VOLATILITY MODEL – SOLUTIONS

1 a) $\widehat{dlog}(P_t) = -0.0003 + u_t$

$$\sigma_t^2 = 0.000 - 0.011\, u_{t-1}^2 + 0.125\, d_{t-1} u_{t-1}^2 + 0.9278\, \sigma_{t-1}^2$$

with $d_{t-1} = 1$ if $u_{t-1} < 0$; 0 otherwise.

b) yes ; No

c) The standardized residuals are not normally distributed.

2 a) $r_t = c + \theta \log \sigma_t + u_t$ EGARCH(1,2)–M

$$\sigma_t^2 = \alpha_0 + \alpha_1 \left| \frac{u_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{u_{t-1}}{\sigma_{t-1}} + \beta_1 \log \sigma_{t-1}^2 + \beta_2 \log \sigma_{t-2}^2$$

b) Yes (lag 1) ; No

c) Not normally distributed

3
$$\begin{aligned}
\sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2 \\
&= \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 (\alpha_0 + \alpha_1 u_{t-2}^2 + \alpha_2 u_{t-3}^2 + \beta_1 \sigma_{t-2}^2) \\
&= \alpha_0 (1+\beta_1) + \alpha_1 u_{t-1}^2 + (\alpha_1 \beta_1 + \alpha_2) u_{t-2}^2 + \beta_1 (\alpha_1 \beta_1 + \alpha_2) u_{t-3}^2 + \dots \\
&= \alpha_0^* + \alpha_1^* u_{t-1}^2 + \alpha_2^* u_{t-2}^2 + \alpha_3^* u_{t-3}^2 + \dots
\end{aligned}$$

4
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2$$

$$u_t^2 + \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-1}^2 + u_t^2$$

$$\begin{aligned}
u_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + v_t \quad \text{with } v_t = u_t^2 - \sigma_t^2 \\
&= \alpha_0 + (\alpha_1 + \beta_1) u_{t-1}^2 - \beta_1 (u_{t-1}^2 - \sigma_{t-1}^2) + \beta_2 \sigma_{t-2}^2 + v_t \\
&= \alpha_0 + (\alpha_1 + \beta_1) u_{t-1}^2 + \beta_2 \sigma_{t-2}^2 - \beta_1 v_{t-1} + v_t \\
&= \alpha_0^* + \alpha_1^* u_{t-1}^2 + \beta_2 u_{t-2}^2 - \beta_2 (u_{t-1}^2 - \sigma_{t-2}^2) - \beta_1 v_{t-1} + v_t
\end{aligned}$$

$$u_t^2 = \alpha_0^* + \alpha_1^* u_{t-1}^2 + \alpha_2^* u_{t-2}^2 + v_t + \beta_1^* v_{t-1} + \beta_2^* v_{t-2}$$

5 $u_t = \sigma_t \varepsilon_t$ $\quad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 u_{t-1}^2$ where is "ε_t^2" should be "u_t^2"

a) $u_t^2 = \alpha_0 + \alpha^* u_{t-1}^2 + \beta_1^* v_{t-1} + v_t$

b) IGARCH(1,1) ; $u_t^2 \sim$ ARIMA(0,1,1)