

Financial Markets and Instruments (Lecture 4)

Tiago Cardão-Pito



UNIVERSIDADE
DE LISBOA



LISBON
SCHOOL OF
ECONOMICS &
MANAGEMENT

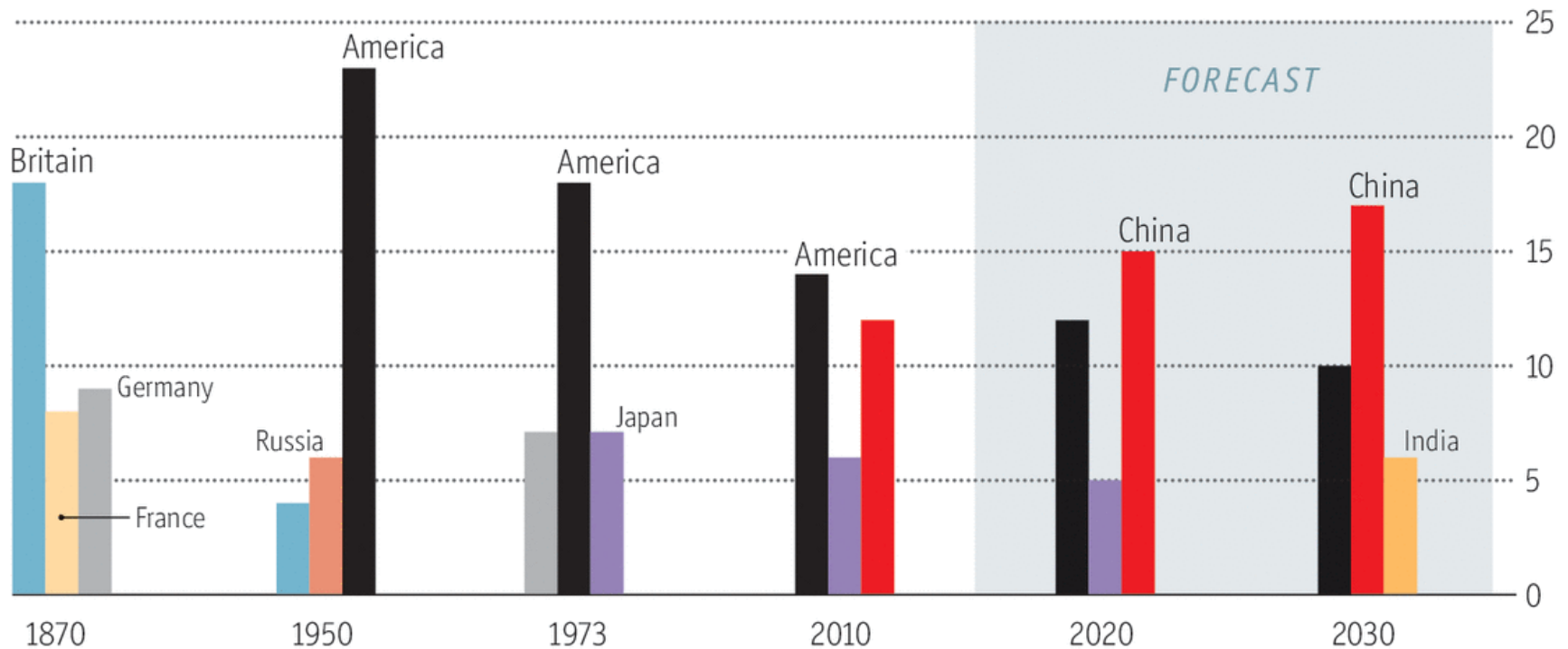
UNIVERSIDADE DE LISBOA

- Class Stocktrack:
- ISEGULisboaFMI2015

- Sources of tension in the financial center of the world

The ebbs and flows of power

% share* of global economic power



Source: "Eclipse: Living in the Shadow of China's Economic Dominance",
by Arvind Subramanian, 2011

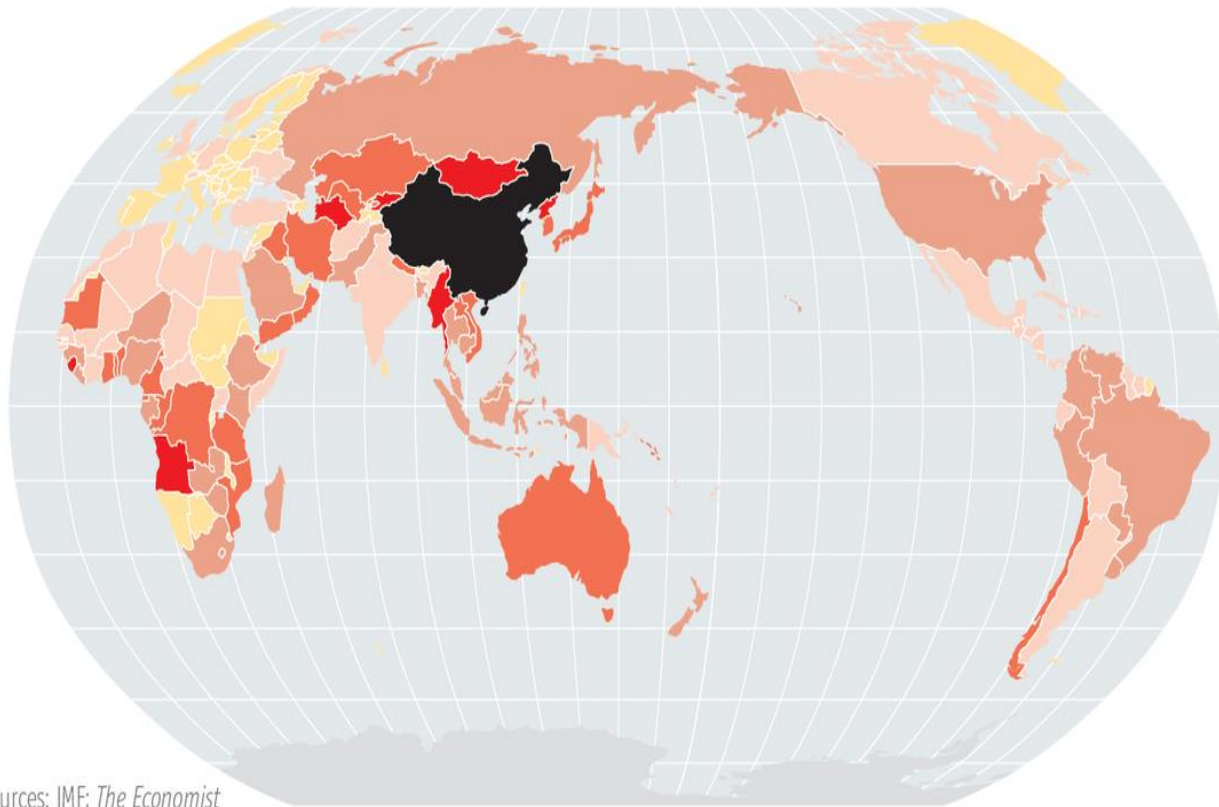
*Weighted by share of world GDP, trade and net capital exports

Economist.com

Merchandise trade with China

As % of countries' total trade, 2014

0-5 5-10 10-20 20-40 Over 40



Sources: IMF; *The Economist*
Economist.com

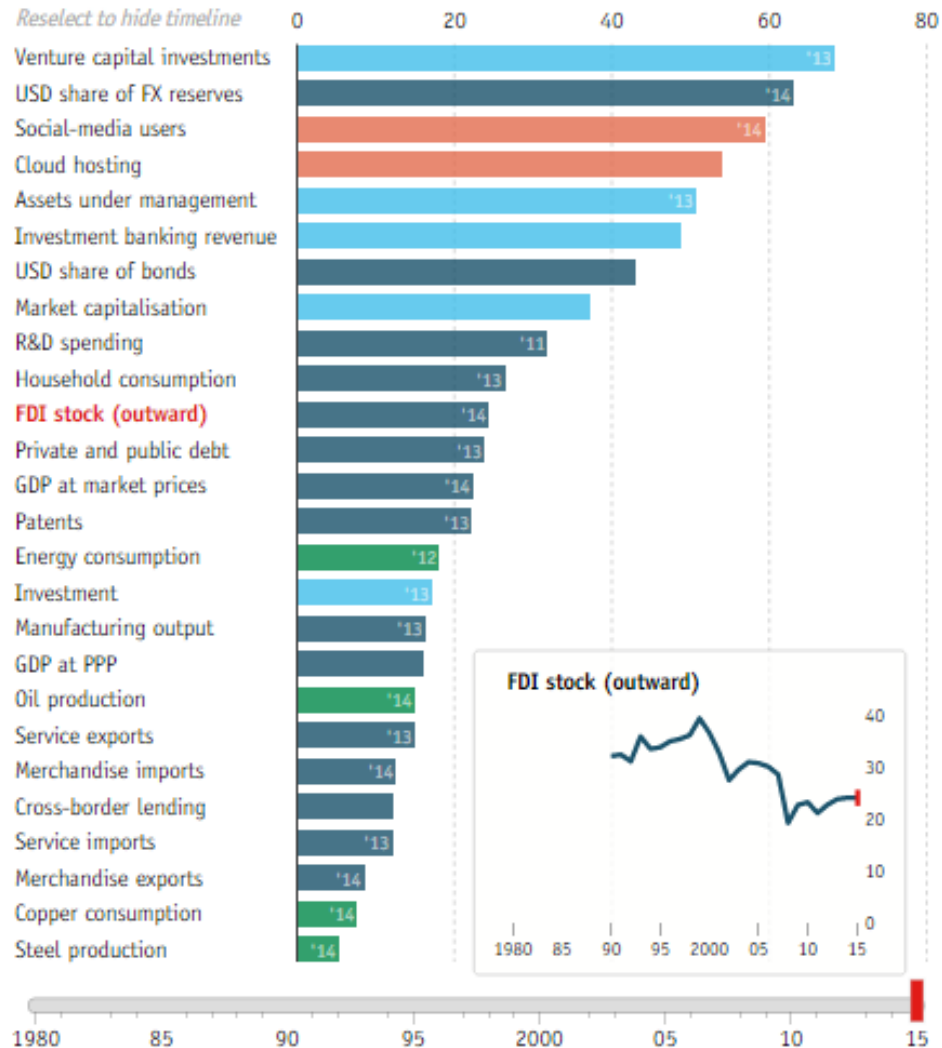
American abacus

United States' share of global total, %, 2015 or latest available previous year

Auto-sort ON

■ Economy
 ■ Financial sector
 ■ Commodities
 ■ Technology

Reselect to hide timeline



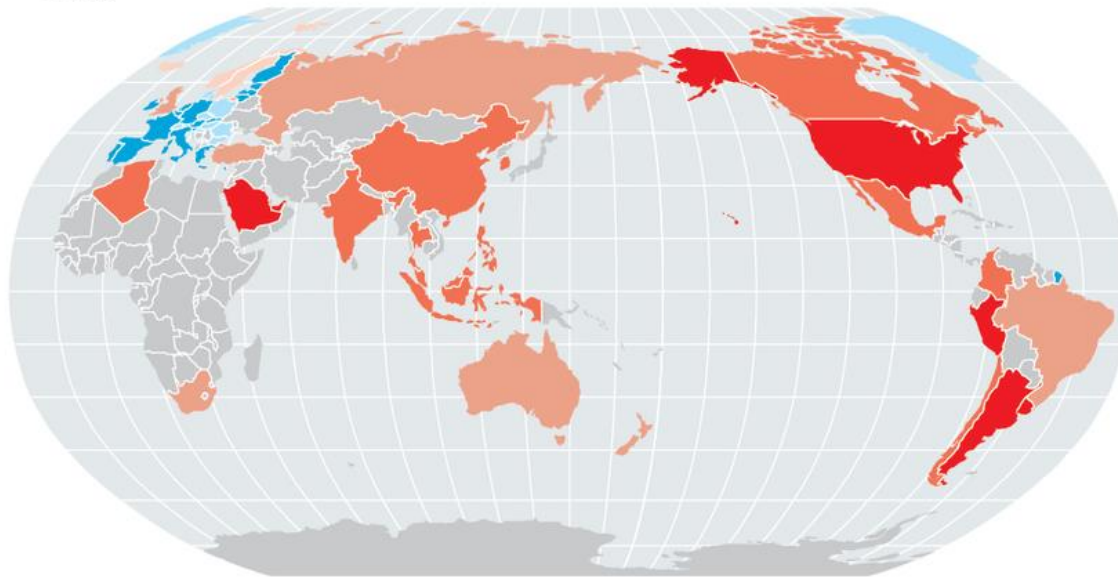
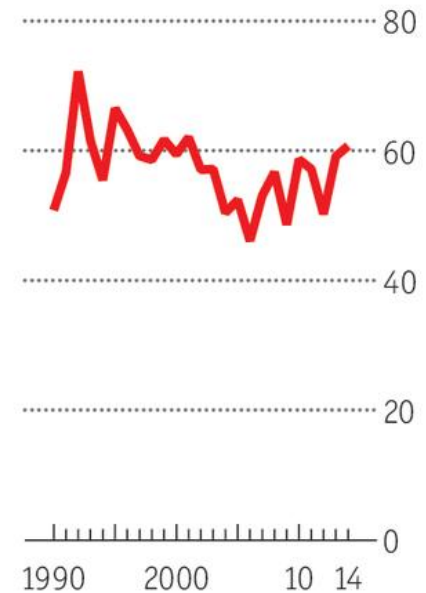
Sources: BIS; Bloomberg; World Federation of Exchanges; Dealogic; DowJones Venture Source; IMF; International Copper Study Group; McKinsey; Synergy Research Group; Towers Watson; UNCTAD; UNESCO; US EIA; WeAreSocial; WIPO; World Bank; World Steel Association; WTO; *The Economist*

