

# Models in Finance - Class 9

Master in Actuarial Science

João Guerra

ISEG

## Main alternative models

- The lognormal model and the Wilkie model are statistically based models, whose structure is derived from past time series together with some intuition regarding reasonable model formulae.
- Alternative to purely statistical models: give more weight to economic theory. For instance, we can consider market efficiency arguments or purchasing power parity arguments in order to guide the construction of stochastic investment models.
- The advantage of using more economic theory is that it gives us a more concrete way of interpreting model output.

# Estimating parameters for asset pricing models

- Estimation of parameters is one of the most time-consuming aspects of stochastic asset modelling.
- Simplest case of parameters estimation: the purely statistical model, where parameters are calibrated entirely to past time series. If the data is available, the calibration can be a straightforward and mechanical process.
- Example: in the lognormal model (which is a purely statistical model) the parameter estimation is easier because we do not consider economic restrictions or economic factors. However, even in this case some critical analysis of the results is required from the modeller.

# Estimating parameters for asset pricing models

- There may not always be as much data as we would like, and the statistical error in estimating parameters may be substantial.
- Furthermore, there is a difficulty in interpreting data which appears to invalidate the model being fitted.
- Example: what should be done when fitting a gaussian model in the presence of large outliers in the data? Should we reject the hypothesis of normality and build the model under some alternative hypothesis? Or should we exclude the outliers?
- Major financial risks lie in the outliers: is it reasonable to ignore them?
- On the other hand, non-normal models are generally more difficult to apply than gaussian models.

## Estimating parameters for asset pricing models

- Question: Why do major financial risks lie in the outliers?
- In practice, a common approach to outliers is to exclude them from the statistical analysis. Then, the model standard deviation may be subjectively increased, in order to give some recognition to the outliers which have been excluded.
- In actuarial modelling it is common practice to use the same data to specify the model structure, to fit the parameters, and to validate the model choice.
- A large number of possible model structures are tested, and testing stops when a model which is found which passes a suitable array of tests.
- Unfortunately, many of these tests (for example, tests of stationarity) have low power, and therefore may not reject incorrect models.
- Indeed, even if the “true” model was not in the class of models being fitted, we would still end up with an apparently acceptable fit, because the rules say we keep generalising until we find one.
- This process of generalisation tends to lead to models which wrap themselves around the data, resulting in an understatement of future risk, and optimism regarding the accuracy of out-of-sample forecasts.

João Guerra (ISEG)

Models in Finance - Class 9

5 / 9

## Estimating parameters for asset pricing models

- Even if the “true” model was not in the class of models being fitted, we would still end up with an apparently acceptable fit, because the rules say we keep generalising until we find one.
- This "generalisation" tends to lead to models which wrap themselves around the data, resulting in an understatement of future risk, and optimism regarding the accuracy of out-of-sample forecasts.
- An alternative approach is the "general to specific" approach: the modeller starts with a very general model, which include all the variables (both current values and lagged values) that could influence the variable being modelled, then the modeller makes the model more specific by eliminating variables (one at a time) that do not affect the significance of the fit to past data.

# Estimating parameters for economic models

- For economic models, the estimation becomes more complex. The objective of such models is to simplify reality by imposing certain stylised facts about how markets would behave in an ideal world.
- This theory may impose constraints, for example on the relative volatilities of bonds and currencies. Observed data may not fit these constraints perfectly.
- In these cases, it is important to prioritise what features of the economy are most important to calibrate accurately for a particular application.

## Problems - part 2 - problem 1

- Problem 1: An investor model has decided to model the rate of wage inflation in year  $t$  with the AR(2) model

$$I_t = 0.02 + 0.3(I_{t-1} - 0.02) + 0.09(I_{t-2} - 0.02) + 0.005Z_t,$$

where  $Z_t \sim N(0, 1)$ . The current value of wage inflation is  $I_t = 0.022$ .

(a) State the key statistical properties and discuss the economic plausibility of this model.

(b) By considering  $\begin{bmatrix} I_t \\ I_{t-1} \end{bmatrix}$ , show that the model can be written in vector form to produce a Markov model.

(c) During the last year, the rate of wage inflation was such that  $I_t = 1.05I_{t-1}$ . Calculate the probability that it will increase by at least 5% again in the coming year. Comment on your answer.

## Problems - part 2 - problem 2

- Problem 2: A government in a developed country is convinced that in order to win a national election, all it needs to do is ensure that the annual rate of inflation is between 1% and 3% at the time of the national election. The central bank has informed that the government that the annual force of inflation  $I_t$  at each month  $t$  can be modelled by:

$$I_t = 0.95I_{t-1} + 0.001Z_t.$$

The current annual rate of inflation is 2.9%.

- Find the distribution of  $I_{12}$ .
- Assuming that the government and the central bank are correct, calculate the probability that the government will win the next election in one year's time.
- Explain, with reasons, whether inflation in this model is mean-reverting and give an example of another financial quantity that would normally be considered to be mean-reverting.