Bonds

Gestão Financeira I
Gestão Financeira

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LISBON
SCHOOL OF
ECONOMICS &
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Outline

- Bond Terminology
- Zero-Coupon Bonds
- Coupon Bonds
- Why Bond Prices Change
- Corporate Bonds

Bond Terminology

– Bond Certificate
  - States the terms of the bond
– Maturity Date
  - Final repayment date
– Term
  - The time remaining until the repayment date
– Coupon
  - Promised interest payments

– Face Value (*FV*)
  - Notional amount used to compute the interest payments
– Coupon Rate
  - Determines the amount of each coupon payment, expressed as an APR
– Coupon Payment (*CPN*)

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

Zero-Coupon Bonds

- A **Zero-Coupon Bond**:
  - Does not make coupon payments
  - Always sells at a **discount** (a price lower than face value), so they are also called **pure discount bonds**
  - **Treasury Bills** are U.S. government zero-coupon bonds with a maturity of up to one year.
- Example: Suppose that a one-year, risk-free, zero-coupon bond with a $100,000 face value has an initial price of $96,618.36. The cash flows would be:

Zero-Coupon Bonds (cont.)

- Yield to Maturity of a Zero-Coupon Bond (*YTM* or *y*):
  - The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.
- Price of a Zero-Coupon bond:

$$P = \frac{\text{Face Value}}{(1 + YTM_n)^n}$$

- Example: For the previous example's one-year zero coupon bond, we have

$$96{,}618.36 = \frac{100{,}000}{(1 + YTM_1)}$$

$$1 + YTM_1 = \frac{100{,}000}{96{,}618.36} = 1.035$$

- Thus, the Yield to Maturity is YTM= 3.5%.

Zero-Coupon Bonds (cont.)

- The Yield to Maturity of an *n*-Year Zero-Coupon Bond is

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$

- Example: Suppose that the following zero-coupon bonds are selling at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

Maturity	1 year	2 years	3 years	4 years
Price	$98.04	$95.18	$91.51	$87.14

$$YTM = (100 / 98.04) - 1 = 0.02 = 2\%$$

$$YTM = (100 / 95.18)^{1/2} - 1 = 0.025 = 2.5\%$$

$$YTM = (100 / 91.51)^{1/3} - 1 = 0.03 = 3\%$$

$$YTM = (100 / 87.14)^{1/4} - 1 = 0.035 = 3.5\%$$

Zero-Coupon Bonds (cont.)

- Risk-Free Interest Rates: A default-free zero-coupon bond that matures on date *n* provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.
  - Risk-Free Interest Rate with Maturity *n*:

$$r_n = YTM_n$$

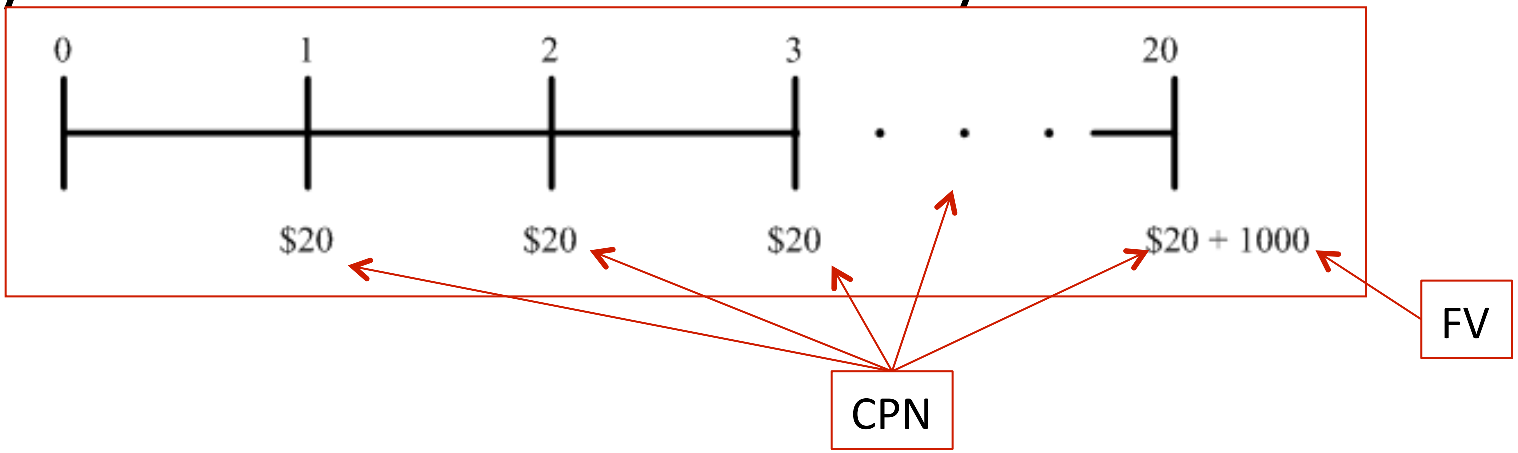
- Spot Interest Rate
    - Another term for a default-free, zero-coupon yield
- Zero-Coupon Yield Curve
    - A plot of the yield of risk-free zero-coupon bonds as a function of the bond's maturity date. (Remember Chapter 5, BDH).