Dollar zone

Intensity of currency's link to the \$, %
2013



Dollar-zone share of world GDP

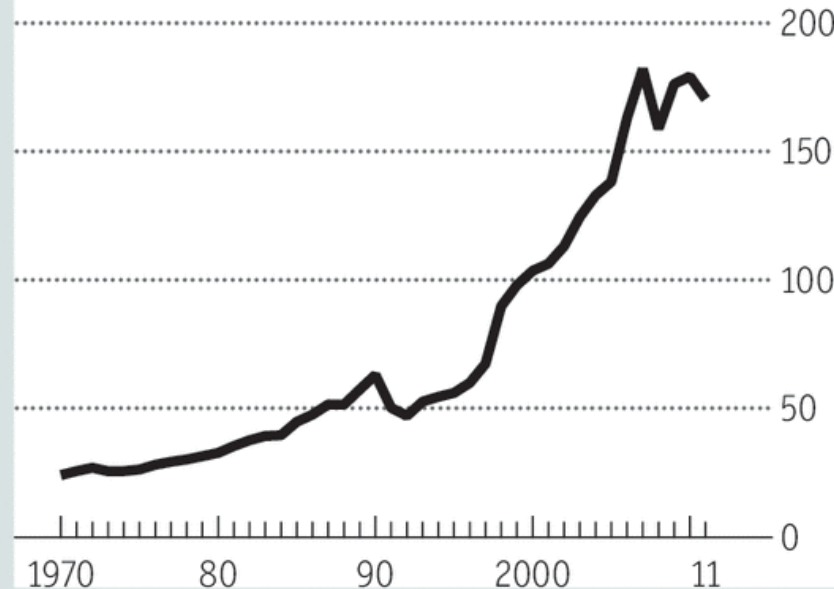


Source: "Currency movements drive reserve composition", by Robert McCauley and Tracy Chan, BIS Quarterly Review, December 2014
Economist.com

Tempestuous

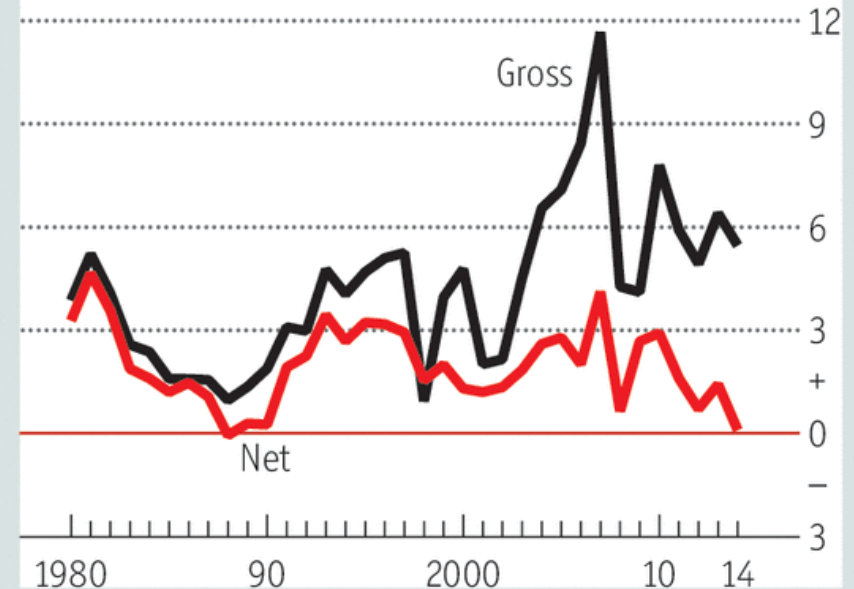
Global external assets

As % of GDP



Emerging-market capital flows

As % of GDP



Sources: "External Wealth of Nations", by Philip Lane and Gian Maria Milesi-Ferretti, 2013; IMF; "Capital Flows are Fickle", by John Bluedorn *et al*, 2013, IMF working paper 13/183

Economist.com

- Shift in World politics.

- **3- The major institutions of the financial sector**

A- Banks

The separation between retail banks and investment banks that existed in previous decades is no longer verifiably today.

Em 1950								
	Payment services	Saving products	Fiduciary services	Lending business	consumer	Underwriting Issuance of equity debt		Insurance and risk management products
Institution								
Depository institutions	X	X	X	X	X			
Insurance companies		X		*				X
Finance companies				*	X			
Securities firms		X	X			X	X	
Pension funds		X						
Mutual funds		X						

Em 2007								
	Payment services	Saving products	Fiduciary services	Lending business	consumer	Underwriting Issuance of equity debt		Insurance and risk management products
Institution								
Depository institutions	X	X	X	X	X	X	X	X
Insurance companies	X	X	X	X	X	X	X	X
Finance companies	X	X	X	X	X	*	*	X
Securities firms	X	X	X	X	X	X	X	X
Pension funds		X	X	X				X
Mutual funds	X	X	X					X

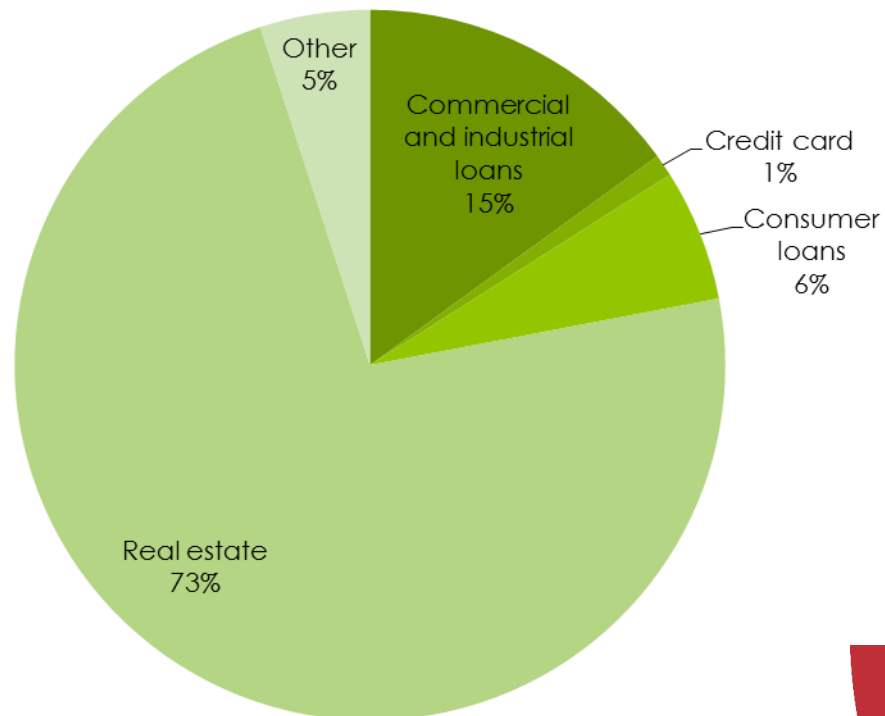
Rank	Bank	Country	Total assets, US\$bn	Balance sheet
1	Industrial & Commercial Bank of China (ICBC)	China	2,953.85	31.03.2013
2	HSBC Holdings	UK	2,681.36	31.03.2013
3	Deutsche Bank	Germany	2,597.36	31.03.2013
4	Credit Agricole Group	France	2,582.42	31.03.2013
5	BNP Paribas	France	2,507.96	31.03.2013
6	Mitsubishi UFJ Financial Group	Japan	2,486.31	31.03.2013
7	Barclays PLC	UK	2,414.78	31.03.2013
8	JPMorgan Chase & Co	USA	2,389.35	31.03.2013
9	China Construction Bank Corporation	China	2,361.60	31.03.2013
10	Japan Post Bank	Japan	2,118.84	31.03.2013
11	Agricultural Bank of China	China	2,295.80	31.03.2013
12	Bank of America	USA	2,174.61	31.03.2013
13	Bank of China	China	2,130.82	31.03.2013
14	Royal Bank of Scotland Group	UK	1,979.14	31.03.2013
15	Citigroup Inc	USA	1,881.73	31.03.2013
16	Mizuho Financial Group	Japan	1,881.03	31.03.2013
17	Banco Santander	Spain	1,637.74	31.03.2013
18	Societe Generale	France	1,592.51	31.03.2013
19	Sumitomo Mitsui Financial Group	Japan	1,576.58	31.03.2013
20	ING Group	Netherlands	1,508.71	31.03.2013

- Banco Best
- Banco Comercial Português (Millennium BCP)
- Novo-Banco (antes Banco Espírito Santo (BES))
- Banif
- Banco Internacional de Crédito (BIC)
- Banco Português de Investimento (BPI)
- Banco Santander Totta
- Barclays Bank Portugal
- BBVA Caixa Económica Montepio Geral (CEMG)
- Caixa Geral de Depósitos (CGD)

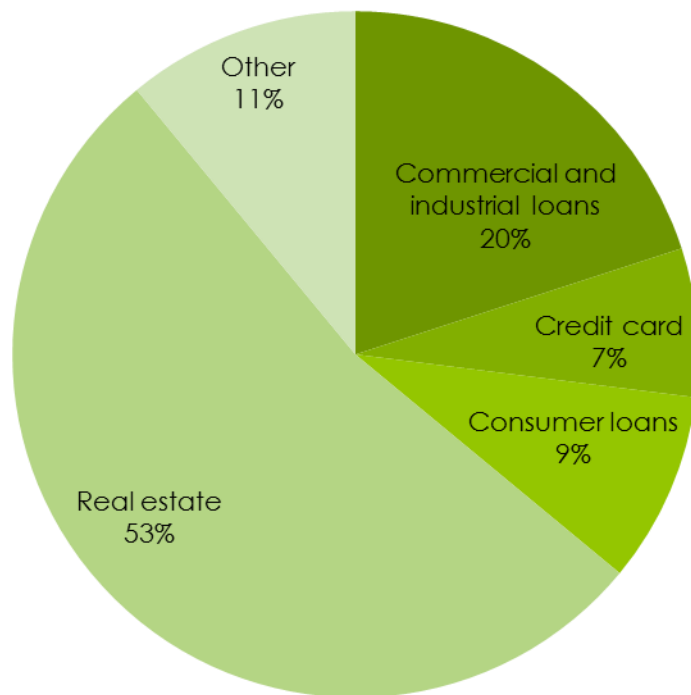
- Interest rate, income and risk.

Break down of loan portfolio US 2007

Small banks



Large banks



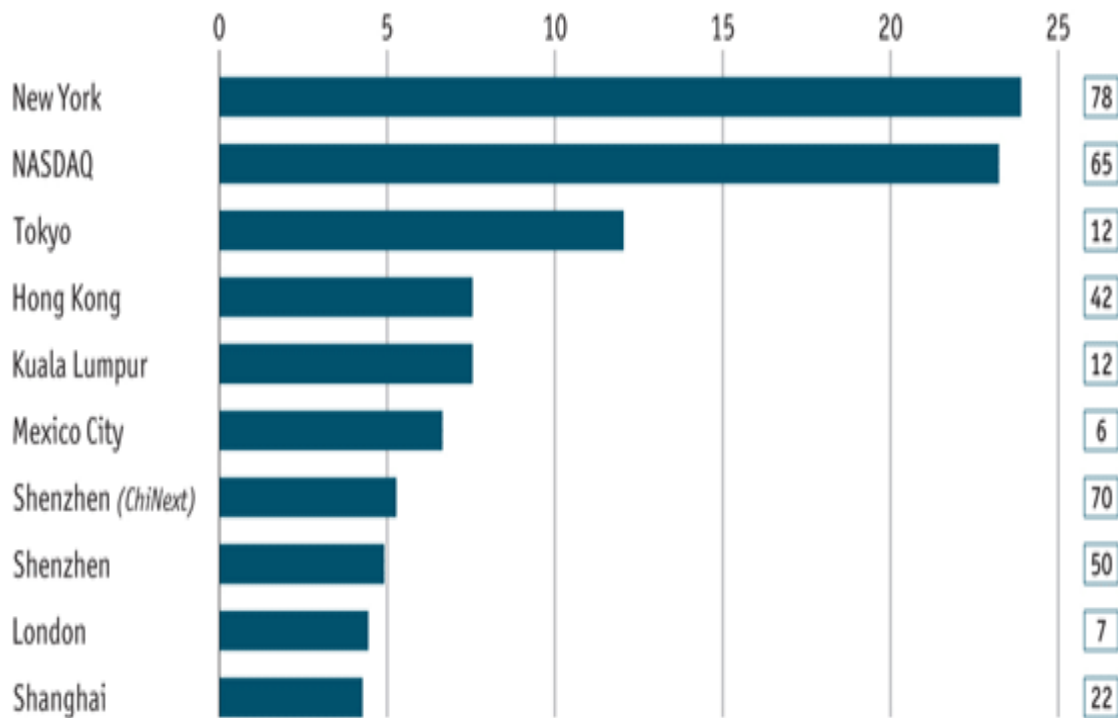
Traditional functions of investment banks

