STATISTICS I - 2nd Year Management Science BSc - 1st semester – **15/01/2014**

Época Normal – Theoretical Part (40 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (8 marks). This answer sheet will be collected 40 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____**nº**_______

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 5 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cros**s (X)**

1. Let A_i $(i = 1,2,3)$, B and C events of a sample space S with positive probability.

	T	F
If A_1, A_2 and A_3, are mutually independent events with positive probability, then they are a partition of the sample space S.		
Let B and C be mutually exclusive events, then $P(B - C) = P(B)$.		
Let B and C be independent events then, $P(\overline{B \cup C}) = P(\overline{B}).P(\overline{C})$		
If event $A \cup B$ occured then we know that event B occured as well.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
Let X be a continuous random variable, then $\forall\, x \in \mathbb{R}, F(x + 0) - F(x) = 0$.		
Let $Y = \varphi(X)$ be a function of X. If X is a continuous random variable, then Y can only be a continuous random variable.		
If $F_X(x)$ is differentiable at x, then we have that $F'_X(x) \geq 0$		
Let X be a mixed random variable and **b** a discontinuity point of $F(x)$, then $P(X \leq b) > P(X < b)$		

3. Let (X, Y)be a two-dimensional random variable.

	T	F
Let (X, Y) be a continuous random vector, and $Cov(X, Y) \neq 0$, then $E[X\|Y] \neq E(X)$.		
If X and Y are independent random variables then $M_{X+Y}(t) = M_X(t) \times M_Y(t)$		
Let (X, Y) be a discrete random variable with joint probability function $f_{(X,Y)}(x, y)$, then $P(Y \leq a) = \sum_{y \leq a} \sum_{x \in D_X} f_{(X,Y)}(x, y) \quad a \in D_Y$.		
Let X be a random variable with $E(X) = \mu$; $Var(X) = \sigma^2$ and $Y = (3X - 2)/\sigma$, then $Var(Y) = 9$.		

Turn please →

4. Let X be a random variable:

	T	F
If the number of events that occur in a minute follows a Poisson distribution with mean λ, then the time, in minutes, between consecutive events follows a Gamma with mean $1/\lambda$.		
If $X \sim N(1, \sigma^2) \Rightarrow Y = 1 - X \sim N(0, \sigma^2)$.		
If $X \sim U(0,1)$, then $P(X > 3) = 1$		
If $X \sim B(n, \theta) \Rightarrow Y = n - X \sim B(n - x, \theta)$.		

5. Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters.

	T	F
$\left(X_{(n)} - X_{(1)}\right)/\sigma$ is a statistic		
The larger the sample size, *n,* the smaller the $Var[\bar{X}]$		
If $T \sim t_{(n)}$, then $P(T < a) < 0.5 \quad (a < 0)$		
$P(X_1 > x, X_2 > x) = [P(X > x)]^2$ for any value of x		

Each of the following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. Let $A, B, C \in \Omega$ with positive probabilities be mutually independent events.
Prove that $P(A - B|C) = P(A)P(\bar{B})$. [Cotação: 15].

7. Let $X \sim Po(\lambda)$, **prove that** $E(X) = \frac{P(X=1)}{P(X=0)}$ and $Var(X) = 2.\frac{P(X=2)}{P(X=1)}$. [Cotação: 15]

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Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cros**s (X)**

1. Let A_i $(i = 1,2,3)$, B and C events of a sample space S with positive probability.

	T	F
If A_1, A_2 and A_3, are mutually exclusive events with positive probability, then they are a partition of the sample space S.		
Let B and C be mutually exclusive events, then $P(B - C) < P(B)$		
Let B and C be independent events then, $P(\overline{B \cup C}) \neq P(\overline{B}).P(\overline{C})$		
If event $A \cup B$ occured then we know that event B occured as well.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
Let X be a discrete random variable, then $\forall\, x \in \mathbb{R},\ F(x + 0) - F(x) = 0$.		
Let $Y = \varphi(X)$ be a function of X. If X is a mixed random variable, then Y can be discrete random variable.		
If $F_X(x)$ is differentiable at x, then we have that $F_X'(x) \geq 0$		
Let X be a mixed random variable and **b** a point where $F(x)$,is continuous, then $P(X \leq b) > P(X < b)$		