Coupon Bonds

- A **Coupon Bond**:
  - Pays face value at maturity
  - Pays regular coupon interest payments
  - Examples:
    - Treasury Notes: U.S. Treasury coupon security with original maturities of 1–10 years
    - Treasury Bonds: U.S. Treasury coupon security with original maturities over 10 years

Coupon Bonds (cont.)

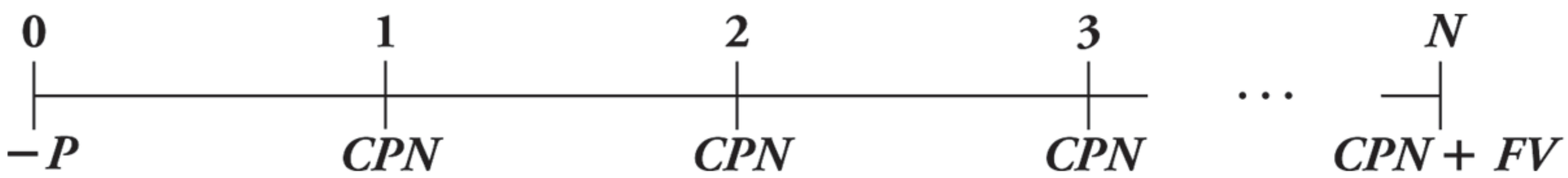
- Example: The U.S. Treasury has just issued a ten-year, $1000 bond with a 4% coupon and semi-annual coupon payments. What cash flows will you receive if you hold the bond until maturity?



- Note that the coupon rate is an APR, and that coupon payment is semi-annual: $CPN = \frac{0.04 * \$1000}{2} = \$20$

Coupon Bonds (cont.)

- **Yield to Maturity**: The YTM is the *single* discount rate that equates the present value of the bond's remaining cash flows to its current price.



- **Yield to Maturity of a Coupon Bond**:

$$P = \underbrace{CPN \times \overbrace{\frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right)}^{\text{Annuity Factor using the YTM } (y)}}_{\text{Present Value of all of the periodic coupon payments}} + \underbrace{\frac{FV}{(1+y)^N}}_{\text{Present Value of the Face Value repayment using the YTM } (y)}$$

Coupon Bonds (cont.)

- Example: Consider the following semi-annual bond:
  - $1000 par value
  - 7 years until maturity
  - 9% coupon rate
  - Price is $1,080.55

– **What is the bond's yield to maturity?**

$$\$1{,}080.55 = \left(\frac{0.09}{2}\times\$1{,}000\right) * \frac{1}{y}\left[1-\frac{1}{(1+y)^{14}}\right]+\frac{\$1{,}000}{(1+y)^{14}}$$

- With a financial calculator, or with excel:
  - y=3.75% (in semi-annual compounding);
  - So the annual Yield to maturity (APR, with semiannual compounding) is y=7.50%.

