Models in Finance - Class 24

Master in Actuarial Science

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ISEG

Credit risk

- Before, we have assumed that bonds are default free.
- This is not a reasonable assumption for corporate bonds and some government bonds.
- It can be reasonable for some government bonds.
- The outcome of a default may be that the contracted payment stream is:
- (i) rescheduled.
- (ii) cancelled by the payment of an amount which is less than the default-free value of the original contract.
- (iii) continues at a reduced rate.
- (iv) totally wiped out.

Credit risk

- The default of a bond can be triggered by a credit event of the type:
- (i) failure to pay capital or a coupon.
- (ii) loss event.
- (iii) bankruptcy.
- (iv) rating downgrade of the bond by a rating agency such as Standard and Poor's or Moody's or Fitch.
- Recovery rate: fraction of the defaulted amount that can be recovered through bankruptcy proceedings or other forms of settlement.

Structural models

- Structural models: explicit models of a corporate entity issuing both equity and debt.
- These models link default events explicitly to the fortunes of the issuing corporate entity.
- These models are simple and cannot be realistically used to price credit risk.
- These models can give an insight into the nature of default and the interaction between bond holders and equity holders.
- Examples of a structural model: the Merton model or First Passage models.

Reduced form models

- Reduced form models: statistical models which use observed market statistics rather than specific data relating to the issuing corporate entity.
- The market statistics most commonly used are the credit ratings issued by credit rating agencies such as Standard and Poor's, Moody's or Fitch.
- These models use market statistics along with data on the default-free market to model the movement of the credit rating of the bonds.
- The output of these models is a distribution of the time to default.

Intensity-based models

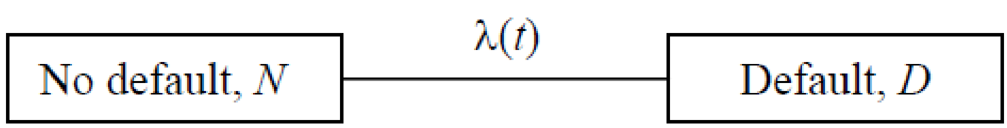
- An intensity-based model is a particular type of reduced form model.
- These models are defined in continuous-time and they model the "jumps" between different states (usually credit ratings) using transition intensities.
- Examples: two-state model for credit ratings with a deterministic transition intensity and the Jarrow-Lando-Turnbull model.

The Merton model

- Consider that a corporate entity has issued both equity and debt such that its total value at time t is $F(t)$.
- The zero-coupon debt is related to a promised repayment amount of L at a future maturity time T. At time T the remainder of the value of the corporate entity will be distributed amongst the equity holders.
- Default situation: if $F(T) < L$.
- If default occurs, the bond holder receive $F(T)$ instead of L and the equity holders receive nothing at all.
- For the equity holders, this is equivalent to have a European call option on the assets of the company with maturity T and a strike price L.
- The Merton model can be used to estimate the risk-neutral probability that the company will default or the credit spread on the debt.

Two-state model with constant intensity

- In continuous time, consider a model with two states: N (not previously defaulted) and D (previously defaulted).
- Assume that the interest rate term structure is deterministic: $r(t) = r$ for all t.
- The transition intensity, under the real world measure P, from N to D is denoted by $\lambda(t)$.



Two state model with constant intensity

- The state D is an absorbing state.
- Let $X(t)$ be the state at time t. The transition intensity $\lambda(t)$ is such that (under P)

$$\begin{aligned} P\left[X(t+dt) = N | X(t) = N\right] &= 1 - \lambda(t)dt + o(dt) \quad \text{as } dt \longrightarrow 0, \\ P\left[X(t+dt) = D | X(t) = N\right] &= \lambda(t)dt + o(dt) \quad \text{as } dt \longrightarrow 0. \end{aligned}$$

- Define the stopping time τ (time of default):

$$\tau = \inf\left\{t : X\left(t\right) = D\right\}.$$

- Define the number of defaults as the counting process $N(t)$:

$$N(t) = \begin{cases} 0 & \text{if } \tau > t, \\ 1 & \text{if } \tau \leq t. \end{cases}$$

Two state model with deterministic intensity

- Assume that if the corporate entity defaults all bond payments will be reduced by a deterministic factor $(1-\delta)$ where δ is the recovery rate.
- If a bond is due to pay 1 at time T, the actual payment at time T will be 1 if $\tau > T$ and δ if $\tau \leq T$.
- Let $B(t,T)$ be the price at time t of a zero-coupon bond. Then there exists a risk-neutral measure Q equivalent to P under which:

$$\begin{aligned} B(t,T) &= e^{-r(T-t)} E_Q\left[\textit{Payoff at } T | \mathcal{F}_t\right] \\ &= e^{-r(T-t)} E_Q\left[1 - (1-\delta) N(T) | \mathcal{F}_t\right]. \end{aligned}$$

