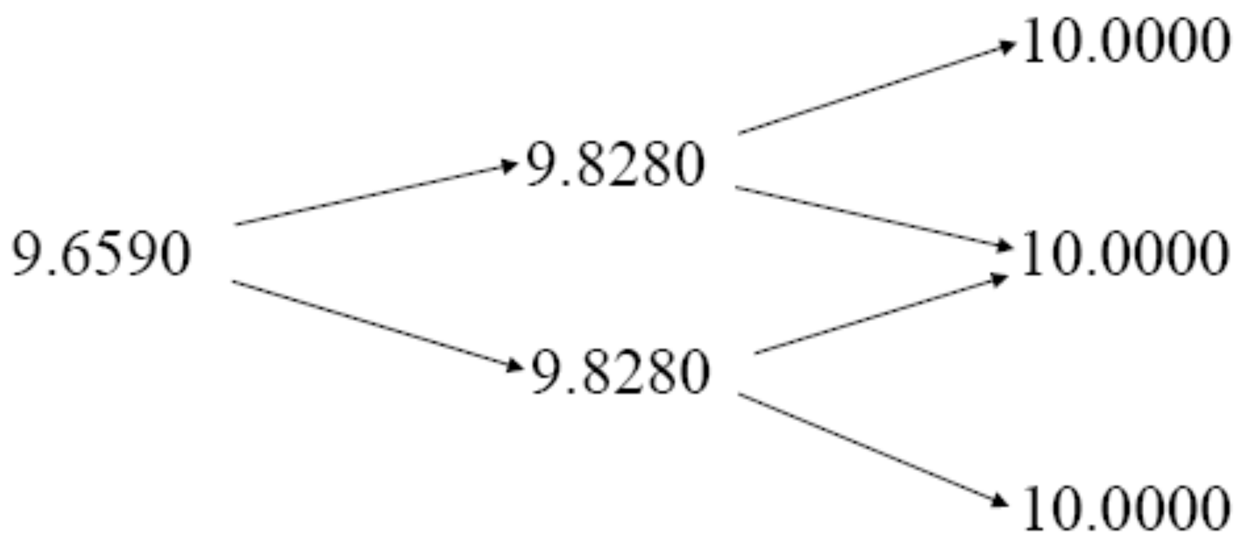
$$p = 0.7879$$

We can now use the tree for V in order to value debt.

Correct Random Walk for V



Tree for Valuation of Debt



In this example debt does not involve credit risk.

Exercise

The spreadsheet MertonExercise.xls contains a 6-step implementation of the model with the following parameters:

- A debt security, with 6 months maturity;
- Par Value 100;
- Flat Interest Rates 8%;
- Equity Value 30;
- Volatility of Shares 40%;
- Six steps to maturity, each 1 month long.