- 1) Investing
- 2) Services associated with security emission
- 3) Market making
- 4) Services associated with financial market transactions
- 5) Managing funds
- 6) Mergers and acquisitions

Initial public offerings

Top exchanges by IPO value*

Jan 1st-Dec 18th 2012, \$bn



Source: Dealogic

*Dual-listings are given full value at each exchange

M&A Financial Advisory League Tables

Global Announced Deals

Ranked by Volume

1/1/13 - 6/30/13	2013				2012		MKT SHARE CHANGE
FIRM	RANK	MKT SHARE	VOLUME USD (Mln)	DEAL COUNT	RANK	MKT SHARE	
Goldman Sachs & Co	1	21.1	204,467	130	1	24.3	(3.2)
JP Morgan	2	20.0	194,049	89	3	20.2	(0.2)
Morgan Stanley	3	16.5	159,476	125	2	23.5	(7.0)
Bank of America Merrill Lynch	4	15.0	145,332	76	8	13.2	1.8
Barclays Capital Group	5	11.1	107,190	76	5	19.2	(8.1)
Credit Suisse Group AG	6	10.9	105,296	75	6	16.1	(5.2)
Lazard Ltd	7	9.5	91,820	87	11	7.8	1.7
Citigroup Inc	8	9.1	88,675	84	7	15.2	(6.1)
Deutsche Bank AG	9	8.9	86,638	67	4	19.6	(10.7)
UBS AG	10	6.2	60,172	73	13	6.0	0.2
Moelis & Co	11	5.4	52,616	33	24	2.0	3.4
Centerview Partners LLC	12	5.0	48,706	5	16	3.1	1.9
Evercore Partners Inc	13	4.1	39,458	49	17	3.1	1.0
Rothschild	14	3.9	38,196	86	10	9.2	(5.3)
LionTree Advisors LLC	15	3.9	38,027	6	142	0.1	3.8
Wells Fargo & Co	16	3.9	37,300	24	31	1.5	2.4
RBC Capital Markets	17	3.7	35,730	62	14	4.9	(1.2)
HSBC Bank PLC	18	3.5	33,655	29	15	4.1	(0.6)
Leonardo & Co	19	3.1	29,837	22	421	0.0	3.1
BNP Paribas SA	20	2.7	26,239	43	12	6.6	(3.9)
TOTAL			\$969,698	12,539		\$1,023,490	

*Includes Mergers, Acquisitions, Divestitures, Self-tenders and Spinoffs. Excludes Open Market Transactions.

*Total Volume represents all announced transactions in US\$ millions.



- B- Public capital markets

World Federation of Exchanges (2013)
 e de Mitchie (2006)

Table I.1 Global securities outstanding: market value 1990–2003 US\$ trillion

Year	Total	Stocks	Bonds
1990	US\$ 27.2 trillion	US\$9.4 trillion (34.6%)	US\$17.8 trillion (65.4%)
2003	US\$ 82.0 trillion	US\$31.2 trillion (38.0%)	US\$50.8 trillion (62.0%)

Source: Bank for International Settlements, Statistics, 2006; World Federation of Exchanges, The Significance of the Exchange Industry, July 2004.

Table I.2 Stock market capitalization as a percentage of GDP

Country	1913	1938	1970	1999
World	56%	65%	55%	97%
UK	109%	192%	199%	225%
France	78%	19%	16%	117%
Germany	45%	18%	16%	67%
USA	41%	56%	66%	152%
Japan	49%	181%	23%	95%
India	2%	7%	6%	46%

Source: R. G. Rajan and L. Zingales, *The Great Reversals: The politics of financial development in the 20th century* [Paris 2000] pp 36, 39.

Domestic equity market capitalization

- ◆ Regional and total WFE domestic equity market capitalization performances at year-end 2012 compared to year-end 2011

Time zone	USD bn end-2012	USD bn end-2011	% change in USD
Americas	23 193	19 789	17.2%
Asia-Pacific	16 929	14 670	15.4%
Europe Africa Middle East	14 447	12 942	11.6%
Total WFE	54 570	47 401	15.1%

Value of bond trading

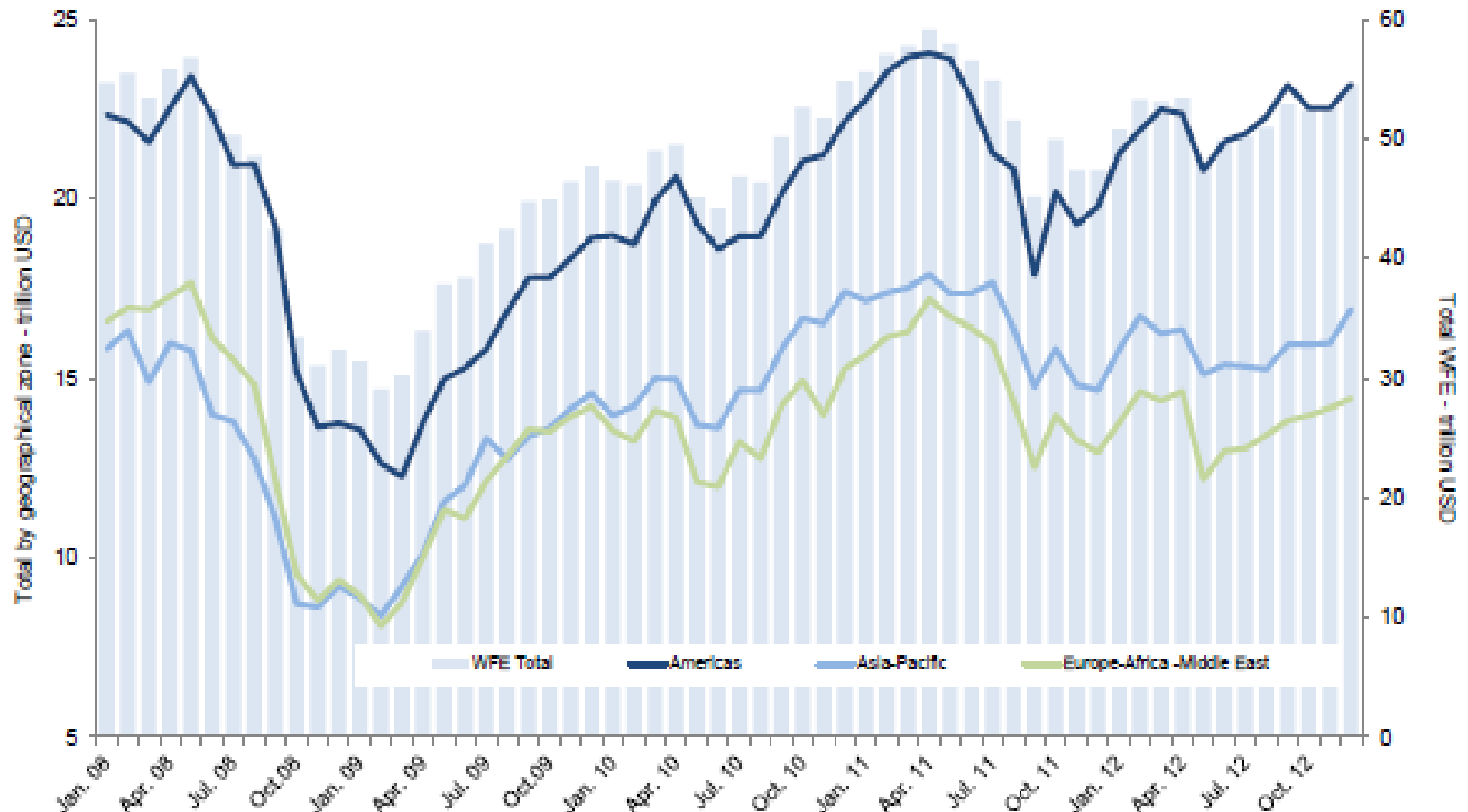
- ◆ Regional and total WFE bond trading value performances in 2012 compared to 2011

Time zone	USD bn	USD bn	% change
	Jan-Dec 2012	Jan-Dec 2011	in USD
Americas	1 050	1 188	-11.6%
Asia-Pacific	1 539	1 004	53.3%
Europe Africa Middle East	23 468	30 369	-22.7%
Total WFE	26 057	32 561	-20.0%

◆ Largest domestic equity market capitalizations at year-end 2012 and 2011

	Exchange	USD bn	USD bn	% change	% change
		end-2012	end-2011	in USD	in local currency
1	NYSE Euronext (US)	14 086	11 796	19.4%	19.4%
2	NASDAQ OMX (US)	4 582	3 845	19.2%	19.2%
3	Tokyo Stock Exchange Group	3 479	3 325	4.6%	17.6%
4	London Stock Exchange Group	3 397	3 266	4.0%	2.4%
5	NYSE Euronext (Europe)	2 832	2 447	15.8%	14.0%
6	Hong Kong Exchanges	2 832	2 258	25.4%	25.2%
7	Shanghai SE	2 547	2 357	8.1%	7.0%
8	TMX Group	2 059	1 912	7.7%	5.3%
9	Deutsche Börse	1 486	1 185	25.5%	23.6%
10	Australian SE	1 387	1 198	15.7%	14.3%

◆ Recent evolution of domestic equity market capitalization by time zones in USD trillion





- C- Other financial Institutions

- 
- a) Investment and hedge funds
 - b) Insurance Companies
 - c) Regulators

C.1 Investment and Hedge Funds

Table 3.2 Ownership of quoted shares in Britain, distribution by sector (%)

Sector	1963	1975	1989	1997
Individuals	54.0	37.5	17.7	20.5
Pension funds	6.4	16.8	34.2	27.9
Insurance cos.	10.0	15.9	17.3	23.1
Others (banks, public sector, unit trusts, overseas, etc.)	29.6	29.6	30.8	28.5

Source: Office for National Statistics. Crown Copyright 1997. Reproduced by permission of the Controller of HMSO and the Office for National Statistics.

Rank	Hedge Fund Managers	Location	Hedge Fund Assets	Date
1	Bridgewater Associates	US, Westport, CT	\$75.3 bn	30-06-2012
2	Man Group	UK, London	\$52.7 bn	30-06-2012
3	JPMorgan Asset Management	US, New York	\$44.0 bn	31-12-2012
4	Brevan Howard Asset Management	UK, London	\$36.7 bn	30-06-2012
5	BlueCrest Capital Management	UK, London	\$31.1 bn	30-06-2012
6	Och-Ziff Capital Management Group	US, New York	\$31.0 bn	01-10-2012
7	Winton Capital Management	UK, London	\$28.4 bn	30-06-2012
8	Highbridge Capital Management 1	US, New York	\$28.0 bn	30-06-2012
9	GLG Partners 2	UK, London	\$26.4 bn	30-06-2012
10	Soros Fund Management	US, New York	\$25.0 bn	31-12-2011
11	BlackRock	US, New York	\$24.2 bn	30-06-2012
12	Baupost Group	US, Boston	\$23.8 bn	30-06-2012
13	Angelo Gordon Co.	US, New York	\$23.0 bn	30-06-2012
14	Paulson & Co.	US, New York	\$22.6 bn	31-12-2011
15	D.E. Shaw & Co.	US, New York	\$22.0 bn	31-12-2011
18	Elliott Management	US, New York	\$20,2 bn	30-06-2012
16	Renaissance Technologies	US, New York	\$20.0 bn	31-12-2011
17	Farallon Capital Management	US, San Francisco	\$19.2 bn	31-12-2011
19	King Street Capital Management	US, New York	\$17.6 bn	31-12-2011
20	AHL 3	UK, London	\$16.7 bn	30-06-2012
21	Davidson Kempner Capital Management	US, New York	\$16.5 bn	31-12-2011
22	GoldenTree Asset Management	US, New York	\$16.3 bn	30-09-2012
23	Adage Capital Management	US, New York	\$16.0 bn	31-12-2011
24	Goldman Sachs Assets Management	US, New York	\$15.3 bn	31-12-2011
25	Moore Capital Management	US, New York	\$15.0 bn	31-12-2011

Source Bloomberg Markets Magazine

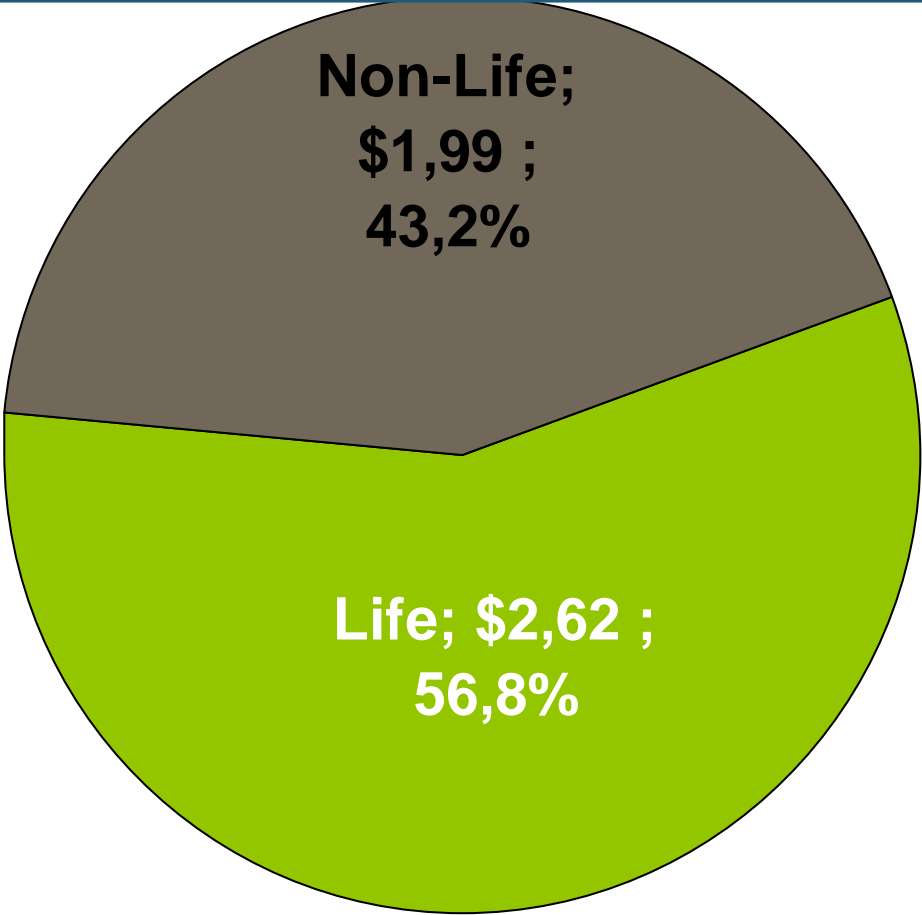
C.2 Insurance companies

- As to the separation between retail and investment banks, today also there is not a clear separation between banks and insurance companies.