Two state model with constant intensity

- It can be proved that:

$$E_Q\left[N\left(T\right)|N(t)=0\right]=E_Q\left[1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right].$$

- Assuming that $\widetilde{\lambda}\left(s\right)$ is deterministic, this implies that:

$$B(t,T)=e^{-r(T-t)}\left[1-(1-\delta)\left(1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right)\right]$$

which is equivalent to:

$$\widetilde{\lambda}\left(t\right)=-\frac{\partial}{\partial s}\log\left[e^{r(s-t)}B(t,s)-\delta\right]$$

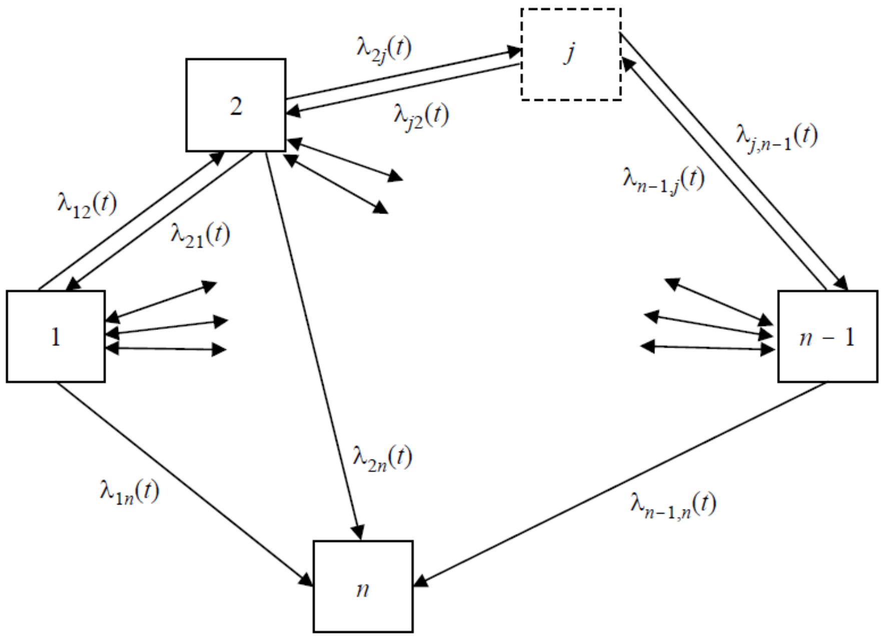
- Note: $\widetilde{\lambda}\left(t\right)$ is the transition intensity under Q.
- From the bond term structures and making an assumption about the recovery rate allows the implied risk-neutral transition intensities to be determined.

The Jarrow-Lando-Turnbull model

- In this model there are $n-1$ credit ratings plus default (n states).
- $\lambda_{ij}\left(t\right)$: transition intensities, under the real-world measure P, from state i to state j at time t.
- If $X(t)$ is the state or credit rating at time t, then, for $i,j=1,\ldots,n-1$,

$$P\left[X(t+dt)=j|X(t)=i\right]=$$
$$=\begin{cases}\lambda_{ij}(t)dt+o(dt) & \text{for } j\neq i\\ 1-\sum_{i\neq j}\lambda_{ij}(t)dt+o(dt)=\lambda_{ii}(t)dt+o(dt) & \text{for } j=i\end{cases}.$$

The Jarrow-Lando-Turnbull model



The Jarrow-Lando-Turnbull model

- The state n (default) is absorbing: $\lambda_{nj}(t) = 0$ for all j and for all t.
- $n \times n$ intensity matrix:

$$\Lambda(t) = [\lambda_{ij}(t)]_{i,j=1}^{n}.$$

- Define, for $s > t$, the transition probabilities:

$$p_{i,j}(t,s) = P[X(s) = j | X(t) = i].$$

- Matrix of transition probabilities:

$$\Pi(t,s) = [p_{ij}(t,s)]_{i,j=1}^{n}.$$

The Jarrow-Lando-Turnbull model

- It can be shown that:

$$\Pi(t,s) = \exp\left[\int_t^s \Lambda(u)\,du\right].$$

- It can be shown that there exists a risk-neutral measure Q equivalent to P such that the price of a zero-coupon bond maturing at time T, which pays 1 if default has not yet occurred and δ if default has occurred and for which the credit rating of the underlying corporate entity is i is given by:

$$V(t,T,X(t)) = B(t,T)\left[1-(1-\delta)P_Q\left[X(T)=n|\mathcal{F}_t\right]\right].$$