equanimity a writing-down of the value of their reserves, or unless one is prepared to forego the possibility of exchange-rate adjustment, any major extension of the gold exchange standard is dependent upon the introduction of guarantees. It is misleading to suggest that the multiple key-currency system is an alternative to a guarantee, as implied by Roosa [6, pp. 5-7 and 9-12].

IV. *Conclusion*

The most noteworthy conclusion to be drawn from this analysis is that the successful operation of a multiple key-currency system would require both exchange guarantees and continuing cooperation between central bankers of a type that would effectively limit their choice as to the form in which they hold their reserves. Yet these are two of the conditions whose undesirability has frequently been held to be an obstacle to implementation of the alternative proposal to create a world central bank. The multiple key-currency proposal represents an attempt to avoid the impracticality supposedly associated with a world central bank, but if both proposals in fact depend on the fulfillment of similar conditions, it is difficult to convince oneself that the sacrifice of the additional liquidity that an almost closed system would permit is worth while. Unless, of course, the object of the exercise is to reinforce discipline rather than to expand liquidity.

JOHN WILLIAMSON*

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* The author is instructor in economics at Princeton University. He acknowledges the helpful comments of Fritz Machlup. Views expressed are those of the author alone.

Corporate Income Taxes and the Cost of Capital: A Correction

The purpose of this communication is to correct an error in our paper "The Cost of Capital, Corporation Finance and the Theory of Investment" (this *Review*, June 1958). In our discussion of the effects of the present method of taxing corporations on the valuation of firms, we said (p. 272):

> The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their

physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that *the market values of firms in each class must be proportional in equilibrium to their expected returns net of taxes (that is, to the sum of the interest paid and expected net stockholder income)*. (Italics added.)

The statement in italics, unfortunately, is wrong. For even though one firm may have an *expected* return after taxes (our $\bar{X}^\tau$) twice that of another firm in the same risk-equivalent class, it will not be the case that the *actual* return after taxes (our X^τ) of the first firm will always be twice that of the second, if the two firms have different degrees of leverage.[1] And since the distribution of returns after taxes of the two firms will not be proportional, there can be no "arbitrage" process which forces their values to be proportional to their expected after-tax returns.[2] In fact, it can be shown—and this time it really will be shown—that "arbitrage" will make values within any class a function not only of expected after-tax returns, but of the tax rate and the degree of leverage. This means, among other things, that the tax advantages of debt financing are somewhat greater than we originally suggested and, to this extent, the quantitative difference between the valuations implied by our position and by the traditional view is narrowed. It still remains true, however, that under our analysis the tax advantages of debt are the *only* permanent advantages so that the gulf between the two views in matters of interpretation and policy is as wide as ever.

I. *Taxes, Leverage, and the Probability Distribution of After-Tax Returns*

To see how the distribution of after-tax earnings is affected by leverage, let us again denote by the random variable X the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, k.[3] From our definition of a risk class it follows that X can be expressed in the form $\bar{X}Z$, where $\bar{X}$ is the expected value of X, and the random variable $Z = X/\bar{X}$, having the same value for all firms in class k, is a drawing from a distribution, say $f_k(Z)$. Hence the

[1] With some exceptions, which will be noted when they occur, we shall preserve here both the notation and the terminology of the original paper. A working knowledge of both on the part of the reader will be presumed.

[2] Barring, of course, the trivial case of universal linear utility functions. Note that in deference to Professor Durand (see his Comment on our paper and our reply, this *Review*, Sept. 1959, *49*, 639–69) we here and throughout use quotation marks when referring to arbitrage.

[3] Thus our X corresponds essentially to the familiar EBIT concept of the finance literature. The use of EBIT and related "income" concepts as the basis of valuation is strictly valid only when the underlying real assets are assumed to have perpetual lives. In such a case, of course, EBIT and "cash flow" are one and the same. This was, in effect, the interpretation of X we used in the original paper and we shall retain it here both to preserve continuity and for the considerable simplification it permits in the exposition. We should point out, however, that the perpetuity interpretation is much less restrictive than might appear at first glance. Before-tax cash flow and EBIT can also safely be equated even where assets have finite lives as soon as these assets attain a steady state age distribution in which annual replacements equal annual depreciation. The subject of finite lives of assets will be further discussed in connection with the problem of the cut-off rate for investment decisions.