Microsoft Office
cel 97-2003 Worksh

Computing a Bond Price from Its Yield to Maturity

- Consider a five-year, $1000 bond with a 2.2% coupon rate and semiannual coupons.
- Suppose the bond's yield to maturity is 2% (expressed as an APR with semiannual compounding).
- What price is the bond trading for?
  - Considering the 6-month yield of 1.0%, the bond price must be

$$P = 11 \times \frac{1}{0.01}\left(1 - \frac{1}{1.01^{10}}\right) + \frac{1000}{1.01^{10}} = \$1009.47$$

Why Bond Prices Change

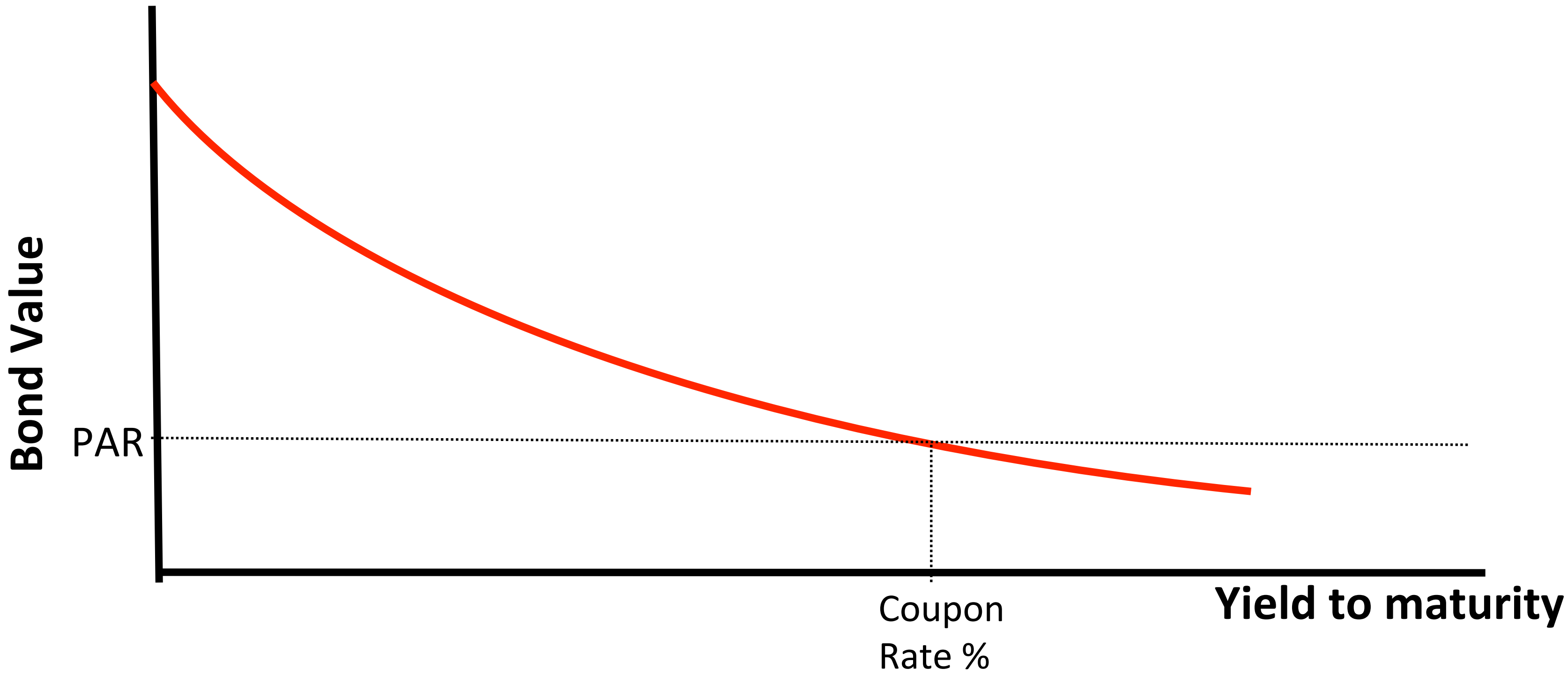
- Zero-coupon bonds always trade for a discount
- Coupon bonds may trade at a discount or at a premium
- Most issuers of coupon bonds choose a coupon rate so that the bonds will initially trade at, or very close to, par
- After the issue date, the market price of a bond changes over time

