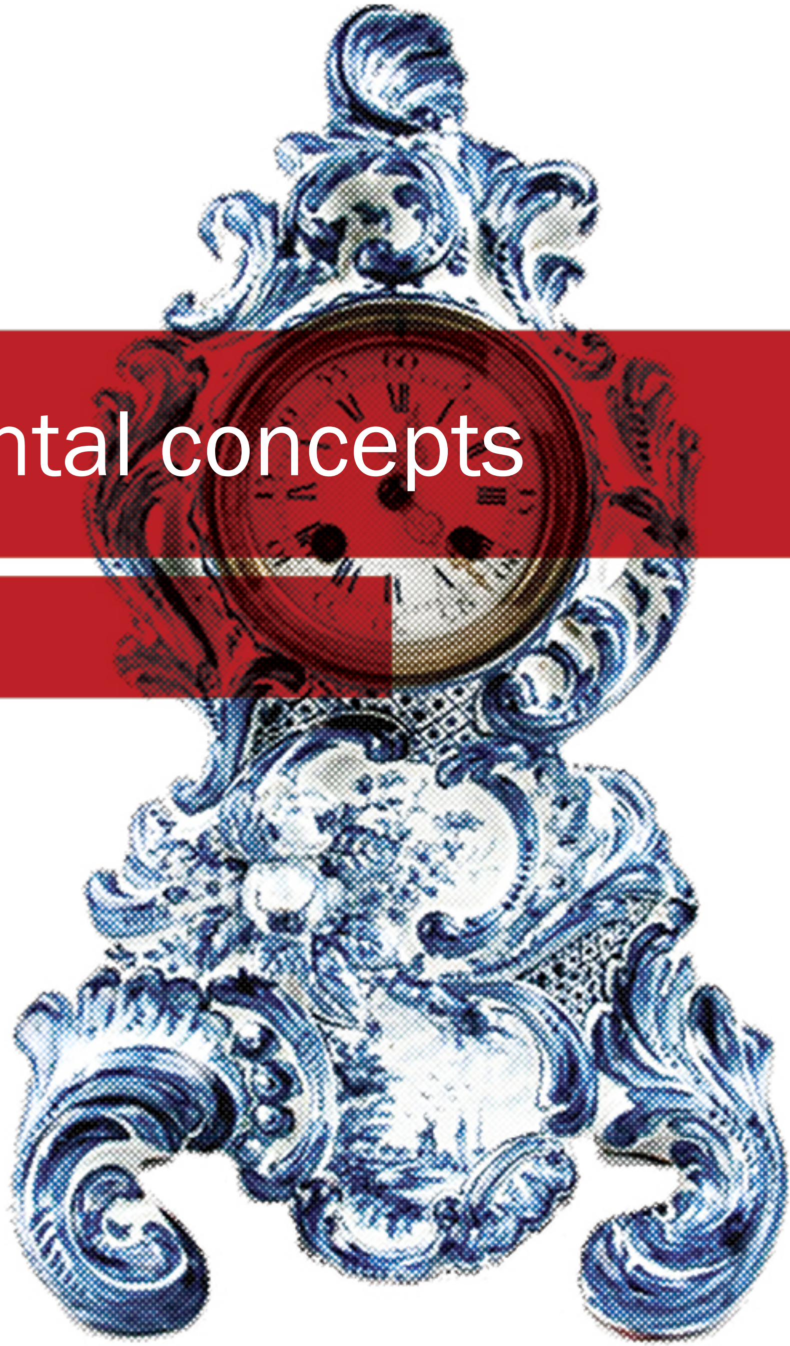
Masters in FINANCE

Game Theory: fundamental concepts

Corporate Investment Appraisal

Fall 2015



ISEG 100 1911-2011 ANOS

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BIBLIOGRAPHY

- Varian, Microeconomic Analysis, Chapter 15 (or any other introductory chapter/book)

Description of a Game

Form of a Game:

- Strategic/Normal;
- Extensive.

Elements of a Game:

- Set of Players;
- Set of Strategies and Actions;
- Set of Payoffs.

These elements are "common knowledge";

We assume agents are Rational.

EXAMPLES OF GAMES

Example 1: "Matching Pennies"

		Player Column	
		Heads	Tails
Player	Heads	1,-1	-1,1
Row	Tails	-1,1	1,-1

This is a "zero sum" game.

Example 2: "The Prisoner's Dilemma"

		Player Column	
		Cooperate	Defect
Player	Cooperate	3,3	0,4
Row	Defect	4,0	1,1

This is a "variable sum" game.

Example 3: Cournot Duopoly

Each company chooses its output: X1, X2

Total Supply: X=X1+X2

Demand: p(x)

Profit of Firm i: p(X1+X2)*Xi – C(Xi)… , i=1,2

Example 4: Bertrand Duopoly

Demand: X(p);

Payoff to Firm 1:

$$\begin{cases} p1 \cdot X(p1) & \text{if} \quad p1<p2 \\ p1 \cdot X(p1)/2 & \text{if} \quad p1=p2 \\ 0 & \text{if} \quad p1>p2 \end{cases}$$

SOLUTION CONCEPTS

Strategies:

Pure;

Mixed.