random variable X^τ, measuring the after-tax return, can be expressed as:

$$(1)\quad X^\tau = (1-\tau)(X-R)+R = (1-\tau)X+\tau R = (1-\tau)\bar{X}Z+\tau R$$

where τ is the marginal corporate income tax rate (assumed equal to the average), and R is the interest bill. Since $E(X^\tau)\equiv\bar{X}^\tau=(1-\tau)\bar{X}+\tau R$ we can substitute $\bar{X}^\tau-\tau R$ for $(1-\tau)\bar{X}$ in (1) to obtain:

$$(2)\qquad X^\tau = (\bar{X}^\tau-\tau R)Z+\tau R = \bar{X}^\tau\left(1-\frac{\tau R}{\bar{X}^\tau}\right)Z+\tau R.$$

Thus, if the tax rate is other than zero, the shape of the distribution of X^τ will depend not only on the "scale" of the stream $\bar{X}^\tau$ and on the distribution of Z, but also on the tax rate and the degree of leverage (one measure of which is $R/\bar{X}^\tau$). For example, if Var $(Z)=\sigma^2$, we have:

$$\text{Var}\,(X^\tau) = \sigma^2(\bar{X}^\tau)^2\left(1-\tau\frac{R}{\bar{X}^\tau}\right)^2$$

implying that for given $\bar{X}^\tau$ the variance of after-tax returns is smaller, the higher τ and the degree of leverage.[4]

II. *The Valuation of After-Tax Returns*

Note from equation (1) that, from the investor's point of view, the long-run average stream of after-tax returns appears as a sum of two components: (1) an uncertain stream $(1-\tau)\bar{X}Z$; and (2) a sure stream τR.[5] This suggests that the equilibrium market value of the combined stream can be found by capitalizing each component separately. More precisely, let ρ^τ be the rate at which the market capitalizes the expected returns net of tax of an unlevered company of size $\bar{X}$ in class k, i.e.,

$$\rho^\tau = \frac{(1-\tau)\bar{X}}{V_U}\quad\text{or}\quad V_U = \frac{(1-\tau)\bar{X}}{\rho^\tau};^6$$

[4] It may seem paradoxical at first to say that leverage *reduces* the variability of outcomes, but remember we are here discussing the variability of total returns, interest plus net profits. The variability of stockholder net profits will, of course, be greater in the presence than in the absence of leverage, though relatively less so than in an otherwise comparable world of no taxes. The reasons for this will become clearer after the discussion in the next section.

[5] The statement that τR—the tax saving per period on the interest payments—is a sure stream is subject to two qualifications. First, it must be the case that firms can always obtain the tax benefit of their interest deductions either by offsetting them directly against other taxable income in the year incurred; or, in the event no such income is available in any given year, by carrying them backward or forward against past or future taxable earnings; or, in the extreme case, by merger of the firm with (or its sale to) another firm that can utilize the deduction. Second, it must be assumed that the tax rate will remain the same. To the extent that neither of these conditions holds exactly then some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets. For simplicity, however, we shall here ignore these possible elements of delay or of uncertainty in the tax saving; but it should be kept in mind that this neglect means that the subsequent valuation formulas overstate, if anything, the value of the tax saving for any given permanent level of debt.

[6] Note that here, as in our original paper, we neglect dividend policy and "growth" in the

and let r be the rate at which the market capitalizes the sure streams generated by debts. For simplicity, assume this rate of interest is a constant independent of the size of the debt so that

$$r = \frac{R}{D} \quad \text{or} \quad D = \frac{R}{r}.^{7}$$

Then we would expect the value of a levered firm of size $\bar{X}$, with a permanent level of debt D_L in its capital structure, to be given by:

$$V_L = \frac{(1-\tau)\bar{X}}{\rho^\tau} + \frac{\tau R}{r} = V_U + \tau D_L.^{8} \tag{3}$$

