SOLUÇÕES DOS EXERCÍCIOS DE ANÁLISE MATEMÁTICA IV

Equações Diferenciais Ordinárias

Capítulo I

7. $x(t) = \frac{C}{t} + t, \ \ C \in \Re$

8. $x(t) = Ct^n + t^n e^t, \ \ C \in \Re$

10. $x(t) = e^{1/t^2}$

11. $x^3(t) = \frac{C(1+t^2)^{3/2}}{t^3} - 1, \ \ C \in \Re$

12. $x(t) = te^t$

13. $x^2(t) e^{t^2/x^2} = K, \ \ K \in \Re$

14. $\frac{t^2}{x^3} - \frac{1}{x} = C, \ \ C \in \Re$

15. $x(t) = \begin{cases} 2(1 - e^{-t}) & se \ \ t \in [0,1] \\ 2e^{-t}(e-1) & se \ \ t > 1 \end{cases}$

19. $t^3 x + tx^3 + 2t + 5x = 9$

22b) $\log t - \log x - \frac{1}{2(tx)^2} = C, \ \ C \in \Re$

23 $a) \ \mu(t) = e^{\int a(t)dt}$

$b) \ x(t) = Ce^{-t^2} - 1, \ \ C \in \Re \ \ \lim_{t \to +\infty} x(t) = -1$

24. $x(t) = -e^{-t}$

25. $x(t) = C + \frac{t^2}{2}, \ \ C \in \Re$ **ou** $x(t) = \left(\frac{t}{A} - \frac{1}{A^2} \right) e^{At} + B, \ \ A \neq 0 \ e \ B \in \Re$

Capítulo II

1.

$x(t)=0\ \forall_{t\in\Re}$ **e** $\sqrt{x}=\frac{1}{2}\int_0^t g(s)ds$ **são ambas soluções do PVI. Falha a condição de Lipschitz.**

$a)\ x(t)=\frac{1}{2}\left(e^t+e^{-t}\right)=\cosh t$

2 $b)\ PVI\begin{cases} x''=x \\ x(0)=1 \\ x'(0)=0 \end{cases}$

$a)\ x_1(t)=1+t+\frac{t^3}{3},\ x_2(t)=1+t+t^2+\frac{2t^3}{3}+\frac{5t^4}{12}+\frac{2t^5}{15}+\frac{t^6}{18}+\frac{t^7}{63}$

3 $b)\ k=10$

$c)\ x(t)=\frac{e^{t^3/3}}{1-\int_0^t e^{l^3/3}dl}$

4.

$x(t)=0\ \forall_{t\in\Re}$ **e** $\sqrt{x}=\frac{1}{2}\int_0^t g(s)ds$ **são ambas soluções do PVI. Falha a condição de Lipschitz.**

5. $x(t)\equiv 1$ **e** $\int_1^x \frac{1}{\sqrt{s^2-1}}ds=t\quad f\notin C^1([\mathbf{1},+\infty))$

6a) $k=6$

9. $\theta\in\left(0,\frac{1}{T}\right)$

Capítulo III

4.

$a)\ x(t) = -2e^{2t} + 3e^{t}$

$b)\ x(t) = (1-A)e^{-t^2} + (1-3t)Ae^{2t},\ \ A \in \Re$

$c)\ x(t) = -te^{-t} + e^{-t}\log|t+1| + te^{-t}\log|t+1| + C_1e^{-t} + C_2te^{-t},\ \ C_1, C_2 \in \Re$

$d)\ x(t) = C_1e^{t^2} + C_2e^{-2t} - \frac{2}{5}\cos 2t - \frac{6}{5}sen2t,\ \ C_1, C_2 \in \Re$

$e)\ x(t) = C_1\cos 2t + C_2 sen2t + \frac{1}{4}tsen2t,\ \ C_1, C_2 \in \Re$

$f)\ x(t) = C_1\cos 3t + C_2 sen3t + e^{3t}\left(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{4}{81}\right),\ \ C_1, C_2 \in \Re$

$g)\ x(t) = e^{3t}\left(C_1 + C_2t - \frac{2}{9}\cos 3t\right),\ \ C_1, C_2 \in \Re$

$h)\ x(t) = K_1e^{3t^2} + K_2e^{2t} + e^{t}\left(\frac{1}{2}t^2 + \frac{3}{2}t + \frac{1}{4}\right),\ \ K_1, K_2 \in \Re$

$i)\ x(t) = K_1\cos t + K_2 sent + \cos t.\log|\cos t| + tsent,\ \ K_1, K_2 \in \Re$

5.

$x(t) = C_1e^{-2t^2} + C_2te^{-2t} + C_3\cos t + C_4 sent$

$a)\ x(0) = C_3\quad x'(0) = C_4\quad x''(0) = -C_3\quad x'''(0) = -C_4$

$b)\ x(0) = C_1\quad x'(0) = -2C_1 + C_2\quad x''(0) = 4(C_1 - C_2)\quad x'''(0) = -8C_1 + 12C_2$

$c)\ impossível$

$d)\ x(0) = C_1 + C_3\quad x'(0) = -2C_1 + C_2 + C_4\quad x''(0) = 4C_1 - 4C_2 - C_3\quad x'''(0) = -8C_1 + 12C_2 - C_4$

6. $x(t) = C_1\cos t + C_2 sent,\ \ C_1, C_2 \in \Re$

7
$b)\ w(t) = At^{-3} + B\quad A \neq 0,\ \ B \in \Re$
$c)\ x(t) = -4t^{-1} + t^3/4\ \ \forall_{t>0}$

8. $x(t) = C_1 + C_2t^2 + \frac{t^3}{3},\ \ C_1, C_2 \in \Re$

9
$a)\ y'(t) = K(y(t) - M(t))$
$b)\ 60m$

Capítulo IV

2. $e^{At} = I + A\left(e^{t} - 1\right)$

5b) $e^{At} = \begin{bmatrix} e^{t} & e^{2t} - e^{t} \\ 0 & e^{2t} \end{bmatrix}$

6c) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \cos 2t + 5sen2t \\ 2\cos 2t - 3sen2t \end{bmatrix}$

7. $a_0 = -4 \quad a_1 = 0$

8. $y(t) = C_1 + C_2 e^{t} sent + C_3 e^{t} \cos t, \quad C_1, C_2, C_3 \in \Re$

9. $e^{At} = \begin{bmatrix} e^{t} & 0 & 0 \\ 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \end{bmatrix}$

10. $z(t) = \phi_1(t)\phi_1^{-1}(0)\phi_2(t)\phi_2^{-1}(0)z_0$

14. $b \in C^{0}\left([0,+\infty)\right) \wedge \int_{0}^{+\infty} |b(s)| ds < +\infty$

15b) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^{t} \\ 2e^{t} - \frac{9}{4}e^{-4t} + \frac{1}{4} \end{bmatrix}$

16. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{3}e^{t} + \frac{1}{3}e^{4t} \\ \frac{5}{4}e^{2t} - \frac{1}{4}e^{-2t} \end{bmatrix}$