Example with 2 players ("Row" and "Column")

Probability of Strategy R of player Row: p_R

(for the various possible strategies R for this player).

Probability of Strategy C of player Column: p_C

(for the various possible strategies C of this player).

Each player forms a "Belief" regarding the other player's strategy:

Row believes that Column plays according to: π_C

Column forms the belief that Row plays according to: π_R

Expected Payoff:

For Row: $$\sum_R \sum_C p_R \pi_C U_R(R,C)$$

For Column: $$\sum_C \sum_R p_C \pi_R U_C(R,C)$$

NASH EQUILIBRIUM

Consists of probability beliefs (π_R, π_C) over strategies, and probability of choosing strategies (*pR* , *pC*), such that:

The beliefs are correct: $p_R = \pi_R$ and $p_C = \pi_C$; and

Each player chooses his/her probabilities (*pR and pC*) so as to maximize his expected utility given his beliefs.

NASH EQUILIBRIUM: AN EXAMPLE

"The Battle of the Sexes":

		Calvin	
		Left (L)	Right (R)
Rhonda	Top (T)	2,1	0,0
	Bottom (B)	0,0	1,2

NE in Pure strategies?

(T,L); (B,R).

NE in Mixed strategies? $p_L = \pi_L = \frac{1}{3}$ and $p_T = \pi_T = \frac{2}{3}$

DOMINANT STRATEGIES

Let *r1* and *r2* be two possible strategies for player Row.

r1* Strictly Dominates *r2 if the payoff to Row associated with strategy *r1* is strictly larger (>) to the payoff associated to *r2*, no matter what choice Column might make.

r1* Weakly Dominates *r2 if the payoff to Row from *r1* is at least as large (≥) as the payoff from *r2*, for all choices that player Column might make and strictly larger for some choice.

DOMINANT STRATEGY EQUILIBRIUM

A **Dominant Strategy Equilibrium** is a choice of strategies by each player such that each strategy (weakly) dominates every other strategy available to *that* player.

All Dominant Strategy Equilibria (DSE) are NE, but not all NE are DSE.

EXAMPLE OF A DSE

Example 2: The Prisoner's Dilemmaa

		Player Column	
		Cooperate	Defect
Player	~~Cooperate~~	~~3,3~~	~~0,4~~
Row	Defect	4,0	1,1

DSE: (Defect, Defect)

ANOTHER EXAMPLE OF A DSE

Example:

		Column	
		Left (L)	Right (R)
Row	Top (T)	2,2	0,2
	Bottom (B)	2,0	1,1

Pure NE?

(T,L);(B,R)

DSE?

(B,R)

SEQUENTIAL GAMES

Example of a Game with simultaneous moves:

		Column	
		Left (L)	Right (R)
Row	Top (T)	1,9	1,9
	Bottom (B)	0,0	2,1

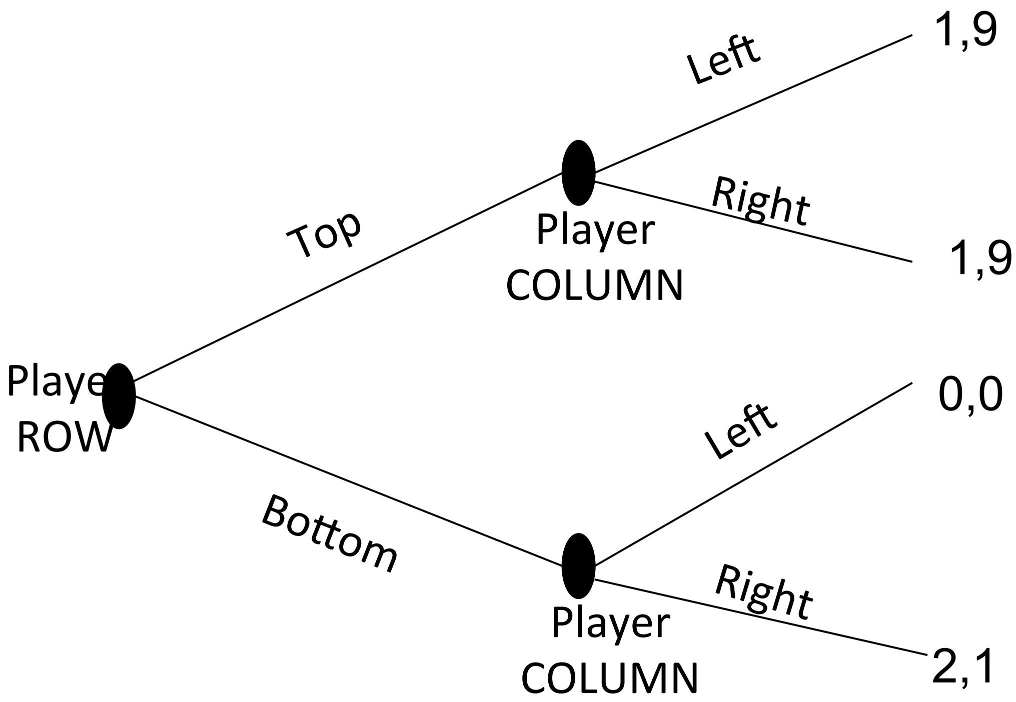
Pure NE:

(T,L);(B,R)

What if Row plays first and Columns plays only after observing Row's move?