In our original paper we asserted instead that, within a risk class, market value would be proportional to expected after-tax return $\bar{X}^\tau$ (cf. our original equation [11]), which would imply:

$$V_L = \frac{\bar{X}^\tau}{\rho^\tau} = \frac{(1-\tau)\bar{X}}{\rho^\tau} + \frac{\tau R}{\rho^\tau} = V_U + \frac{r}{\rho^\tau}\tau D_L. \tag{4}$$

We will now show that if (3) does not hold, investors can secure a more efficient portfolio by switching from relatively overvalued to relatively undervalued firms. Suppose first that unlevered firms are overvalued or that

$$V_L - \tau D_L < V_U.$$

An investor holding m dollars of stock in the unlevered company has a right to the fraction m/V_U of the eventual outcome, i.e., has the uncertain income

$$Y_U = \left(\frac{m}{V_U}\right)(1-\tau)\bar{X}Z.$$

Consider now an alternative portfolio obtained by investing m dollars as follows: (1) the portion,

$$m\left(\frac{S_L}{S_L + (1-\tau)D_L}\right),$$

is invested in the stock of the levered firm, S_L; and (2) the remaining portion,

$$m\left(\frac{(1-\tau)D_L}{S_L + (1-\tau)D_L}\right),$$

sense of opportunities to invest at a rate of return greater than the market rate of return. These subjects are treated extensively in our paper, "Dividend Policy, Growth and the Valuation of Shares," *Jour. Bus.*, Univ. Chicago, Oct. 1961, 411–33.

[7] Here and throughout, the corresponding formulas when the rate of interest rises with leverage can be obtained merely by substituting $r(L)$ for r, where L is some suitable measure of leverage.

[8] The assumption that the debt is permanent is not necessary for the analysis. It is employed here both to maintain continuity with the original model and because it gives an upper bound on the value of the tax saving. See in this connection footnote 5 and footnote 9.

is invested in its bonds. The stock component entitles the holder to a fraction,

$$\frac{m}{S_L + (1 - \tau)D_L},$$

of the net profits of the levered company or

$$\left(\frac{m}{S_L + (1 - \tau)D_L}\right)[(1 - \tau)(\bar{X}Z - R_L)].$$

The holding of bonds yields

$$\left(\frac{m}{S_L + (1 - \tau)D_L}\right)[(1 - \tau)R_L].$$

Hence the total outcome is

$$Y_L = \left(\frac{m}{(S_L + (1 - \tau)D_L)}\right)[(1 - \tau)\bar{X}Z]$$

and this will dominate the uncertain income Y_U if (and only if)

$$S_L + (1 - \tau)D_L \equiv S_L + D_L - \tau D_L \equiv V_L - \tau D_L < V_U.$$

Thus, in equilibrium, V_U cannot exceed $V_L - \tau D_L$, for if it did investors would have an incentive to sell shares in the unlevered company and purchase the shares (and bonds) of the levered company.

Suppose now that $V_L - \tau D_L > V_U$. An investment of m dollars in the stock of the levered firm entitles the holder to the outcome

$$\begin{aligned} Y_L &= (m/S_L)[(1 - \tau)(\bar{X}Z - R_L)] \\ &= (m/S_L)(1 - \tau)\bar{X}Z - (m/S_L)(1 - \tau)R_L. \end{aligned}$$

Consider the following alternative portfolio: (1) borrow an amount $(m/S_L)(1-\tau)D_L$ for which the interest cost will be $(m/S_L)(1-\tau)R_L$ (assuming, of course, that individuals and corporations can borrow at the same rate, r); and (2) invest m plus the amount borrowed, i.e.,

$$m + \frac{m(1 - \tau)D_L}{S_L} = m\frac{S_L + (1 - \tau)D_L}{S_L} = (m/S_L)[V_L - \tau D_L]$$

in the stock of the unlevered firm. The outcome so secured will be

$$(m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\bar{X}Z.$$

Subtracting the interest charges on the borrowed funds leaves an income of

$$Y_U = (m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\bar{X}Z - (m/S_L)(1 - \tau)R_L$$

which will dominate Y_L if (and only if) $V_L - \tau D_L > V_U$. Thus, in equilibrium, both $V_L - \tau D_L > V_U$ and $V_L - \tau D_L < V_U$ are ruled out and (3) must hold.