Dynamic Behavior of Bond Prices

- A bond may be selling at:
  - a Discount (below par): A bond is selling at a **discount** if the price is less than the face value.
  - Par: A bond is selling at **par** if the price is equal to the face value.
  - a Premium (above par): A bond is selling at a **premium** if the price is greater than the face value.

Dynamic Behavior of Bond Prices

When the bond price is …	greater than the face value	equal to the face value	less than the face value
We say the bond trades	"above par" or "at a premium"	"at par"	"below par" or "at a discount"
This occurs when	Coupon Rate > Yield to Maturity	Coupon Rate = Yield to Maturity	Coupon Rate < Yield to Maturity



Dynamic Behavior of Bond Prices (cont.)

- Interpretation:
  - If a coupon bond trades *at a discount*, an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond.
  - If a coupon bond trades *at a premium* it will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond.

Dynamic Behavior of Bond Prices (cont.)

- Example: Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity is 5%:
  - What is the price of each bond per $100 face value?
  - Which bond trades at a premium, which trades at a discount, and which trades at par?

$$\text{P}(10\%\text{ coupon}) = (0.1 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$176.86$$

Trades at a premium

$$\text{P}(5\%\text{ coupon}) = (0.05 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$100.00$$

Trades at par

$$\text{P}(3\%\text{ coupon}) = (0.03 \times \$100) \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{\$100}{1.05^{30}} = \$69.26$$

Trades at a discount

The effect of Time on Bond Prices

- Holding all other things constant, a bond's yield to maturity will not change over time.
- Holding all other things constant, the price of a discount or of a premium bond will move towards par value over time.
- If a bond's yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity even if you sell the bond early.

The effect of Time on Bond Prices (cont.)

- Example: Consider a 30-year bond with
  - a 10% coupon rate
  - Annual payments
  - $100 face value

– What is the initial price of this bond if it has a 5% yield to maturity?

