

Group 1

1. Consider a consumer whose utility function is $u(x_1, x_2) = \ln(x_1 + 2x_2)$.
 - a) (0.5 marks) Formulate the consumer choice problem.
 - b) (2 marks) Find this consumer's demand.
 - c) (0.5 marks) Determine the indirect utility function.
 - d) (1 mark) Determine the expenditure function.

2. (1 mark) Let the "at least as good as relation" be a preference relation (a binary relation satisfying completeness and transitivity). Show that the "strict preference relation" is transitive.

Group 2

Filipe plays soccer with a second league team. If he does not suffer any injury by the end of the season, he will get a professional contract with his team, which is worth 10000 euros. If he is injured, he will get a contract as a fitness trainer, which is only worth 100 euros. The probability of suffering an injury is 10%. He is likely to get through the season unscathed with a 90% probability. Assume that Filipe's von Neuman-Morgenstern utility function is $u(x) = \sqrt{x}$.

- a) (0.5 marks) How high is the expected value of this gamble?
- b) (0.5 marks) Is Filipe risk averse, risk neutral, or risk lover?
- c) (1 mark) How high is the expected utility of the gamble described above?
- d) (1.5 marks) Assuming that Filipe could buy insurance at price p that would pay him 9900 euro in the case of an accident, how high could the maximal value of p be for Filipe to still afford the insurance?
- e) (1.5 marks) How high is the certainty equivalent for the above gamble? How high is the risk premium?

Group 3

1. Let $f(x) = 10x - x^2/2$. Determine:
 - a) (1.5 marks) the input demand function;
 - b) (1 mark) the output supply function;
 - c) (1 mark) the profit function.

2. (1.5 marks) Prove that if the production function $f(x_1, x_2)$ is homogeneous of degree 1, it can be written as: $f(x_1, x_2) = MP_1(x_1)x_1 + MP_2(x_2)x_2$, where $MP_i(x_i)$ is the marginal product of input i .

Group 4

1. In a perfectly competitive market there are J firms. Each firm produces output q according to an identical cost function $c(q) = k + q^2$, where $k > 0$. Market demand is given by $Q_d = a - p$. Assume $a > 2\sqrt{k}$.

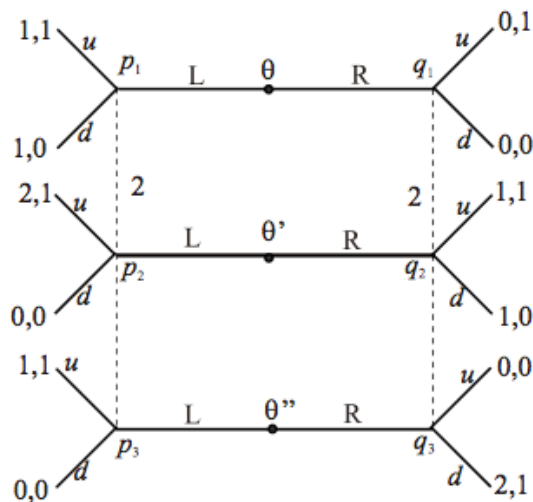
- (1.5 marks) Determine the profit-maximizing output of an individual firm.
- (1 mark) Determine the market price and amount produced by all firms in the short-run.
- (1 mark) Compute the number of firms that are active in this market in the long-run equilibrium (ignoring any integer constraints).

2. Consider a consumer whose income is y_0 and consider an inferior (but not Giffen) good q , whose price falls from p_0 to p_1 (i.e., $p_0 > p_1$).

- (0.5 marks) Define compensating variation (CV).
- (1 mark) Graphically represent the CV in the space (q, p) .

Group 5

1. (5 marks) The game below starts with a move by Nature (not represented in the tree), who chooses player 1's type. Player 1 can be of type θ , θ' or θ'' , each type has equal probability of occurring. Player 1 observes his type, but player 2 only knows the *a priori* probability of each type. However, player 2 observes the action chosen by player 1 (L or R).



Check whether there is a weak perfect Bayesian equilibrium in which all types of player 1 choose L. Specify the profile of equilibrium strategies, as well as the equilibrium beliefs.

Group 6

1. Consider the market for used cars, where cars are transferred from one person to another after a period of use and the car's inevitable wear and tear. Assume that the quality of a car is given by θ , which follows a uniform distribution in $[0, 1]$, where 0 is the lowest possible quality level and 1 is the highest quality level. Assume that both buyers and sellers are risk neutral and let v_s and v_b be the valuation of a car of quality 1, for buyers and sellers, respectively.

- a) (2 marks) Assume that both sides of the market observe θ . Under which conditions is the seller of a car of quality θ willing to sell his car? Under which conditions is a buyer willing to buy a car of quality θ ? Which values can the price of such a car take?
- b) (2 marks) Now suppose that only the seller can observe quality, while a buyer knows the distribution of θ . Under which conditions is the seller of a car of quality θ willing to sell his car? Under which conditions is a buyer willing to buy a car of quality θ ?

2. (1 mark) Explain (i) the meaning and (ii) the importance of the "single-crossing property" of indifference curves in signaling models.