III. *Some Implications of Formula (3)*

To see what is involved in replacing (4) with (3) as the rule of valuation, note first that both expressions make the value of the firm a function of leverage and the tax rate. The difference between them is a matter of the size and source of the tax advantages of debt financing. Under our original formulation, values within a class were strictly proportional to expected earnings after taxes. Hence the tax advantage of debt was due solely to the fact that the deductibility of interest payments implied a higher level of after-tax income for any given level of before-tax earnings (i.e., higher by the amount τR since $\overline{X}^{\tau}=(1-\tau)\overline{X}+\tau R$). Under the corrected rule (3), however, there is an additional gain due to the fact that the extra after-tax earnings, τR, represent a sure income in contrast to the uncertain outcome $(1-\tau)\overline{X}$. Hence τR is capitalized at the more favorable certainty rate, $1/r$, rather than at the rate for uncertain streams, $1/\rho^{\tau}$.[9]

Since the difference between (3) and (4) is solely a matter of the rate at which the tax savings on interest payments are capitalized, the required changes in all formulas and expressions derived from (4) are reasonably straightforward. Consider, first, the before-tax earnings yield, i.e., the ratio of expected earnings before interest and taxes to the value of the firm.[10] Dividing both sides of (3) by V and by $(1-\tau)$ and simplifying we obtain:

$$\frac{\overline{X}}{V}=\frac{\rho^{\tau}}{1-\tau}\left[1-\tau\frac{D}{V}\right] \tag{31.c}$$

which replaces our original equation (31) (p. 294). The new relation differs from the old in that the coefficient of D/V in the original (31) was smaller by a factor of r/ρ^{τ}.

Consider next the after-tax earnings yield, i.e., the ratio of interest payments plus profits after taxes to total market value.[11] This concept was discussed extensively in our paper because it helps to bring out more clearly the differences between our position and the traditional view, and because it facilitates the construction of empirical tests of the two hypotheses about the valuation process. To see what the new equation (3) implies for this yield we need merely substitute $\overline{X}^{\tau}-\tau R$ for $(1-\tau)\overline{X}$ in (3) obtaining:

[9] Remember, however, that in one sense formula (3) gives only an upper bound on the value of the firm since $\tau R/r=\tau D$ is an exact measure of the value of the tax saving only where both the tax rate and the level of debt are assumed to be fixed forever (and where the firm is certain to be able to use its interest deduction to reduce taxable income either directly or via transfer of the loss to another firm). Alternative versions of (3) can readily be developed for cases in which the debt is not assumed to be permanent, but rather to be outstanding only for some specified finite length of time. For reasons of space, we shall not pursue this line of inquiry here beyond observing that the shorter the debt period considered, the closer does the valuation formula approach our original (4). Hence, the latter is perhaps still of some interest if only as a lower bound.

[10] Following usage common in the field of finance we referred to this yield as the "average cost of capital." We feel now, however, that the term "before-tax earnings yield" would be preferable both because it is more immediately descriptive and because it releases the term "cost of capital" for use in discussions of optimal investment policy (in accord with standard usage in the capital budgeting literature).

[11] We referred to this yield as the "after-tax cost of capital." Cf. the previous footnote.

$$V = \frac{\bar{X}^\tau - \tau R}{\rho^\tau} + \tau D = \frac{\bar{X}^\tau}{\rho^\tau} + \tau \frac{\rho^\tau - r}{\rho^\tau} D, \tag{5}$$

from which it follows that the after-tax earnings yield must be:

$$\frac{\bar{X}^\tau}{V} = \rho^\tau - \tau(\rho^\tau - r)D/V. \tag{11.c}$$

This replaces our original equation (11) (p. 272) in which we had simply $\bar{X}^\tau/V = \rho^\tau$. Thus, in contrast to our earlier result, the corrected version (11.c) implies that even the after-tax yield is affected by leverage. The predicted rate of decrease of $\bar{X}^\tau/V$ with D/V, however, is still considerably smaller than under the naive traditional view, which, as we showed, implied essentially $\bar{X}^\tau/V = \rho^\tau\text{-}(\rho^\tau - r)D/V$. See our equation (17) and the discussion immediately preceding it (p. 277).[12] And, of course, (11.c) implies that the effect of leverage on $\bar{X}^\tau/V$ is *solely* a matter of the deductibility of interest payments whereas, under the traditional view, going into debt would lower the cost of capital regardless of the method of taxing corporate earnings.