We represent the game in its Extensive Form using a Game Tree.

SEQUENTIAL GAMES



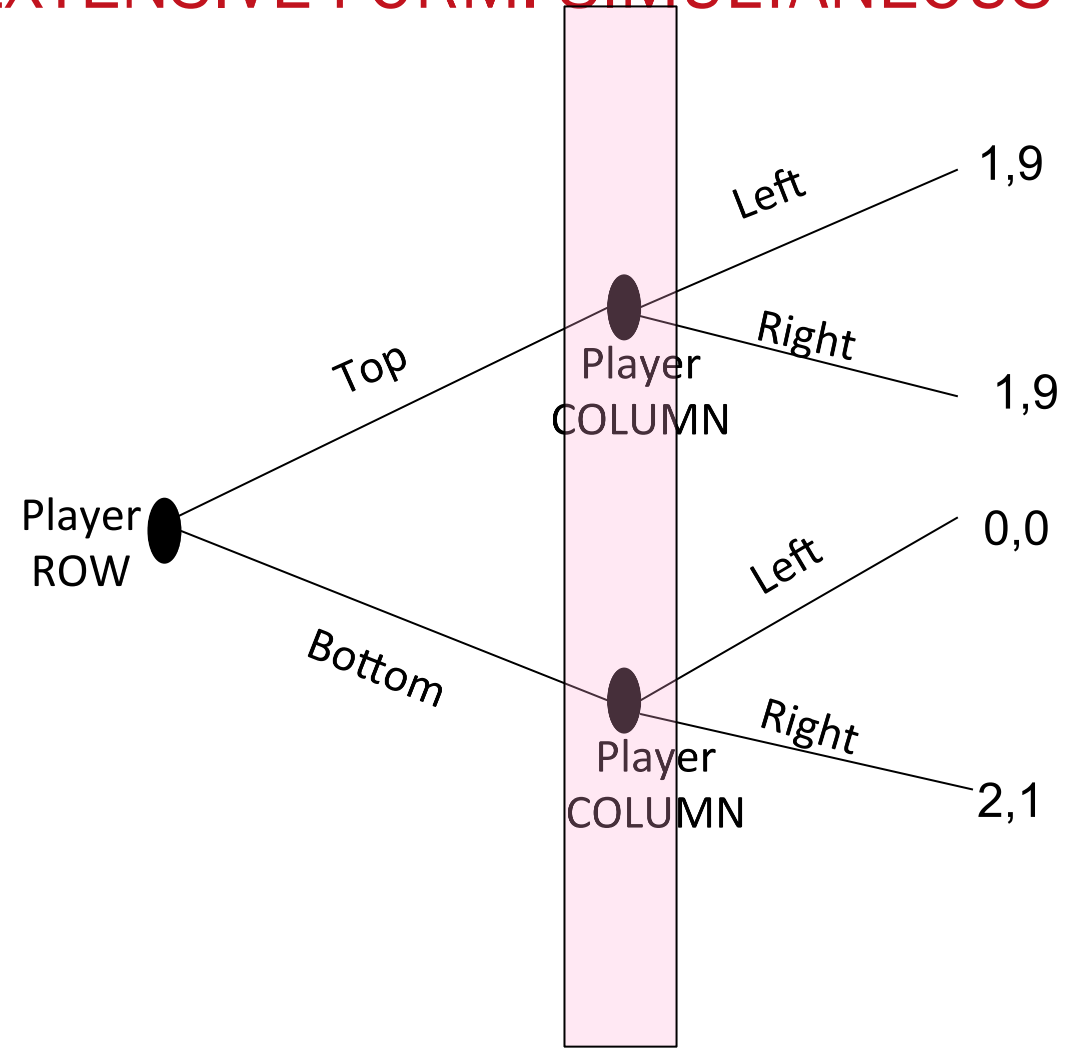
SUB-GAME PERFECT EQUILIBRIUM

Only one of the NE in Pure Strategies *is also a Nash Equilibrium in each of the sub-games*. A NE with this property is known as a **Subgame perfect equilibrium** (SPE).

For the previous example, SPE?

(B,R)

EXTENSIVE FORM: SIMULTANEOUS MOVES



BAYES-NASH EQUILIBRIUM (Perfect Bayesian Equilibrium)

By attributing beliefs formed by each player regarding the other players' behavior, we will try to maximize each player's expected payoff.

By so doing, we end up finding the several NE in pure and mixed strategies.

In this example these are the possible equilibria:

$$\left(p_T = 1; p_L \in \left[\tfrac{1}{2}, 1\right]\right); \left(p_T = 0; p_L = 0\right)$$