3. Let (X, Y) be a two-dimensional random variable.

	T	F
Let (X, Y) be a discrete random vector and and $Cov(X, Y) \neq 0$, then $E[X\|y_i]$ varies with y_i.		
If X and Y are independent random variables then $M_{X+Y}(t) = M_X(t) + M_Y(t)$		
Let (X, Y) be a continuous random variable with joint probability density function $f_{(X,Y)}(x, y)$, then $P(X \leq a) = \int_{-\infty}^{a}\int_{-\infty}^{+\infty} f_{(X,Y)}(x, y)dydx \quad a \in D_X$		
Let X be a random variable with $E(X) = \mu$; $Var(X) = \sigma^2$ and $Y = (3X - 2)/\sigma$, then $Var(Y) \neq 9$.		

Turn please →

4. Let X be a random variable:

	T	F
If the number of events that occur in a minute follows a Poisson distribution with mean λ, then the mean time, in minutes, between events follows a Gamma with mean λ.		
If $X \sim N(\mu, \sigma^2) \Rightarrow Y = -X \sim N(-\mu, \sigma^2)$.		
If $X \sim U(0,1)$, then $P(X < 3) = 1$		
If $X \sim B(n, \theta) \Rightarrow Y = n - X \sim B(n, \theta)$.		

5. Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with finite mean.

	T	F
If $T \sim t_{(n)}$, then $P(T < a) < 0.5 \quad (a > 0)$		
When $n \to \infty$, $Var[\bar{X}] \to 0$.		
$(X_1, X_2, \cdots, X_n)$ is a statistic.		
$P(X_1 > x, X_2 > x) = P(X > x)$ for any value of x		

Each of the following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. Let *A*, *B*,*C* ⊂ Ω with positive probabilities be mutually independent.

Prove that $P(A - B|C) = P(A)P(\bar{B})$. [Cotação: 15]

7. Let $X \sim Po(\lambda)$, **prove that** $E(X) = \frac{P(X=1)}{P(X=0)}$ and $Var(X) = 2.\frac{P(X=2)}{P(X=1)}$. [Cotação: 15]

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E N – Practical Part (80 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the provided space.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:________

Espaço reservado para a classificação

1 a) (10) b) (20)	**2** a)(20) b)(20)	**3** a)(10) b)(20)	**4** a)(10) b)(10)	**P-**
________	________	________	________	________

1. NANIOTEC is a firm that deals with technological development. It develops nanotecnologic and biotecnologic products. It is known that 30% of its employees are nanotecnology experts. It is also known that if an employee is a nanotecnology expert the probability that she\he is also a biotechnolgy expert is 40% and if she\he is not a nanotecnology expert the probability that she\he is a biotechnolgy expert is 80%.

a) In a random sample of 16 employees compute the probability that at least half of them being nanotecnology experts.

0,9513 ☐ 0,0257 ☐ 0,8990 ☐ 0,0744 ☐

b) If we know that an employee is a biotecnology expert find the probability that she\he is a nanotecnology expert too?

2. Let (X, Y) be a two dimensional random variable with joint probability density function given by

$$f(x, y) = kx^2y \ \ (0 < x < 1; \ \ 0 < y < 1)$$

a) Prove that $k = 6$ and compute the probability that $P(X < 0.5)$

b) Find the expected value for Y when $X = 0.6$. What can you say about the independence between X and Y?

3. The duration (in minutes) of advertisements in a television chanel is a random variable following an uniform distribution in the interval (1,1.5). In a day 36 of these news services are randomly selected.

a) Compute the probability that one of the news services lasts more than 1.2 minutes?

0,4 ☐ 0,6 ☐ 0,2 ☐ 0,8 ☐

b) Find the probability that the mean duration of the advertisements, in the sample, be less than 1.2 minutes?

4. The number of planes that land in a certain airport follows a Poisson process with a mean rate of 6 per hour.

a) Find the probability that in a 10 minutes interval less than 2 planes land at that airport.

0.9197 ☐ 0.1839 ☐ 0.3679 ☐ 0.7358 ☐

b) Compute the probability that the time between two consecutive landings do not excede 5 minutes?

0.3935 ☐ 1.0000 ☐ 0.6065 ☐ 0.0000 ☐