

# Masters in FINANCE

## RISKY DEBT - II Anderson and Sundaresan (1996)

Corporate Investment Appraisal

Fall 2014



100 ANOS A PENSAR NO FUTURO



## 0. General View

Integrates literature of:

corporate finance;  
pricing (options).

Strategic Concerns in a valuation model.

How?

Game in extensive form, determined by:

Covenants/clauses of the debt contract;  
Bankruptcy law (and enforcement).

Determine “sub-game perfect” *Equilibrium*, with endogenous:  
cash-flow allocation;  
“boundaries of reorganization” of the firm (i.e., for which values of the parameters is control transferred from shareholders to creditors).

Shareholders (owner-manager) and creditors play *non-cooperatively*.

*Complete Information* about the structure of the game and cash flows.

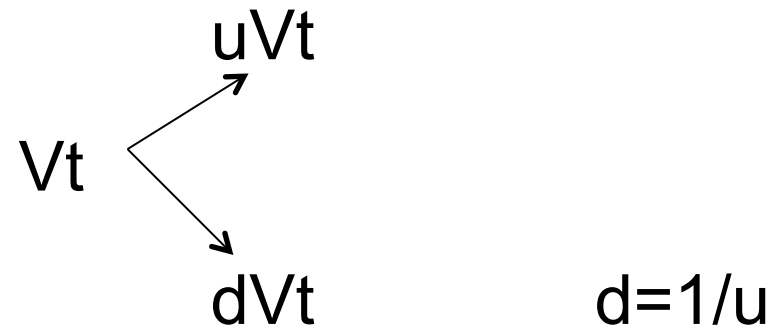
Objectives: How to design an optimal debt contract? (e.g., cash-payout ratio, level of leverage, tax effects).

## Results:

Possibility of Strategic Debt Service;  
Higher Default Premium than in previous studies;  
And many others...

# 1. Model

Technology:



$V_t$  is the present value of all cash flows (future and current).

Cash Flows:  $f_t = \beta V_t$

$\beta$  is the payout ratio;

a high  $\beta$  corresponds to a “cash cow” project;

a low  $\beta$  corresponds to growth opportunities.

Risk neutral Probability “up”:

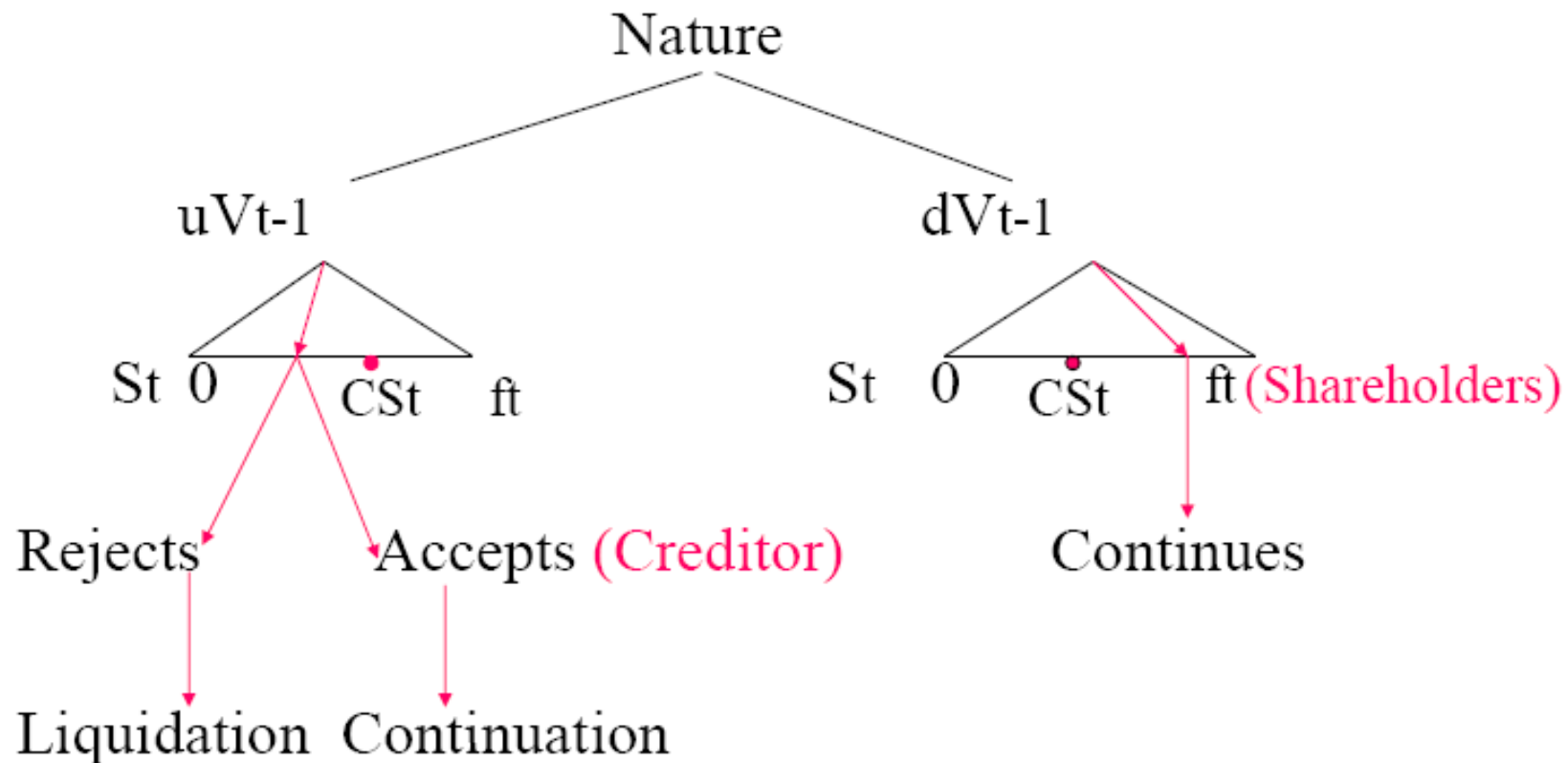
$$p = \frac{R(1 - \beta) - d}{u - d}$$

Liquidation Cost:  $K$  (fixed)

