Hillier + Lieberman (2010) (9th ed.)

of the same two machines, Vincent needs to decide and inform the customers this afternoon about how many of each product he will agree to make over the next two days.

Each tow bar requires 3.2 hours on machine 1 and 2 hours on machine 2. Each stabilizer bar requires 2.4 hours on machine 1 and 3 hours on machine 2. Machine 1 will be available for 16 hours over the next two days and machine 2 will be available for 15 hours. The profit for each tow bar produced would be $130 and the profit for each stabilizer bar produced would be $150.

Vincent now wants to determine the mix of these production quantities that will maximize the total profit.

(a) Formulate an IP model for this problem.
(b) Use a graphical approach to solve this model.
C (c) Use the computer to solve the model.

11.1-7. Reconsider Prob. 8.2-21 involving a contractor (Susan Meyer) who needs to arrange for hauling gravel from two pits to three building sites.

Susan now needs to hire the trucks (and their drivers) to do the hauling. Each truck can only be used to haul gravel from a single pit to a single site. In addition to the hauling and gravel costs specified in Prob. 8.2-21, there now is a fixed cost of $150 associated with hiring each truck. A truck can haul 5 tons, but it is not required to go full. For each combination of pit and site, there are now two decisions to be made: the number of trucks to be used and the amount of gravel to be hauled.

(a) Formulate an MIP model for this problem.
C (b) Use the computer to solve this model.

11.2-1. Read the referenced article that fully describes the OR study summarized in the application vignette presented in Sec. 11.2. Briefly describe how integer programming was applied in this study. Then list the various financial and nonfinancial benefits that resulted from this study.

11.2-2. Select one of the actual applications of BIP by a company or governmental agency mentioned in Sec. 11.2. Read the article describing the application in the referenced issue of *Interfaces*. Write a two-page summary of the application and its benefits.

11.2-3. Select three of the actual applications of BIP by a company or governmental agency mentioned in Sec. 11.2. Read the articles describing the applications in the referenced issues of *Interfaces*. For each one, write a one-page summary of the application and its benefits.

11.3-1.* The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.

A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs).

	Product			
	1	2	3	4
Start-up cost	$50,000	$40,000	$70,000	$60,000
Marginal revenue	$ 70	$ 60	$ 90	$ 80

Let the continuous decision variables x_1, x_2, x_3, and x_4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

1. No more than two of the products can be produced.
2. Either product 3 or 4 can be produced only if either product 1 or 2 is produced.
3. Either $5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6{,}000$
   or $4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6{,}000$.

(a) Introduce auxiliary binary variables to formulate a mixed BIP model for this problem.
C (b) Use the computer to solve this model.

11.3-2. Suppose that a mathematical model fits linear programming except for the restriction that $|x_1 - x_2| = 0$, or 3, or 6. Show how to reformulate this restriction to fit an MIP model.

11.3-3. Suppose that a mathematical model fits linear programming except for the restrictions that

1. At least one of the following two inequalities holds:

$$3x_1 - x_2 - x_3 + x_4 \le 12$$
$$x_1 + x_2 + x_3 + x_4 \le 15.$$

2. At least two of the following three inequalities holds:

$$2x_1 + 5x_2 - x_3 + x_4 \le 30$$
$$-x_1 + 3x_2 + 5x_3 + x_4 \le 40$$
$$3x_1 - x_2 + 3x_3 - x_4 \le 60.$$

Show how to reformulate these restrictions to fit an MIP model.

11.3-4. The Toys-R-4-U Company has developed two new toys for possible inclusion in its product line for the upcoming Christmas season. Setting up the production facilities to begin production would cost $50,000 for toy 1 and $80,000 for toy 2. Once these costs are covered, the toys would generate a unit profit of $10 for toy 1 and $15 for toy 2.

The company has two factories that are capable of producing these toys. However, to avoid doubling the start-up costs, just one factory would be used, where the choice would be based on maximizing profit. For administrative reasons, the same factory would be used for both new toys if both are produced.

Toy 1 can be produced at the rate of 50 per hour in factory 1 and 40 per hour in factory 2. Toy 2 can be produced at the rate of 40 per hour in factory 1 and 25 per hour in factory 2. Factories 1 and 2, respectively, have 500 hours and 700 hours of production time available before Christmas that could be used to produce these toys.