- Insurance companies are in the business of risk.
- There are two major types of insurance contracts. Existem dois grandes tipos de seguro:
 - A) Life and health (also called life).
 - B) Property and its damages (also called non-life).

Distribution of Global Insurance Premiums, 2012 (\$ Trillions)

Total Premium Volume = \$4.613 Trillion*

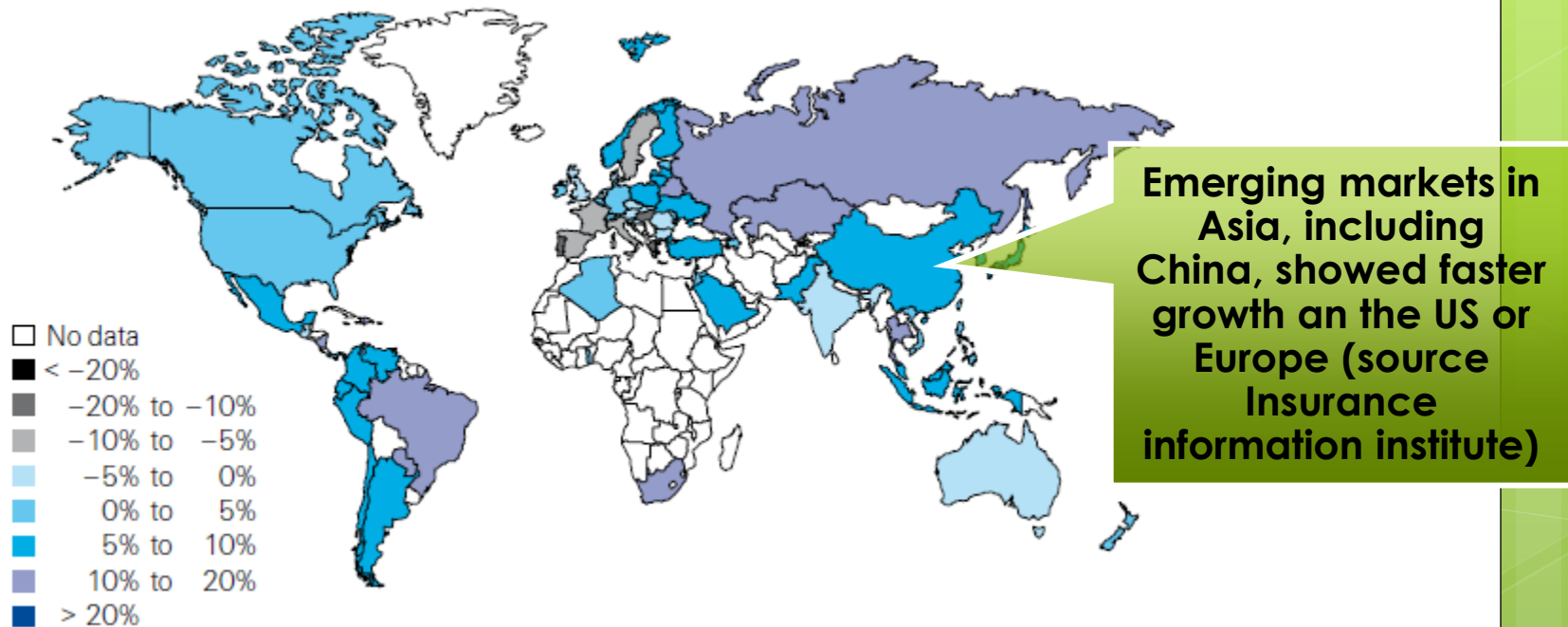


Life insurance accounted for nearly 57% of global premium volume in 2012 vs. 43% for Non-Life (source Insurance information institute)

Source: Swiss Re, *sigma*, No. 3/2013; Insurance Information Institute.

Rank	Insurance company	Country	Assets (US\$b, 31/12/2011)	Market cap (US\$b, 31/3/2012)
1	Japan Post Insurance	Japan	1,258.33 *	-
2	AXA	France	945.571	39.017
3	Allianz	Germany	830.804	54.245
4	Metlife	US	799.625	39.719
5	Nippon Life Insurance Company	Japan	649.402 *	-
6	Prudential Financial	US	624.521	29.667
7	Zenkyoren (JA-Kyosairen)	Japan	581.492**	-
8	American International Group (AIG)	US	555.773	58.480
9	Generali	Italy	547.924	24.132
10	Legal & General	UK	507.935	11.933
11	Aviva	UK	485.637	14.985
12	Manulife Financial	Canada	452.243	24.431
13	Aegon	Netherlands	447.575	10.589
14	ING Insurance (ING Verzekeringen N.V.)	Netherlands	434.377	31.863
15	Prudential	UK	425.322	29.612
16	TIAA-CREF	US	420.070	-
17	CNP Assurances	France	415.758	9.257
18	Berkshire Hathaway	US	392.647	201.135
19	Zurich Insurance Group	Switzerland	385.869	39.559
20	Dai-ichi Life Insurance	Japan	382.803	13.889
21	Ping An Insurance	China	359.728	51.463
22	Meiji Yasuda Life Insurance Company	Japan	352.698 *	-
23	Munich Re	Germany	320.654	26.998
24	Sumitomo Life Insurance	Japan	312.460	-
25	Hartford Financial Services	US	302.666	-

Global Real (Inflation Adjusted) Premium Growth (Life and Non-Life): 2012



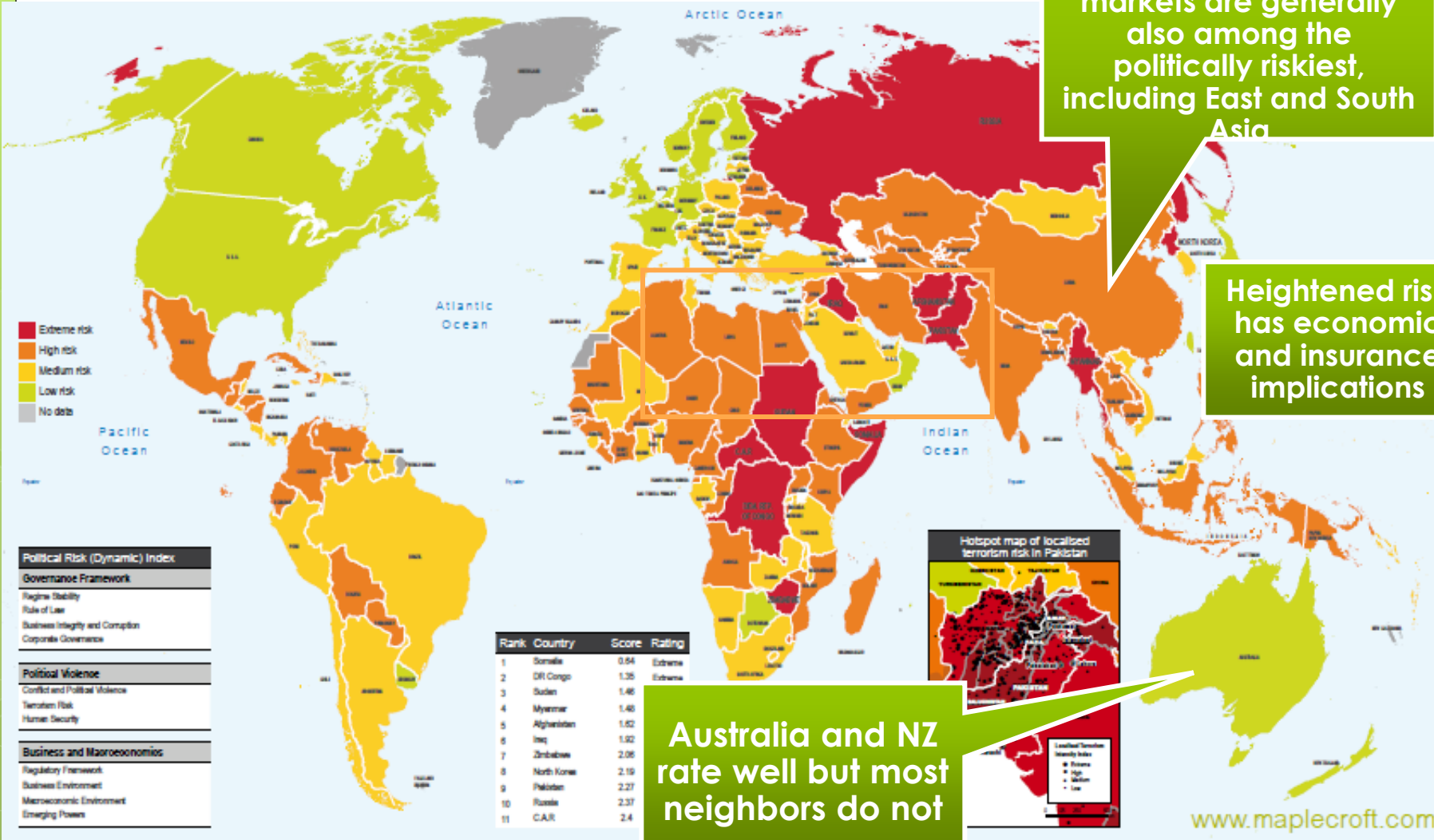
Market	Life	Non-Life	Total
Advanced	1.8	1.5	1.7
Emerging	4.9	8.6	6.8
World	2.3	2.6	2.4

Political Risk in 2011/12: Greatest Business Opportunities Are Often in Risky Nations according to Insurance Information Institute

The fastest growing markets are generally also among the politically riskiest, including East and South Asia

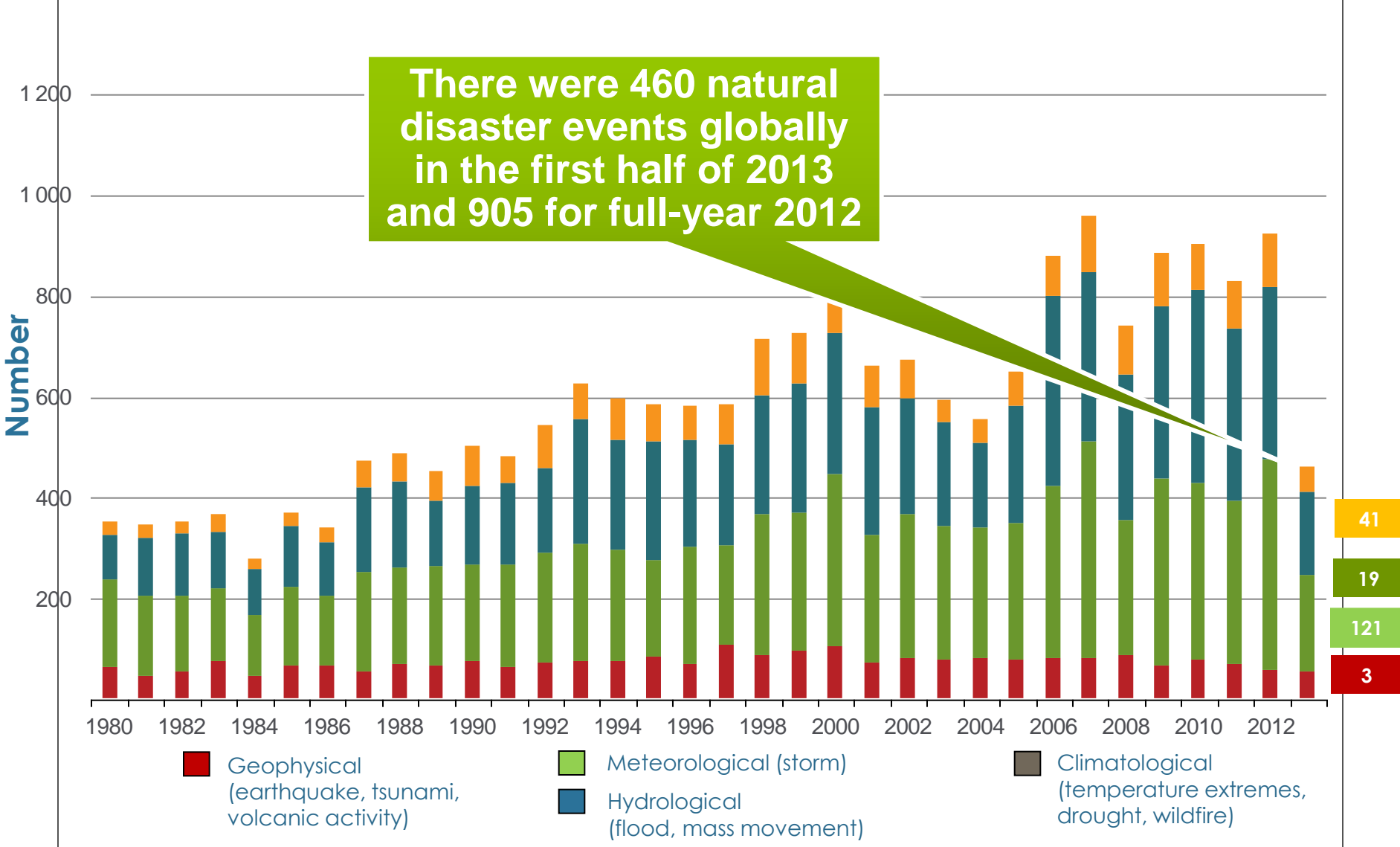
Heightened risk has economic and insurance implications

