Solutions to problems - Part 2

1.

(a) The wage inflation is modelled by an autoregressive process of order 2: AR(2). It is natural that the wage inflation depends much more on the inflation on the previous year than on the second previous year: that is why the first autoregressive parameter (0.3) is larger than the second parameter (0.09).

The process is mean reverting, with a long term mean of 0.02 or 2%.

The model allows for negative values (deflation), which can happen with small probability.

The standard normal random variables Z_t represent the random component of inflation. The independence of these random variables can be a drawback of the model, since there can be dependence in the random oscillations of inflation and these random oscillations can be non-normal.

(b)

$$\begin{bmatrix} I_t \\ I_{t-1} \end{bmatrix} = \begin{bmatrix} 0.0122 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.3 & 0.09 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_{t-1} \\ I_{t-2} \end{bmatrix} + \begin{bmatrix} 0.005 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Z_t \\ Z_{t-1} \end{bmatrix}$$

or in vector-matrix form: $\mathbf{I}_t = \mathbf{b} + \mathbf{A}\mathbf{I}_{t-1} + \mathbf{B}\mathbf{Z}_t$.

(c) The current force of inflation is $I_t = 0.022$ and during the last year incrased 5%. So,

$$I_{t-1} = \frac{0.022}{1.05} = 0.02095.$$

Replacing these values in the equation, we have:

$$I_{t+1} = ... = 0.020686 + 0.005Z_{t+1}.$$

Therefore, the probability of increasing again 5% (such that $I_{t+1} = 0.022 \times 1.05 = 0.0231$) is

$$P\left[I_{t+1} \geq 0.0231\right] = ... = 0.315.$$

The value $I_t = 0.022$ is larger than the long term mean, therefore one expects by the mean-reverting effect that the inflation decreases next year, that is why the probability is less than 0.5.

2.

(a) $I_0 = \ln(1 + 0.029) = \ln(1.029)$

We assume that the random variables Z_t have a distribution $N(0,1)$.

Replacing I_{11} and then $I_{10,}$ etc..., we see that

$$I_{12} = 0.001\left[Z_{12} + 0.95Z_{11} + 0.95^2 Z_{10} + ... + 0.95^{11}Z_1\right] + 0.95^{12}I_0$$

$$Var\left[I_{12}\right] = 0.001^2\left[1 + 0.95^2 + 0.95^4 + ... + 0.95^{22}\right] = 7.2614 \times 10^{-6}$$

So I_{12} has normal distribution $N\left[0.015448; 7.2614 \times 10^{-6}\right]$

(b) We assume that the random variables Z_t have a distribution $N(0,1)$.

$$P[\ln(1.01) < I_{12} < \ln(1.03)] =$$
$$= \Phi(5.2365) - \Phi(-2.0402) = 0.97934$$

(c) We assume that the random variables Z_t have a distribution $N(0,1)$. We can write the model has

$$I_t - 0 = 0.95(I_{t-1} - 0) + 0.001 Z_t,$$

and the model is mean reverting with a long run mean of 0.

Another market variable that can be considered to be mean reverting is the interest rate.