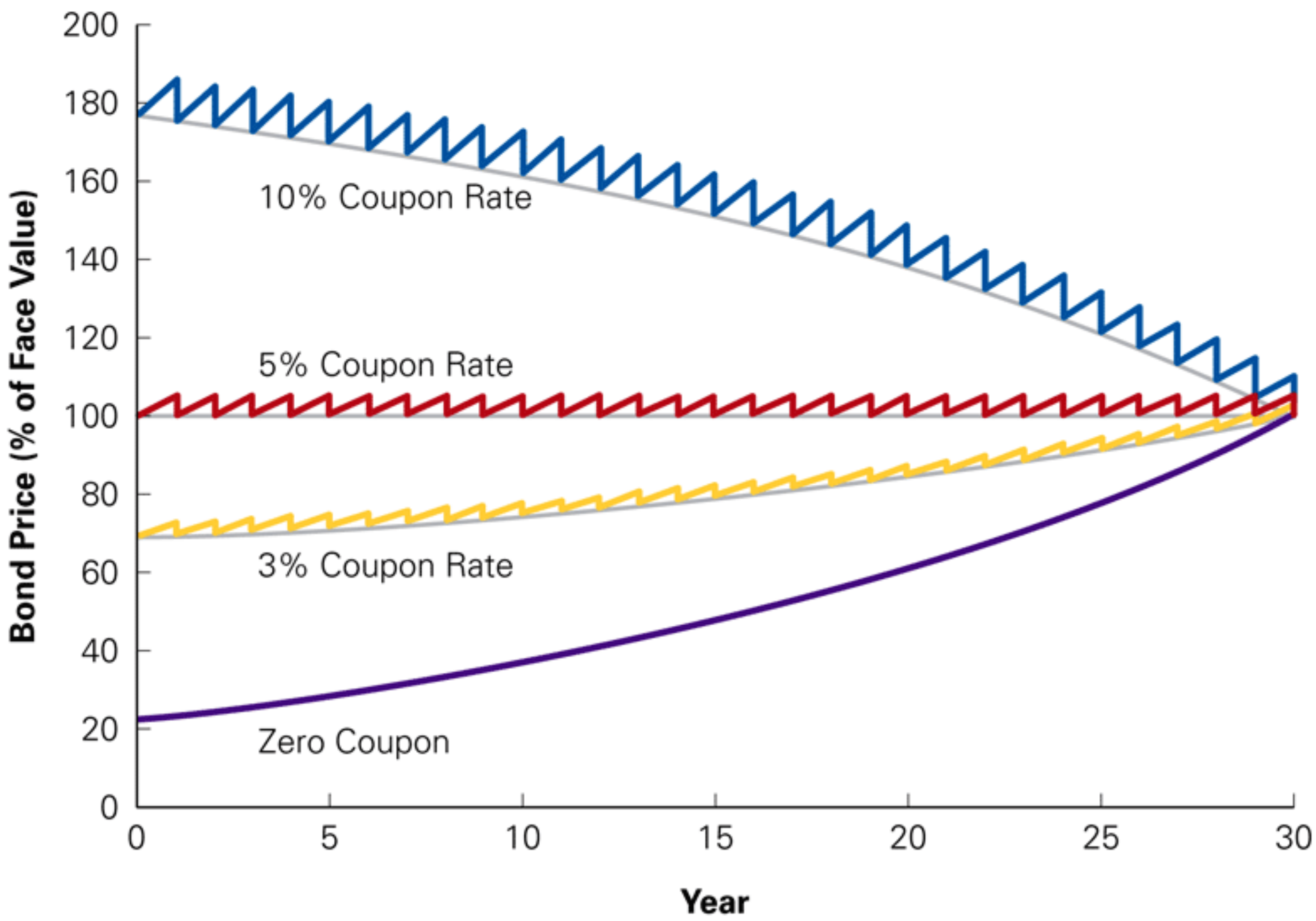
$$P = 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$$

– If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?

$$\text{P(just before first coupon)} = 10 + 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$185.71$$

$$\text{P(just after first coupon)} = 10 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$175.71$$

The effect of Time on Bond Prices (cont.)



The effect of Interest Rate Changes on Bond Prices

- There is an inverse relationship between interest rates and bond prices.
  - As interest rates and bond yields rise, bond prices fall.
  - As interest rates and bond yields fall, bond prices rise.
- The sensitivity of a bond's price to changes in interest rates is measured by the bond's **duration**.
  - Bonds with high durations are highly sensitive to interest rate changes.
  - Bonds with low durations are less sensitive to interest rate changes.

The effect of Interest Rate Changes on Bond Prices (cont.)

- Example: Consider two bonds
  - A 15-year zero-coupon bond;
  - A 30-year coupon bond with annual coupons of 10%.
  - By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?

Yield to maturity	Price of 15-yr zero-coupon bond	Price of 30-yr 10% annual coupon bond
5%	$\frac{100}{1.05^{15}} = \$48.10$	$10 \times \frac{1}{1.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$
6%	$\frac{100}{1.06^{15}} = \$41.73$	$10 \times \frac{1}{1.06}\left(1 - \frac{1}{1.06^{30}}\right) + \frac{100}{1.06^{30}} = \$155.06$
% Change in price due to 1% change in ytm	$\frac{(41.73 - 48.10)}{48.10} = -13.2\%$	$\frac{(155.06 - 176.86)}{176.86} = -12.3\%$

- Even though the 30-year bond has a longer maturity, the fact that it pays coupons reduces its sensitivity to changes in the interest rate, when compared to a zero-coupon bond.

The effect of Interest Rate Changes on Bond Prices (cont.)