Australia and NZ rate well but most neighbors do not



Natural Disasters Worldwide, 1980 – 2013* (Number of Events)

There were 460 natural disaster events globally in the first half of 2013 and 905 for full-year 2012



*Through June 30, 2013.
Source: MR NatCatSERVICE

Top 16 Most Costly World Insurance Losses,

12/01/09 - 9pm

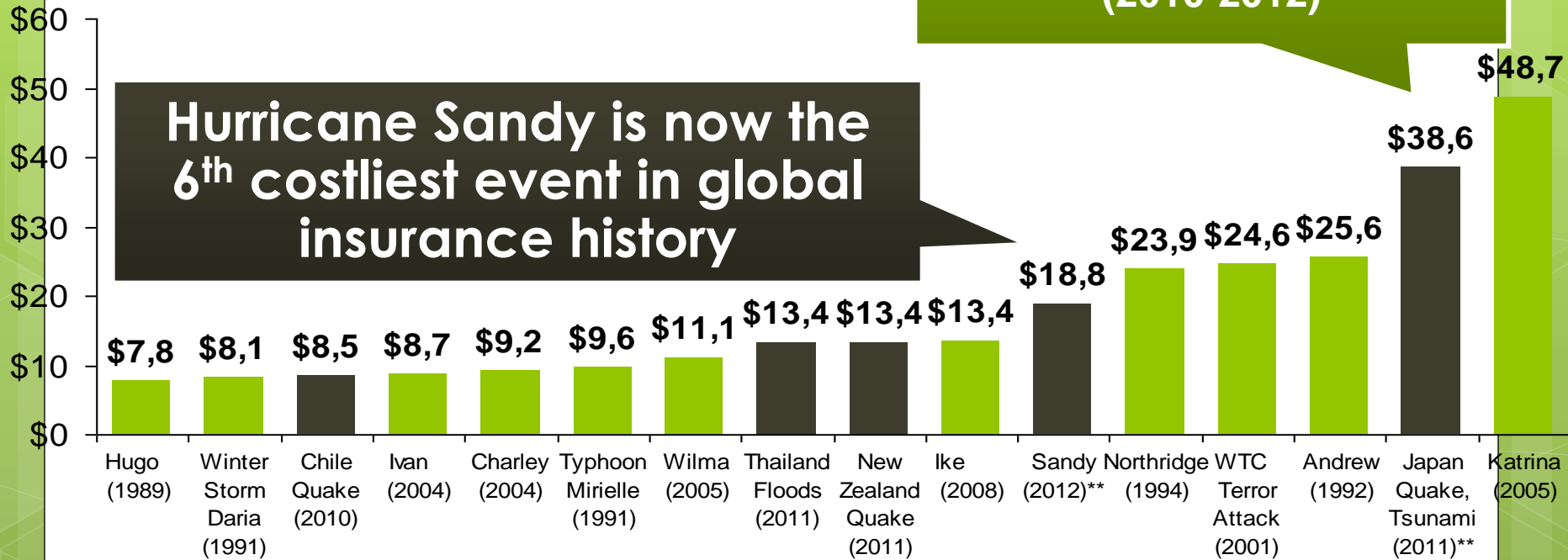
1970-2012*

(Insured Losses, 2012 Dollars, \$ Billions)

2012 insured CAT Losses totaled \$60B; Economic losses totaled \$140B, according to Swiss Re

5 of the top 14 most expensive catastrophes in world history have occurred within the past 3 years (2010-2012)

Hurricane Sandy is now the 6th costliest event in global insurance history



*Figures do not include federally insured flood losses.

**Estimate based on PCS value of \$18.75B as of 4/12/13.

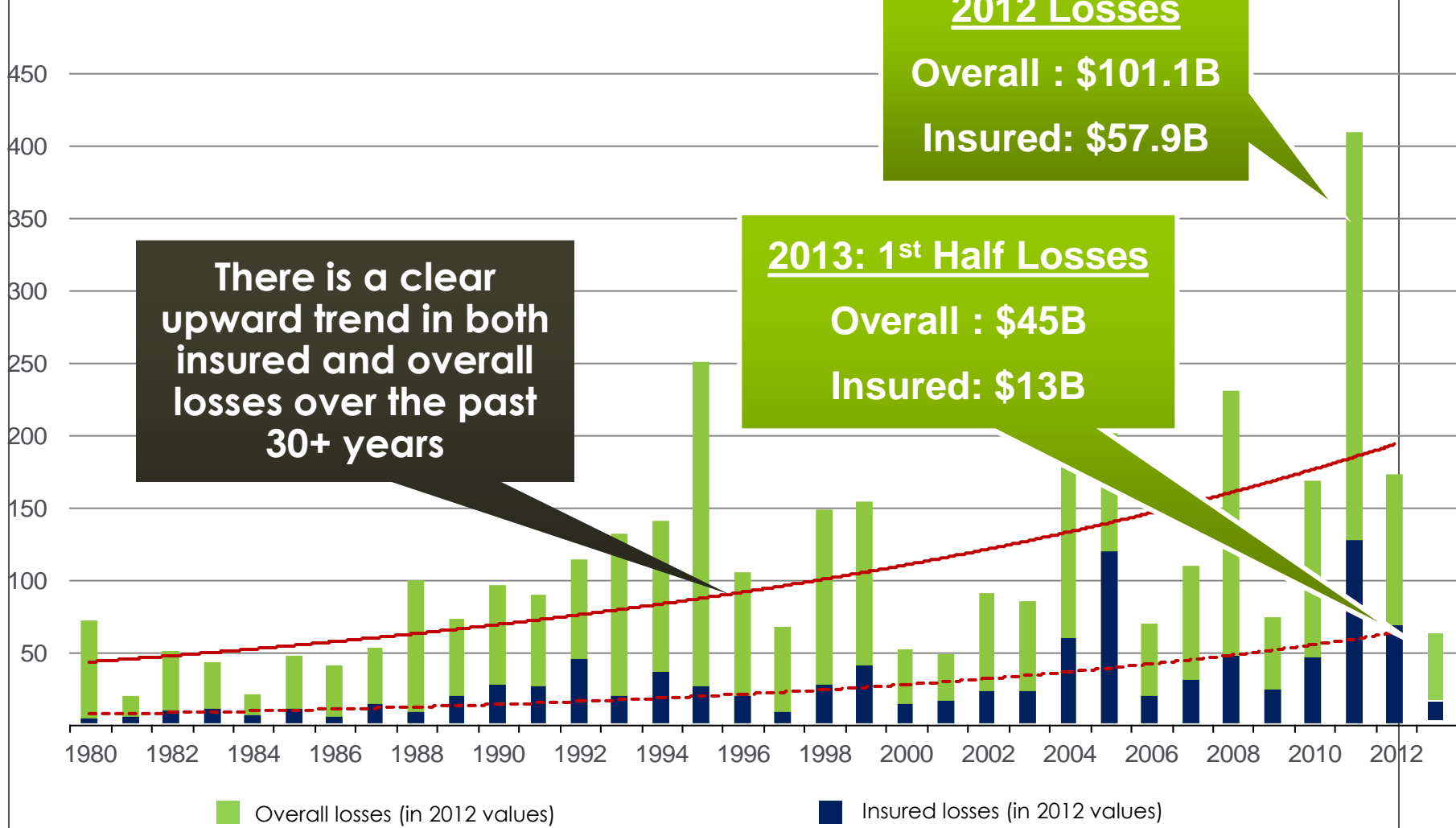
Sources: Munich Re; Swiss Re; Insurance Information Institute research.

Losses Due to Natural Disasters Worldwide, 1980–2013*

(Overall & Insured Losses)

(Overall and Insured Losses)

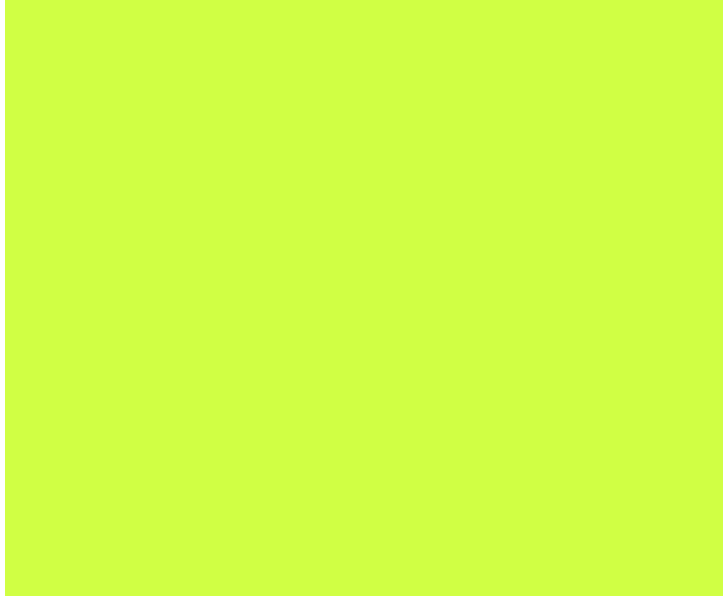
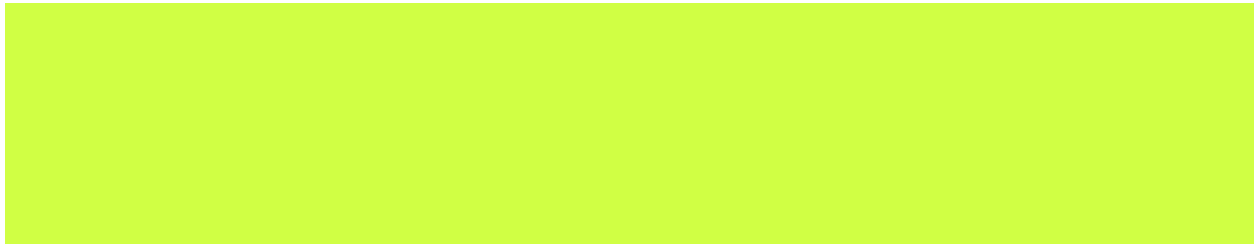
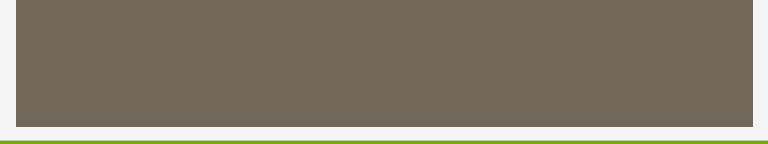
(2012 Dollars, \$ Billions)



*Through June 30, 2013.

Source: MR NatCatSERVICE

C.3 Regulators



Transaction costs

Typology of orders

- Normal order: buy and/or sell
- limit order.
- short sale.
- stop order.

Typology of transaction costs

- 1- Direct costs of transaction (financial intermediation and taxes).
- 2- Bid and ask spread.
- Note that economy of scale from large operators allow reducing costs.