Finally, we have the matter of the after-tax yield on *equity* capital, i.e., the ratio of net profits after taxes to the value of the shares.[13] By subtracting D from both sides of (5) and breaking $\bar{X}^\tau$ into its two components—expected net profits after taxes, $\bar{\pi}^\tau$, and interest payments, $R = rD$—we obtain after simplifying:

$$S = V - D = \frac{\bar{\pi}^\tau}{\rho^\tau} - (1 - \tau)\left(\frac{\rho^\tau - r}{\rho^\tau}\right)D. \tag{6}$$

From (6) it follows that the after-tax yield on equity capital must be:

$$\frac{\bar{\pi}^\tau}{S} = \rho^\tau + (1 - \tau)[\rho^\tau - r]D/S \tag{12.c}$$

which replaces our original equation (12), $\bar{\pi}^\tau/S = \rho^\tau + (\rho^\tau - r)D/S$ (p. 272). The new (12.c) implies an increase in the after-tax yield on equity capital as leverage increases which is smaller than that of our original (12) by a factor of $(1-\tau)$. But again, the linear increasing relation of the corrected (12.c) is still fundamentally different from the naive traditional view which asserts the cost of equity capital to be completely independent of leverage (at least as long as leverage remains within "conventional" industry limits).

IV. *Taxes and the Cost of Capital*

From these corrected valuation formulas we can readily derive corrected measures of the cost of capital in the capital budgeting sense of the minimum prospective yield an investment project must offer to be just worth

[12] The i_k^* of (17) is the same as ρ^τ in the present context, each measuring the ratio of net profits to the value of the shares (and hence of the whole firm) in an unlevered company of the class.

[13] We referred to this yield as the "after-tax cost of equity capital." Cf. footnote 9.

undertaking from the standpoint of the present stockholders. If we interpret earnings streams as perpetuities, as we did in the original paper, then we actually have two equally good ways of defining this minimum yield: either by the required increase in before-tax earnings, $d\overline{X}$, or by the required increase in earnings net of taxes, $d\overline{X}(1-\tau)$.[14] To conserve space, however, as well as to maintain continuity with the original paper, we shall concentrate here on the before-tax case with only brief footnote references to the net-of-tax concept.

Analytically, the derivation of the cost of capital in the above sense amounts to finding the minimum value of $d\overline{X}/dI$ for which $dV=dI$, where I denotes the level of new investment.[15] By differentiating (3) we see that:

$$\frac{dV}{dI} = \frac{1-\tau}{\rho^\tau}\frac{d\overline{X}}{dI} + \tau\frac{dD}{dI} \geq 1 \qquad \text{if } \frac{d\overline{X}}{dI} \geq \frac{1-\tau\dfrac{dD}{dI}}{1-\tau}\rho^\tau. \tag{7}$$

Hence the before tax required rate of return cannot be defined without reference to financial policy. In particular, for an investment considered as being financed entirely by new equity capital $dD/dI=0$ and the required rate of return or marginal cost of equity financing (neglecting flotation costs) would be:

$$\rho^S = \frac{\rho^\tau}{1-\tau}.$$

This result is the same as that in the original paper (see equation [32], p. 294) and is applicable to any other sources of financing where the remuneration to the suppliers of capital is not deductible for tax purposes. It applies, therefore, to preferred stock (except for certain partially deductible issues of public utilities) and would apply also to retained earnings were it not for the favorable tax treatment of capital gains under the personal income tax.

For investments considered as being financed entirely by new debt capital $dI=dD$ and we find from (7) that:

$$\rho^D = \rho^\tau \tag{33.c}$$

which replaces our original equation (33) in which we had:

$$\rho^D = \rho^S - \frac{\tau}{1-\tau}r. \tag{33}$$

[14] Note that we use the term "earnings net of taxes" rather than "earnings after taxes." We feel that to avoid confusion the latter term should be reserved to describe what will actually appear in the firm's accounting statements, namely the net cash flow including the tax savings on the interest (our $\overline{X}^\tau$). Since financing sources cannot in general be allocated to particular investments (see below), the after-tax or accounting concept is not useful for capital budgeting purposes, although it can be extremely useful for valuation equations as we saw in the previous section.