Bond Characteristic	Effect on Interest Rate Risk
Longer term to maturity	Increase
Higher coupon payments	Decrease

Corporate Bonds

- **Credit Risk**
  - U.S. Treasury securities are widely regarded to be risk-free
  - Credit risk is the risk of default, so that the bond's cash flows are not known with certainty
- Corporations with higher default risk will need to pay higher coupons to attract buyers to their bonds
- **Corporate Bonds** are bonds issued by corporations;
  - These bonds involve Risk of default, also known as Credit Risk.
  - The yield of bonds with credit risk will be higher than that of otherwise identical default-free bonds.
  - A bond's expected return will be less than the yield to maturity if there is a risk of default.
    - A higher yield to maturity does not necessarily imply that a bond's expected return is higher.

Corporate Bonds

- Corporate Bond Yields
  - Yield to maturity of a defaultable bond is not equal to the expected return of investing in the bond
  - A higher yield to maturity does not necessarily imply that a bond's expected return is higher

Corporate Bonds: Bond Ratings

- Bond Ratings
  - Several companies rate the creditworthiness of bonds
    - Two best-known are Standard & Poor's and Moody's
  - These ratings help investors assess creditworthiness

Corporate Bonds: Bond Ratings (cont.)

Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

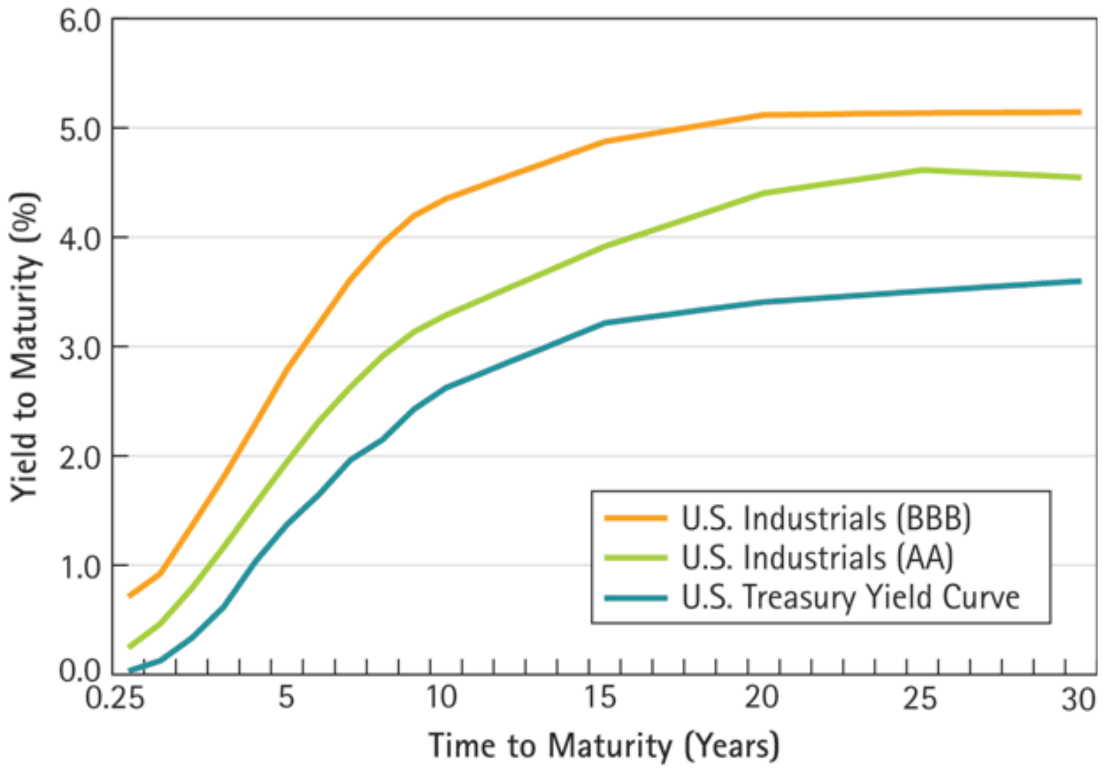
*Ratings: Moody's/Standard & Poor's

Source: www.moodys.com

Corporate Bonds

Corporate Yield Curves

– We can plot a yield curve for corporate bonds just as we can for Treasuries
– The credit spread is the difference between the yields of corporate bonds and Treasuries



Source: Bloomberg.