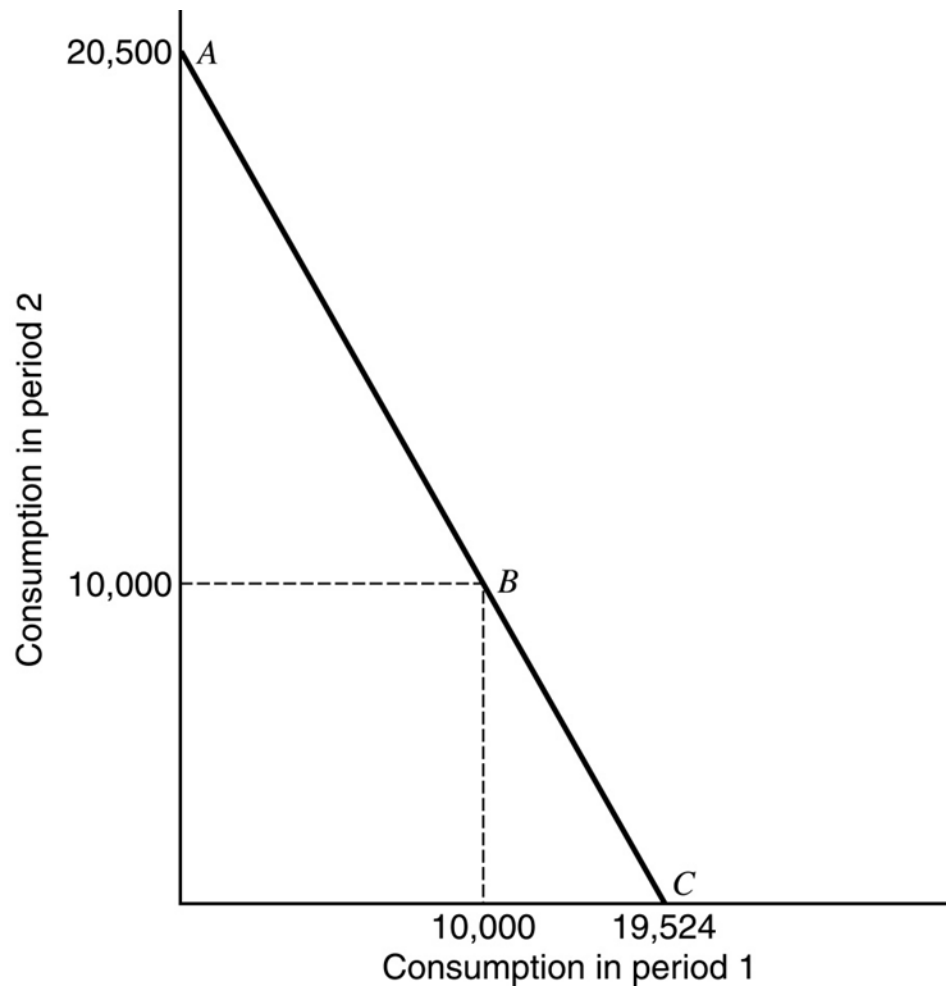
- 4. Managing investment portfolios.

- 4.1 Investment and risk.
- 4.2 Portfolio and efficient investment theory
- 4.3 Techniques for selecting investments into shares.

- Introduction to the economic theory of rational choice.

- Supor que um investidor terá um rendimento de \$ 10000 no período 1 e no período 2.
- Suponha também que o investidor pode emprestar ou pedir dinheiro emprestado à taxa de 5%.

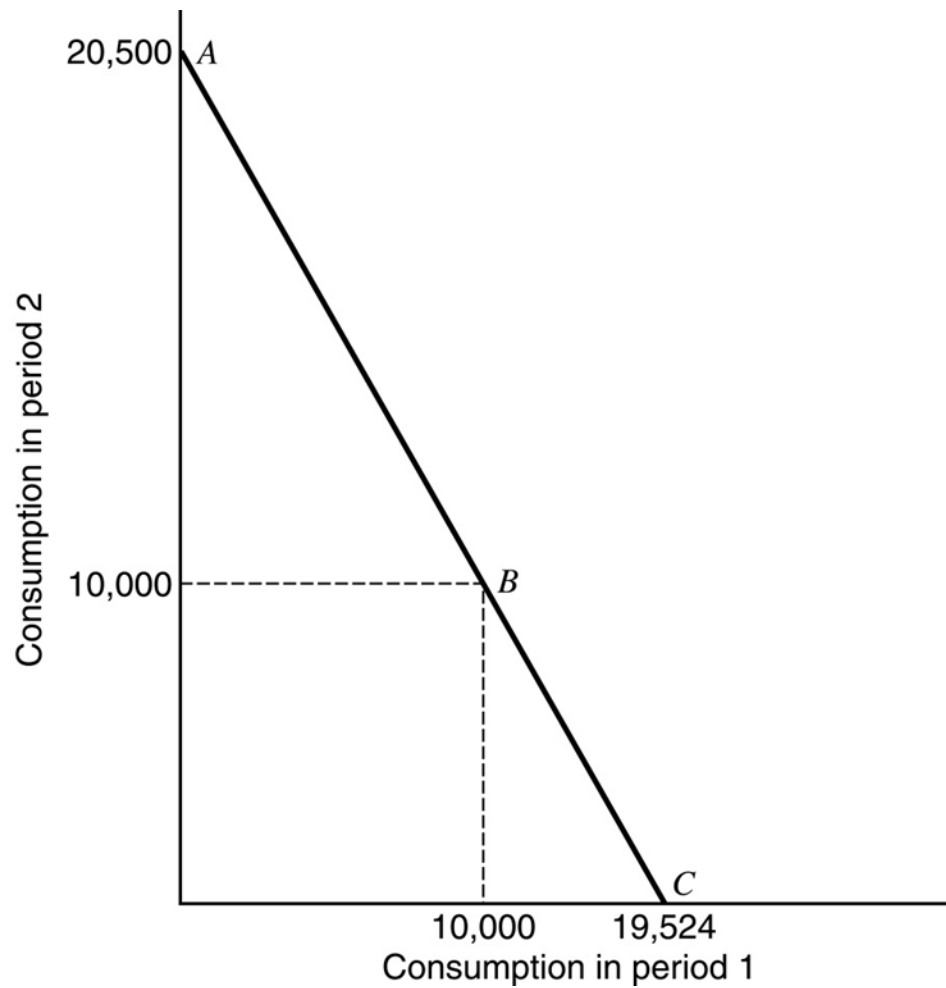
- Deste modo, o máximo que o investidor pode consumir é de
- $10000 + 10000 * 1.05 = 20500$ no período 2
- E de $10000 + 10000 * (1.05)^{-1} = 19524$ no período 1.



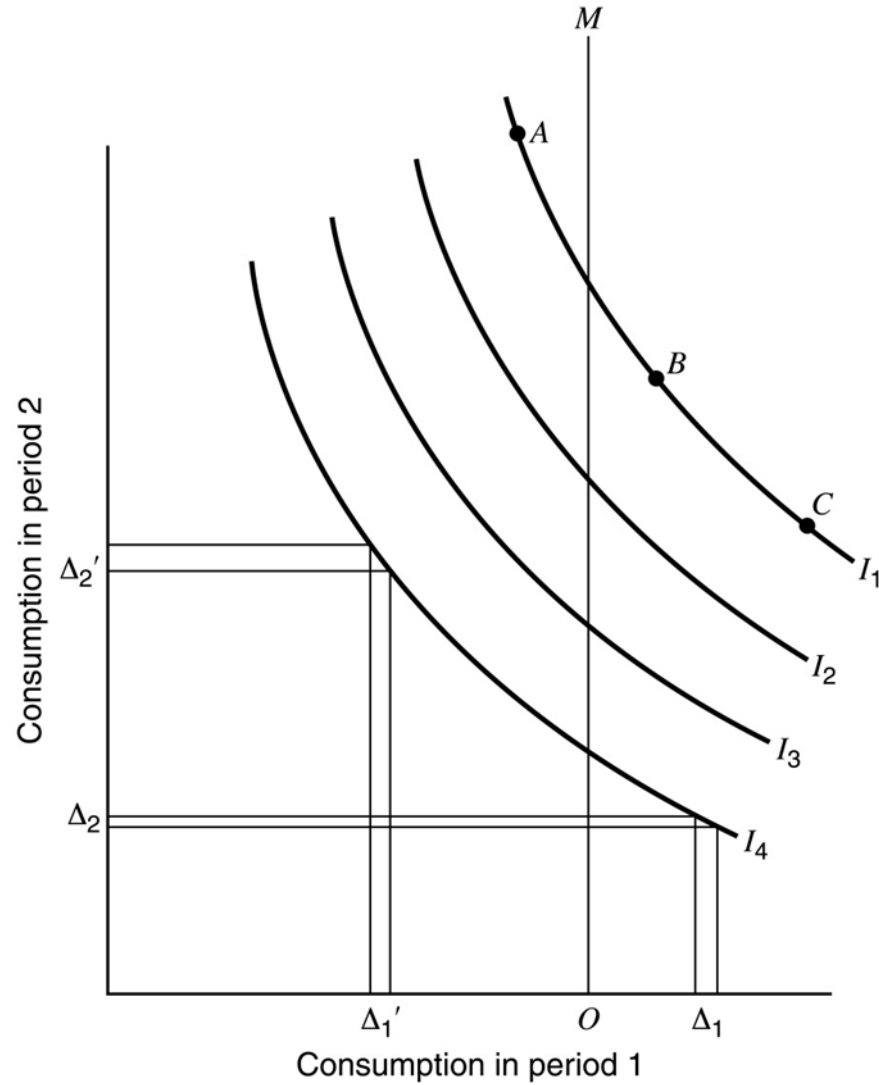
$$\begin{bmatrix} \text{Period 2} \\ \text{consumption} \end{bmatrix} = \begin{bmatrix} \text{Period 2} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{Amount} \\ \text{saved in 1} \end{bmatrix} [1 + 0.05]$$

$$C_2 = \$10,000 + (10,000 - C_1)(1.05)$$

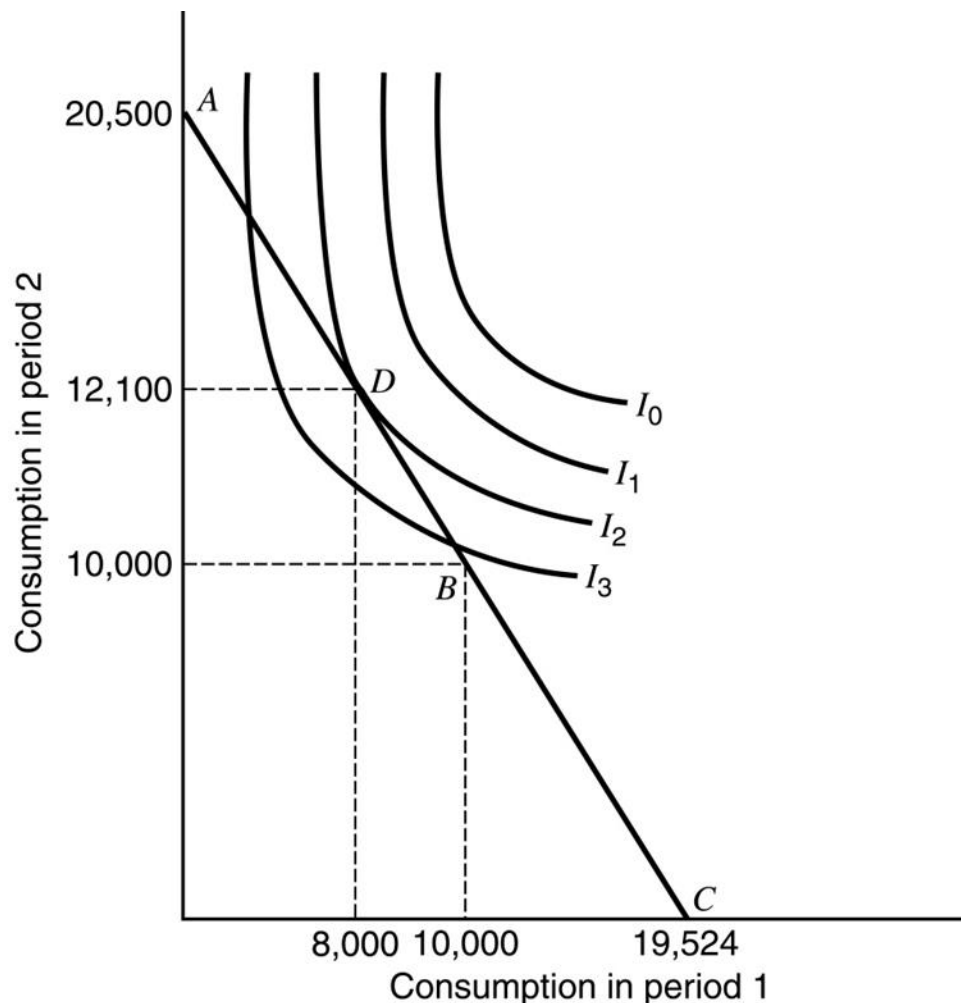
$$C_2 = \$20,500 - (1.05)C_1$$



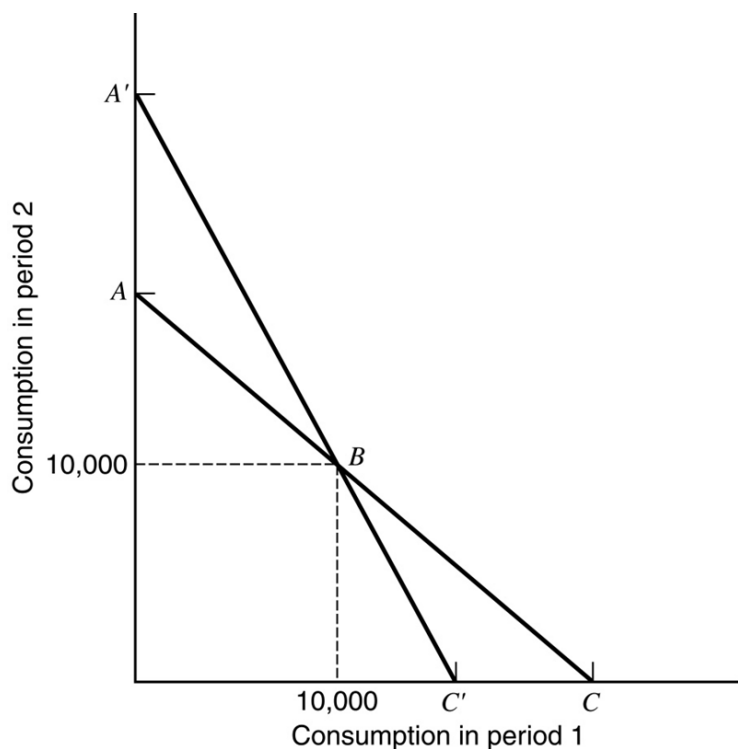
- A teoria económica da decisão racional pressupõe que os investidores escolhem entre oportunidades de investimento através da identificação de curvas de preferência racional, também chamadas funções de utilidade ou curvas de indiferença.