[15] Remember that when we speak of the minimum required yield on an investment we are referring in principle only to investments which increase the *scale* of the firm. That is, the new

Thus for borrowed funds (or any other tax-deductible source of capital) the marginal cost or before-tax required rate of return is simply the market rate of capitalization for net of tax unlevered streams and is thus independent of both the tax rate and the interest rate. This required rate is lower than that implied by our original (33), but still considerably higher than that implied by the traditional view (see esp. pp. 276–77 of our paper) under which the before-tax cost of borrowed funds is simply the interest rate, r.

Having derived the above expressions for the marginal costs of debt and equity financing it may be well to warn readers at this point that these expressions represent at best only the hypothetical extremes insofar as costs are concerned and that neither is directly usable as a cut-off criterion for investment planning. In particular, care must be taken to avoid falling into the famous "Liquigas" fallacy of concluding that if a firm intends to float a bond issue in some given year then its cut-off rate should be set that year at ρ^D; while, if the next issue is to be an equity one, the cut-off is ρ^S. The point is, of course, that no investment can meaningfully be regarded as 100 per cent equity financed if the firm makes any use of debt capital—and most firms do, not only for the tax savings, but for many other reasons having nothing to do with "cost" in the present static sense (cf. our original paper pp. 292–93). And no investment can meaningfully be regarded as 100 per cent debt financed when lenders impose strict limitations on the maximum amount a firm can borrow relative to its equity (and when most firms actually plan on normally borrowing less than this external maximum so as to leave themselves with an emergency reserve of unused borrowing power). Since the firm's long-run capital structure will thus contain both debt and equity capital, investment planning must recognize that, over the long pull, *all* of the firm's assets are really financed by a mixture of debt and equity capital even though only one kind of capital may be raised in any particular year. More precisely, if L^* denotes the firm's long-run "target" debt ratio (around which its actual debt ratio will fluctuate as it "alternately" floats debt issues and retires them with internal or external equity) then the firm can assume, to a first approximation at least, that for any particular investment $dD/dI = L^*$. Hence, the relevant marginal cost of capital for investment planning, which we shall here denote by ρ^*, is:

$$\rho^* = \frac{1 - \tau L^*}{1 - \tau}\rho^\tau = \rho^S - \frac{\tau}{1 - \tau}\rho^D L^* = \rho^S(1 - L^*) + \rho^D L^*.$$

That is, the appropriate cost of capital for (repetitive) investment decisions over time is, to a first approximation, a weighted average of the costs of debt and equity financing, the weights being the proportions of each in the "target" capital structure.[16]

assets must be in the same "class" as the old. See in this connection, J. Hirshleifer, "Risk, the Discount Rate and Investment Decisions," *Am. Econ. Rev.*, May 1961, *51*, 112–20 (especially pp. 119–20). See also footnote 16.

[16] From the formulas in the text one can readily derive corresponding expressions for the required net-of-tax yield, or net-of-tax cost of capital for any given financing policy. Specifi-

V. *Some Concluding Observations*

Such, then, are the major corrections that must be made to the various formulas and valuation expressions in our earlier paper. In general, we can say that the force of these corrections has been to increase somewhat the estimate of the tax advantages of debt financing under our model and consequently to reduce somewhat the quantitative difference between the estimates of the effects of leverage under our model and under the naive traditional view. It may be useful to remind readers once again that the existence of a tax advantage for debt financing—even the larger advantage of the corrected version—does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt in their capital structures. For one thing, other forms of financing, notably retained earnings, may in some circumstances be cheaper still when the tax status of investors under the personal income tax is taken into account. More important, there are, as we pointed out, limitations imposed by lenders (see pp. 292–93), as well as many other dimensions (and kinds of costs) in real-world problems of financial strategy which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety. These additional considerations, which are typically grouped under the rubric of "the need for preserving flexibility," will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power. The tax advantage of debt may well tend to lower the optimal size of that reserve, but it is hard to believe that advantages of the size contemplated under our model could justify any substantial reduction, let alone their complete elimination. Nor do the data