Corporate Yield Curves for Various Ratings, July 2013

Corporate Bonds: Credit Spreads and Bond Prices

Problem:

- Your firm has a credit rating of A.
- You notice that the credit spread for 10-year maturity debt is 90 basis points (0.90%).
- Your firm's ten-year debt has a coupon rate of 5% (semi-annual payment).
- You see that new 10-year Treasury notes are being issued at par with a coupon rate of 4.5%.
- What should the price of your outstanding 10-year bonds be?
  - If the credit spread is 90 basis points, then the yield to maturity (YTM) on your debt should be the YTM on similar treasuries plus 0.9%.
  - The fact that new 10-year treasuries are being issued at par with coupons of 4.5% means that with a coupon rate of 4.5%, these notes are selling for $100 per $100 face value.
  - **Thus their YTM is 4.5% and your debt's YTM should be 4.5% + 0.9% = 5.4%.**

Corporate Bonds: Credit Spreads and Bond Prices (Cont.)

- The cash flows on your bonds are \$5 per year for every \$100 face value, paid as \$2.50 every 6 months.
- The 6-month rate corresponding to a 5.4% yield is 5.4%/2 = 2.7%.

$$2.50 \times \frac{1}{0.027}\left(1 - \frac{1}{1.027^{20}}\right) + \frac{100}{1.027^{20}} = \$96.94$$

Appendix B: The Yield Curve and the Law of One Price

- Using the Law of One Price and the yields of default-free zero-coupon bonds, one can determine the price and yield of any other default-free bond.
  - The yield curve provides sufficient information to evaluate all such bonds.
- Example: Replicating a three-year $1000 bond that pays 10% annual coupon using three zero-coupon bonds:

	0	1	2	3
Coupon bond:		$100	$100	$1100
1-year zero:		$100		
2-year zero:			$100	
3-year zero:				$1100
Zero-coupon Bond portfolio:		$100	$100	$1100

The Yield Curve and Bond Arbitrage (cont.)

- Yields and Prices (per $100 Face Value) for default free Zero Coupon Bonds:
  - Example: Assume additionally that we know

Maturity	1 year	2 years	3 years	4 years
YTM	3.50%	4.00%	4.50%	4.75%
Price	$96.62	$92.45	$87.63	$83.06

  - By the Law of One Price, the three-year default free 10% annual coupon bond must trade for a price of $1153.

Zero-Coupon Bond	Face Value Required	Cost
1 year	100	96.62
2 years	100	92.45
3 years	1100	$11 \times 87.63 = 963.93$
	Total Cost:	$1153.00

Valuing a Default-free Coupon Bond using zero-coupon yields

- The price of a coupon bond must equal the present value of its coupon payments and face value.

$$
\begin{aligned}
PV &= PV(\text{Bond Cash Flows}) \\
&= \frac{CPN}{1+YTM_1} + \frac{CPN}{(1+YTM_2)^2} + \cdots + \frac{CPN+FV}{(1+YTM_n)^n}
\end{aligned}
$$

- Example:

$$
P = \frac{100}{1.035} + \frac{100}{1.04^2} + \frac{100+1000}{1.045^3} = \$1153
$$

Computing the Yield to Maturity of a Default-free Coupon Bond

- Given the yields for default free zero-coupon bonds, we can price a default free coupon bond.
- Once we have the price of a coupon bond, we can compute its yield to maturity.
- Example:

$$P = 1153 = \frac{100}{(1+y)} + \frac{100}{(1+y)^2} + \frac{100 + 1000}{(1+y)^3}$$

- Using a calculator or excel we can determine the yield to maturity, *y=4.44%*.