Contracted Debt Service, date  $t$ :  $CSt$

Actual Debt Service, date  $t$ :  $S_t \in [0, f_t]$

- Game from date (t-1) to date (t): an example



## Equilibrium:

*At terminal date T:*

$V_T$  is observed;

Shareholder decides  $S_T$ ;

If  $S_T \geq CS_T$ , game ends with payoffs:  $(V_T - S_T; S_T)$  for shareholder and creditor, respectively;

If  $S_T < CS_T$ , the creditor may accept or reject;

If the creditor accepts, the payoffs are:  $(V_T - S_T; S_T)$ ;

If the creditor rejects, the payoffs are:  $(0, \max(V_T - K, 0))$ ;

In equilibrium the Value of Equity is:

$$E(V_T) = V_T - B(V_T)$$

And the Value of Debt is:

$$B(V_T) = \min(CS_T, \max(V_T - K, 0))$$



Argument for equilibrium:

- If the shareholders decide  $S_T \geq C_T$ , payoffs are:  
 $(V_T - S_T; S_T)$ . In this case they would choose  $S_T = C_T$ .
- If the shareholders decide  $S_T < C_T$ , then:  
Creditor accepts if:  $S_T \geq \max(V_T - K, 0)$ .  
In this case, the shareholders would choose:  
 $S_T = \max(V_T - K, 0)$
- Hence, shareholders choose to pay whichever minimizes the Value of Debt.

Moving backwards in time... until date t:

- Realization of  $V_t$  (and of  $f_t$ );
- Shareholders choose  $S_t$ ;
  - If  $S_t \geq C_{S_t}$ , the game continues to date  $(t+1)$ ;
  - If  $S_t < C_{S_t}$ , creditors decide to:

Reject, obtaining:  $\max(V_t - K, 0)$ ; or

Accept, getting:

$$S_t + \frac{pB(uV_t) + (1-p)B(dV_t)}{R}$$

They choose the largest of the two.

Shareholders anticipate this decision when choosing  $S_t$ .

For relatively high values of  $V_t$ , there will be no default;

For relatively low values of  $V$ , they choose “strategic default”, paying an amount that leaves creditors just indifferent.

If no liquidation takes place, in state  $V_t$ , the Debt Service is:

$$S(V_t) = \min \left( CS_t, \max \left( 0, \max(V_t - K, 0) - \frac{pB(uV_t) + (1-p)B(dV_t)}{R} \right) \right)$$

The Value of Debt is:

$$B(V_t) = S(V_t) + \frac{pB(uV_t) + (1-p)B(dV_t)}{R}$$

And the Value of Equity:

$$E(V_t) = f_t - S(V_t) + \frac{pE(uV_t) + (1-p)E(dV_t)}{R}$$

In some cases, *forced liquidation* will occur:  $S(V_t) > f_t$ ; the Debt Value being:

$$B(V_t) = \max(0, \min(V_t - K, CS_t + P_t))$$

And Equity Value:  $E(V_t) = V_t - K - B(V_t)$

## 2. Valuation

- For “straight debt”, with fixed coupon and 100% reimbursement at maturity  $T$  ( $CSt=cP$  and  $t<T$ ;  $CSt=(c+1)P$  if  $t=T$ ).
- With  $c=0$  and  $K=0$ , this is the case of Merton (1974). Useful to compare for “calibration”. Denominate a ratio “ $d$ ” of *quasi*-debt:
  - Obtain the same results as Merton in terms of risk premium.
  - When we consider  $K \neq 0$ : the spread in this model changes significantly!! Much more so than in Merton. (Check the tables).

The analysis is extended in order to include non-zero coupon debt.

The paper also makes adjustments with taxes so as to consider the Tax Shield of Debt in the valuation, with tax-deductible coupons:

$$E(V_t) = (f_t - S(V_t))(1 - \tau) + \frac{pE(uV_t) + (1 - p)E(dV_t)}{R}; t < T$$

$$E(V_T) = (1 - \beta)V_T + (f_T - s_T cP)(1 - \tau) - s_T P; s_T = \frac{S(V_T)}{(1 + c)P}$$

In case of forced liquidation, the taxes are deducted before computing the liquidation value – this is the only difference in the way in which the value of Debt is computed.

As  $T \cdot \beta \cdot V_t$  is small relative to  $V_t$ , taxes don't have too large an effect in the strategies for "St".

But Taxes do affect to a large extent the value of "E". They constitute an important factor for the "design".

## Security Design Problem:

$$\max_{c, T, P, g} E(V_0; \sigma^2, \beta, R, K, \tau)$$

*s.t.*

$$D \leq B(V_0; \sigma^2, \beta, R, K, \tau)$$

$c$  = coupon

$T$  = maturity

$P$  = face value

$g$  = number of "grace periods" (no reimbursement of principal)

$A_t$  = amortization at date  $t$

$$A_t = \begin{cases} 0 & \text{if } t \leq g \\ \frac{P}{T - g} & \text{if } t > g \end{cases}$$

## Some Results:

High growth (Low Beta) use low coupons;

Low growth use high coupons;

When the Tax Rate rises, tendency to choose higher coupons;

Highly levered firms use low coupons;

etc...