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cally, let $\bar{\rho}(L)$ denote the required net-of-tax yield for investment financed with a proportion of debt $L=dD/dI$. (More generally L denotes the proportion financed with tax deductible sources of capital.) Then from (7) we find:

$$\bar{\rho}(L) = (1-\tau)\frac{d\bar{X}}{dI} = (1-L\tau)\rho^{\tau} \tag{8}$$

and the various costs can be found by substituting the appropriate value for L. In particular, if we substitute in this formula the "target" leverage ratio, L^*, we obtain:

$$\bar{\rho}^* \equiv \bar{\rho}(L^*) = (1-\tau L^*)\rho^{\tau}$$

and $\bar{\rho}^*$ measures the average net-of-tax cost of capital in the sense described above.

Although the before-tax and the net-of-tax approaches to the cost of capital provide equally good criteria for investment decisions when assets are assumed to generate perpetual (i.e., non-depreciating) streams, such is not the case when assets are assumed to have finite lives (even when it is also assumed that the firm's assets are in a steady state age distribution so that our X or EBIT is approximately the same as the net cash flow before taxes). See footnote 3 above. In the latter event, the correct method for determining the desirability of an investment would be, in principle, to discount the net-of-tax stream at the net-of-tax cost of capital. Only under this net-of-tax approach would it be possible to take into account the deductibility of depreciation (and also to choose the most advantageous depreciation policy for tax purposes). Note that we say that the net-of-tax approach is correct "in principle" because, strictly speaking, nothing in our analysis (or anyone else's, for that matter) has yet established that it is indeed legitimate to "discount" an uncertain stream. One can hope that subsequent research will show the analogy to discounting under the certainty case is a valid one; but, at the moment, this is still only a hope.

indicate that there has in fact been a substantial increase in the use of debt (except relative to preferred stock) by the corporate sector during the recent high tax years.[17]

As to the differences between our modified model and the traditional one, we feel that they are still large in quantitative terms and still very much worth trying to detect. It is not only a matter of the two views having different implications for corporate financial policy (or even for national tax policy). But since the two positions rest on fundamentally different views about investor behavior and the functioning of the capital markets, the results of tests between them may have an important bearing on issues ranging far beyond the immediate one of the effects of leverage on the cost of capital.

FRANCO MODIGLIANI AND MERTON H. MILLER*

[17] See, e.g., Merton H. Miller, "The Corporate Income Tax and Corporate Financial Policies," in *Staff Reports to the Commission on Money and Credit* (forthcoming).

* The authors are, respectively, professor of industrial management, School of Industrial Management, Massachusetts Institute of Technology, and professor of finance, Graduate School of Business, University of Chicago.

Consumption, Savings and Windfall Gains: Comment

In her recent article in this *Review* [3], Margaret Reid attempted to answer previous articles by Bodkin [1] and Jones [2] challenging the validity of the permanent income hypothesis. Bodkin and Jones used income and expenditure data for those consumer units who had received the soldiers' bonus (National Service Life Insurance dividends) during 1950, the year of the urban consumption survey [4]. These bonuses were regarded as windfall gains for the purposes of their analyses.

Professor Reid used data from the same survey, but her windfall gains were represented by "other money receipts." These are defined as "inheritances and occasional large gifts of money from persons outside the family . . . and net receipts from the settlement of fire and accident policies" [4, Vol. 1, p. xxix]. She assumed that the soldiers' bonus was included, and that it accounted for about one-half of other money receipts. Here she made an unfortunate mistake in interpreting the data for the main critical purpose of her article.

The soldiers' bonus is not part of "other money receipts" (O) but rather a part of "disposable money income" (Y). It is the main part of an item in the disposable money income category called "military pay, allotments, and pensions" [4, Vol. 11, p. xxix].

This would appear to alter completely the relationship of Professor Reid's main findings to the Bodkin results and to change the windfall interpretation of the O variable. Surely, fire and accident policy settlements are not windfall income, but rather a (partial) recovery of real assets previously lost. Likewise, inheritances are probably best considered as a long-anticipated increase in assets—not an increase in transitory income.

The discovery of this error probably does not affect whatever importance Professor Reid's secondary finding may have: ". . . the need, in any study of