Sendo assim possível de se encontrar o ponto de equilíbrio de acordo com esta teoria.



Supor um caso com mais que um activo e riscos



Supor que existe um outro investimento que geraria 10000 tanto no momento 1 como 2, mas no entanto está associado a uma taxa de juro de depósito ou empréstimo de 10%

Neste caso, a escolha a fazer deveria situar-se ao longo de $A'BC$

- Problema da incerteza no comportamento dos investimentos, retornos, e dos próprios investidores.

Características do conjunto de oportunidades com risco associado

Return	Probability	Event
12	$\frac{1}{3}$	1
9	$\frac{1}{3}$	2
6	$\frac{1}{3}$	3

Retorno esperado

$$\frac{\sum_{j=1}^3 R_{y_j}}{3} = \frac{R_{y_1} + R_{y_2} + R_{y_3}}{3} = \frac{12 + 9 + 6}{3}$$

$$\bar{R}_i = \sum_{j=1}^M \left[\frac{R_{ij}}{M} \right]$$

>>>>> no caso de os rendimentos serem igualmente esperados.

$$\bar{R}_i = \sum_{j=1}^M R_{ij} P_{ij}$$

>>>>>> no caso geral.

Recordar características dos valores esperados

$$E(R_1 + R_2) = \bar{R}_1 + \bar{R}_2$$

$$E(cR_1) = c\bar{R}_1$$

Medidas de dispersão (variabilidade)

Event	Probability	Asset 1	Asset 2	Asset 3
A	$\frac{1}{3}$	14	28	42
B	$\frac{1}{3}$	10	20	30
C	$\frac{1}{3}$	<u>6</u>	<u>12</u>	<u>18</u>
	Expected Return	10	20	30

Variância da População

$$\sigma_i^2 = \sum_{j=1}^M \left[\frac{(R_{ij} - \bar{R}_i)^2}{M} \right] :$$

>>>>> no caso de os rendimentos serem igualmente esperados.

$$\sigma_i^2 = \sum_{j=1}^M P_{ij} (R_{ij} - \bar{R}_i)^2$$

>>>>>> no caso geral.

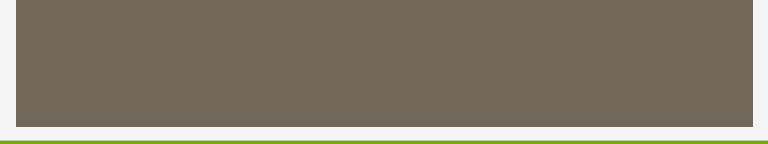
Variância da Rendibilidade de um Activo (Amostra)

$$\sigma_i^2 = \sum_{j=1}^M \left[\frac{(R_{ij} - \bar{R}_i)^2}{M - 1} \right]$$

Market Condition	Return ^a				Rainfall	Return ^a Asset 4
	Asset 1	Asset 2	Asset 3	Asset 5		
Good	15	16	1	16	Plentiful	16
Average	9	10	10	10	Average	10
Poor	3	4	19	4	Poor	4
.....						
Mean return	9	10	10	10		10
Variance	24	24	54	24		24
Standard deviation	4.9	4.9	7.35	4.90		4.9

^aThe alternative returns on each asset are assumed equally likely and, thus, each has a probability of $\frac{1}{3}$.

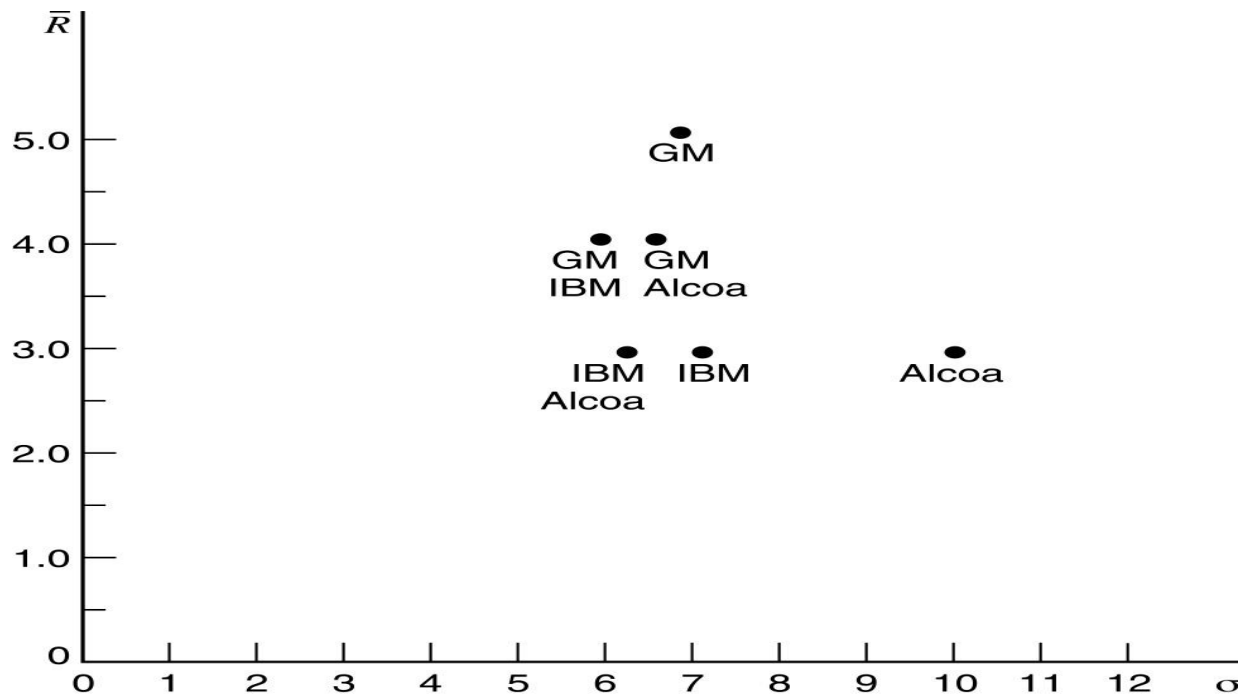
- Então e se combinarmos activos diferentes?



Condition of Market	Asset 2	Asset 3	Combination of Asset 2 (60%) and Asset 3 (40%)
Good	\$1.16	\$1.01	\$1.10
Average	1.10	1.10	1.10
Poor	1.04	1.19	1.10

Month	IBM	Alcoa	GM	$\frac{1}{2}$ IBM + $\frac{1}{2}$ Alcoa	$\frac{1}{2}$ GM + $\frac{1}{2}$ Alcoa	$\frac{1}{2}$ GM + $\frac{1}{2}$ IBM
1	12.05	14.09	25.20	13.07	19.65	18.63
2	15.27	2.96	2.86	9.12	2.91	9.07
3	-4.12	7.19	5.45	1.54	6.32	0.67
4	1.57	24.39	4.56	12.98	14.48	3.07
5	3.16	0.06	3.72	1.61	1.89	3.44
6	-2.79	6.52	0.29	1.87	3.41	-1.25
7	-8.97	-8.75	5.38	-8.86	-1.69	-1.80
8	-1.18	2.82	-2.97	0.82	-0.08	-2.08
9	1.07	-13.97	1.52	-6.45	-6.23	1.30
10	12.75	-8.06	10.75	2.35	1.35	11.75
11	7.48	-0.70	3.79	3.39	1.55	5.64
12	-.94	8.80	1.32	3.93	5.06	0.19
\bar{R}	2.95	2.95	5.16	2.95	4.05	4.05
σ	7.15	10.06	6.83	6.32	6.69	6.02

Correlation Coefficient: IBM and Alcoa = 0.05;
 GM and Alcoa = 0.22; IBM and GM = 0.48



Rendibilidade de uma Carteira

$$R_{Pj} = \sum_{i=1}^N X_i R_{ij}$$

Rendibilidade Esperada de uma Carteira

$$\bar{R}_p = E(R_p) = E\left(\sum_{i=1}^N X_i R_{ij}\right) = \sum_{i=1}^N X_i \bar{R}_i$$

$$\bar{R}_p = \left(\frac{0.6}{1}\right) (0.1) + \left(\frac{0.4}{1}\right) (0.1) = 0.1$$

$$(X+Y)^2 = X^2 + XY + XY + Y^2 = X^2 + 2XY + Y^2$$

Variância de Rendibilidade de uma Carteira com dois activos

$$\sigma_p^2 = E(R_p - \bar{R}_p)^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \sigma_{12}$$

Covariância de Rendibilidade de uma Carteira com dois activos

$$\sigma_{12} = \sum_{j=1}^M \left[\frac{(R_{1j} - \bar{R}_1)(R_{2j} - \bar{R}_2)}{M} \right] = \sigma_1 \sigma_2 \rho_{12}$$

Coeficiente de Correlação entre Rendibilidades de dois activos

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$

Variância de Rendibilidade de uma Carteira com n activos

$$\sigma_p^2 = \sum_{j=1}^N X_j^2 \sigma_j^2 + \sum_{j=1}^N \sum_{\substack{k=1 \\ j \neq k}}^N X_j X_k \sigma_{jk}$$

ou

$$\sigma_p^2 = \sum_{j=1}^N X_j^2 \sigma_j^2 + \sum_{j=1}^N \sum_{\substack{k=1 \\ j \neq k}}^N X_j X_k \rho_{jk} \sigma_j \sigma_k$$

Market Condition	Return ^a				Rainfall	Return ^a Asset 4
	Asset 1	Asset 2	Asset 3	Asset 5		
Good	15	16	1	16	Plentiful	16
Average	9	10	10	10	Average	10
Poor	3	4	19	4	Poor	4
.....						
Mean return	9	10	10	10		10
Variance	24	24	54	24		24
Standard deviation	4.9	4.9	7.35	4.90		4.9

^aThe alternative returns on each asset are assumed equally likely and, thus, each has a probability of $\frac{1}{3}$.

Condition of Market	Deviations Security 1	Deviations Security 2	Product of Deviations	Deviations Security 1	Deviations Security 3	Product of Deviations
Good	(15 – 9)	(16 – 10)	36	(15 – 9)	(1 – 10)	–54
Average	(9 – 9)	(10 – 10)	0	(9 – 9)	(10 – 10)	0
Poor	(3 – 9)	(4 – 10)	<u>36</u>	(3 – 9)	(19 – 10)	<u>–54</u>
			72			–108

	1	2	3	4	5
1		24 (+1)	-36 (-1)	0 (0)	24 (+1)
2			-36 (-1)	0 (0)	24 (+1)
3				0 (0)	-36 (-1)
4					0 (0)
5					

Table 4-7 Covariance and Correlation Coefficients (in Brackets)
Between Assets

- Variância de uma carteira de um euro em que se investe 60 centimos no activo 2 e 40 cêntimos no activo 3

- $\sigma^2 = \left(\frac{0.6}{1}\right)^2 24 + \left(\frac{0.4}{1}\right)^2 54 + 2\left(\frac{0.4}{1}\right) \left(\frac{0.4}{1}\right) (-36)$
=0

Efeitos da diversificação da carteira da diminuição da variância do rendimento esperado

Number of Securities	Expected Portfolio Variance
1	46.619
2	26.839
4	16.948
6	13.651
8	12.003
10	11.014
12	10.354
14	9.883
16	9.530
18	9.256
20	9.036
25	8.640
30	8.376
35	8.188
40	8.047
45	7.937
50	7.849
75	7.585

IE



LISBON
SCHOOL OF
ECONOMICS &
MANAGEMENT

UNIVERSIDADE DE LISBOA

Number of Securities	Expected Portfolio Variance
100	7.453
125	7.374
150	7.321
175	7.284
200	7.255
250	7.216
300	7.190
350	7.171
400	7.157
450	7.146
500	7.137
600	7.124
700	7.114
800	7.107
900	7.102
1000	7.097
Infinity	7.058

United States	73
U.K.	65.5
France	67.3
Germany	56.2
Italy	60.0
Belgium	80.0
Switzerland	56.0
Netherlands	76.1
International stocks	89.3

Table 4-9 Percentage of the Risk on an Individual Security that Can Be Eliminated by Holding a Random Portfolio of Stocks within Selected National Markets and among National Markets