It is not known whether these two toys would be continued after Christmas. Therefore, the problem is to determine how many units (if any) of each new toy should be produced before Christmas to maximize the total profit.

(a) Formulate an MIP model for this problem.

C **(b)** Use the computer to solve this model.

11.3-5.* Northeastern Airlines is considering the purchase of new long-, medium-, and short-range jet passenger airplanes. The purchase price would be \$67 million for each long-range plane, \$50 million for each medium-range plane, and \$35 million for each short-range plane. The board of directors has authorized a maximum commitment of \$1.5 billion for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after capital recovery costs are subtracted) would be \$4.2 million per long-range plane, \$3 million per medium-range plane, and \$2.3 million per short-range plane.

It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, each medium-range plane is equivalent to $1\frac{1}{3}$ short-range planes, and each long-range plane is equivalent to $1\frac{2}{3}$ short-range planes in terms of their use of the maintenance facilities.

The information given here was obtained by a preliminary analysis of the problem. A more detailed analysis will be conducted subsequently. However, using the preceding data as a first approximation, management wishes to know how many planes of each type should be purchased to maximize profit.

(a) Formulate an IP model for this problem.

C **(b)** Use the computer to solve this problem.

(c) Use a binary representation of the variables to reformulate the IP model in part (*a*) as a BIP problem.

C **(d)** Use the computer to solve the BIP model formulated in part (*c*). Then use this optimal solution to identify an optimal solution for the IP model formulated in part (*a*).

11.3-6. Consider the two-variable IP example discussed in Sec. 11.5 and illustrated in Fig. 11.3.

(a) Use a binary representation of the variables to reformulate this model as a BIP problem.

C **(b)** Use the computer to solve this BIP problem. Then use this optimal solution to identify an optimal solution for the original IP model.

11.3-7. The Fly-Right Airplane Company builds small jet airplanes to sell to corporations for the use of their executives. To meet the needs of these executives, the company's customers sometimes order a custom design of the airplanes being purchased. When this occurs, a substantial start-up cost is incurred to initiate the production of these airplanes.

Fly-Right has recently received purchase requests from three customers with short deadlines. However, because the company's production facilities already are almost completely tied up filling previous orders, it will not be able to accept all three orders. Therefore, a decision now needs to be made on the number of airplanes the company will agree to produce (if any) for each of the three customers.

The relevant data are given in the next table. The first row gives the start-up cost required to initiate the production of the airplanes for each customer. Once production is under way, the marginal net revenue (which is the purchase price minus the marginal production cost) from each airplane produced is shown in the second row. The third row gives the percentage of the available production capacity that would be used for each airplane produced. The last row indicates the maximum number of airplanes requested by each customer (but less will be accepted).

	Customer		
	1	**2**	**3**
Start-up cost	\$3 million	\$2 million	0
Marginal net revenue	\$2 million	\$3 million	\$0.8 million
Capacity used per plane	20%	40%	20%
Maximum order	3 planes	2 planes	5 planes

Fly-Right now wants to determine how many airplanes to produce for each customer (if any) to maximize the company's total profit (total net revenue minus start-up costs).

(a) Formulate a model with both integer variables and binary variables for this problem.

C **(b)** Use the computer to solve this model.

11.4-1. Reconsider the Fly-Right Airplane Co. problem introduced in Prob. 11.3-7. A more detailed analysis of the various cost and revenue factors now has revealed that the potential profit from producing airplanes for each customer cannot be expressed simply in terms of a *start-up cost* and a fixed *marginal net revenue* per airplane produced. Instead, the profits are given by the following table.

Airplanes Produced	**Profit from Customer**		
	1	**2**	**3**
0	0	0	0
1	−\$1 million	\$1 million	\$1 [illegible]
2	\$2 million	\$5 million	\$3 [illegible]
3	\$4 million		\$5 million
4			\$6 million
5			\$7 million

(a) Formulate a BIP model for this problem that includes constraints for *mutually exclusive alternatives*.

C **(b)** Use the computer to solve the model formulated in part (*a*). Then use this optimal solution to identify the optimal number of airplanes to produce for each customer.