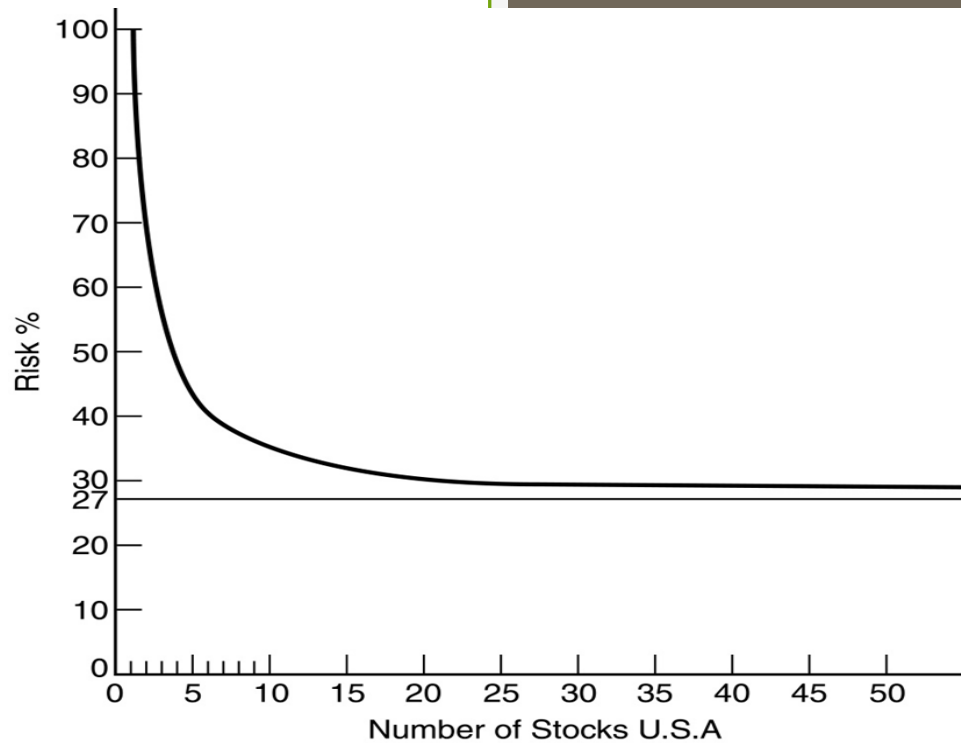


FIGURE 4-2 The effect of number of securities on risk of the portfolio in the United States.

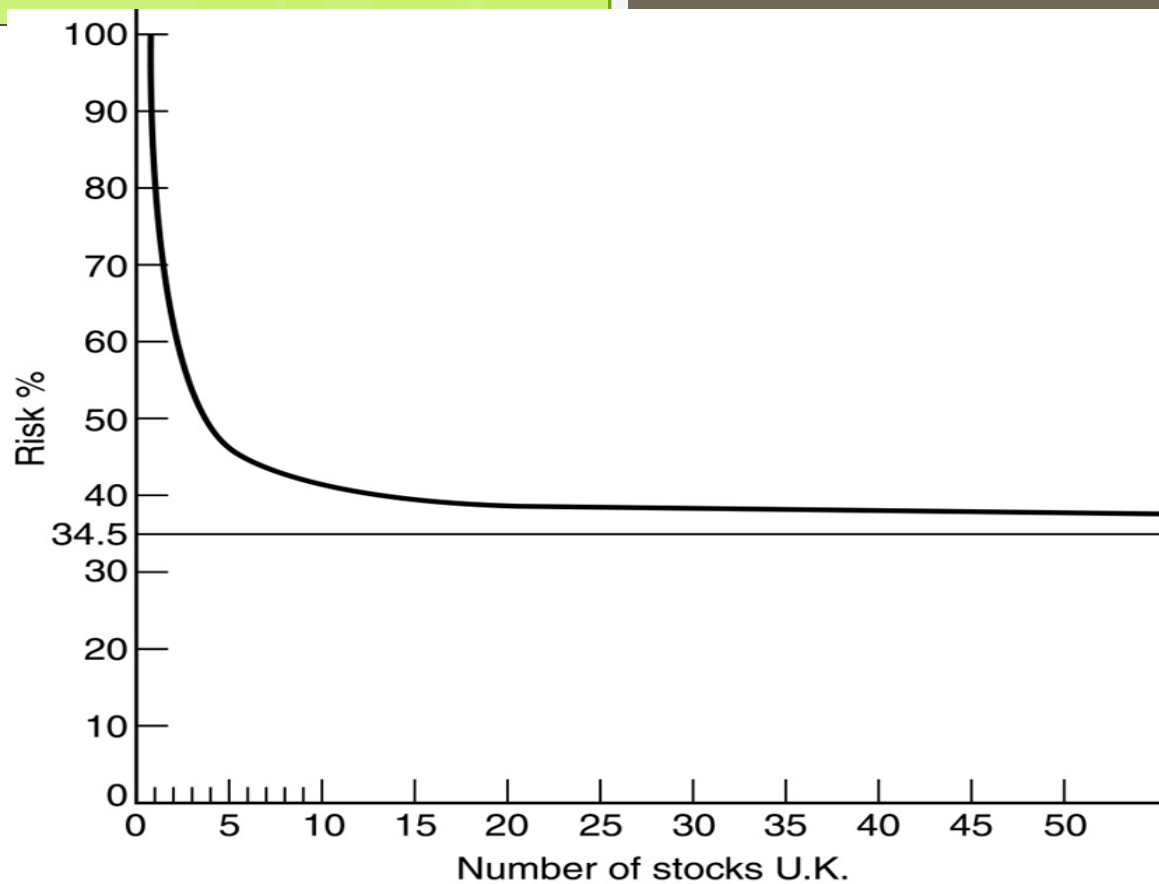


FIGURE 4-3 The effect of securities on risk in the U. K. .

Date	Standard Deviations		Correlation Coefficients
	Bonds	Stocks	
77-81	9.70%	14.54%	0.34
82-86	6.63%	14.66%	0.41
87-91	4.72%	15.40%	0.49
77-91	7.46%	14.87%	0.41

Table 4-10 Historical Data on Bonds and Stocks

Proportion Stocks	Proportion Bonds	Mean Return	Standard Deviation
1	0	12.5	14.90
0.9	0.1	11.85	13.63
0.8	0.2	11.2	12.38
0.7	0.3	10.55	11.15
0.6	0.4	9.9	9.95
0.5	0.5	9.25	8.80
0.4	0.6	8.6	7.70
0.3	0.7	7.95	6.69
0.2	0.8	7.3	5.82
0.1	0.9	6.65	5.16
0	1	6	4.80

Table 4-11 Mean Return and Standard Deviation for Combinations of Stocks and Bonds

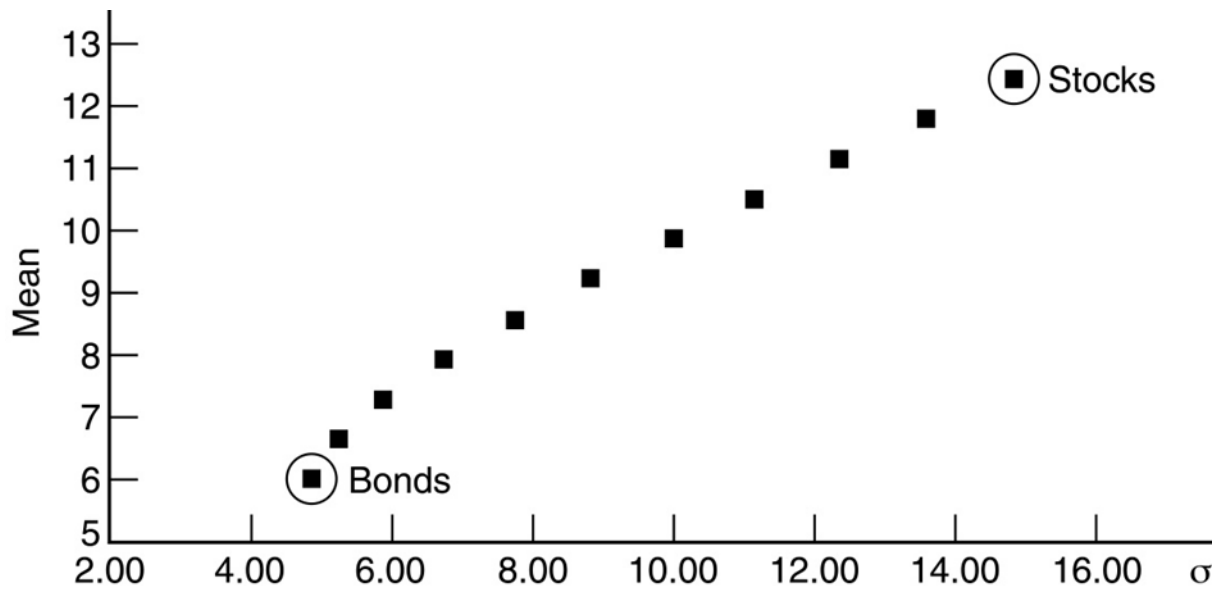


FIGURE 4-4 Combinations of bonds and stocks.

Carteiras com índice S&P e Portfolio Internacional

$$R_{S\&P} = 2.5\% \quad \sigma_{S\&P} = 14.9\% \quad \rho_{S\&P, I} = 0.53$$
$$\bar{R}_I = 10.5\% \quad \sigma_I = 14.0\%$$

Proportion S&P	Proportion International	Mean Return	Standard Deviation
1	0	12.5	14.90
0.9	0.1	12.3	13.93
0.8	0.2	12.1	13.11
0.7	0.3	11.9	12.46
0.6	0.4	11.7	12.01
0.5	0.5	11.5	11.79
0.45	0.55	11.4	11.76
0.4	0.6	11.3	11.80
0.3	0.7	11.1	12.04
0.2	0.8	10.9	12.50
0.1	0.9	10.7	13.17
0	1	10.5	14.00

Table 4-12 Mean Return and Standard Deviation for Combinations of Domestic and International Stocks

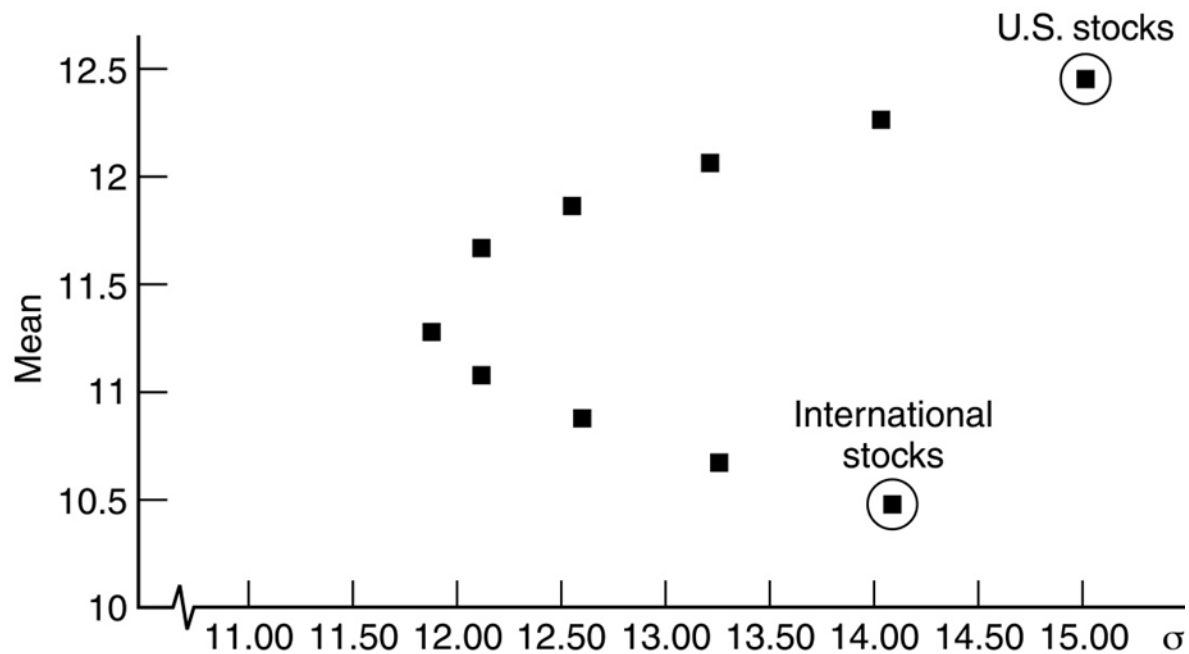


FIGURE 4-5 Combinations of U.S. stocks and international stocks.

Casos Particulares:

1. Se:

$$\sigma_{jk} = 0 \Rightarrow \sigma_p^2 = \sum_{j=1}^N X_j^2 \sigma_j^2$$

2. Se:

$$X_i = 1/N$$

2.1.

$$\sigma_{ik} = 0 \Rightarrow \sigma_p^2 = \frac{1}{N} \sum_{j=1}^N \left[\frac{\sigma_j^2}{N} \right] = \frac{1}{N} \bar{\sigma}_j^2$$

2.2.

$$\sigma_{ik} \neq 0 \Rightarrow \sigma_p^2 = \frac{1}{N} \sum_{j=1}^N \frac{\sigma_j^2}{N} + \left(\frac{N-1}{N} \right) \sum_{j=1}^N \sum_{\substack{k=1 \\ j \neq k}}^N \frac{\sigma_{jk}}{N(N-1)} = \frac{1}{N} \bar{\sigma}_j^2 + \frac{N-1}{N} \bar{\sigma}_{jk}$$