STATISTICS I - 2nd Year Management Science BSc - 1st semester – **28/01/2014**

Época Recurso – Theoretical Part (40 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (8 marks). This answer sheet will be collected 40 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____**Nº**________

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A and B be events of a sample space S with positive probability.

	T	F
$P[(A \cap B) \cup (A \cap B')] = P(A)$		
If A e B are mutually exclusive events then P(A|B)=0		
If $A \subset B$, then $P(A \cup B) = P(A)$		
If the class of events A_1, A_2 and A_3 with positive probabilities is said to be a sample space partition, then A_1, A_2 and A_3 are independent events.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	V	F
Let a and c $(a < c)$ be real numbers. If X is a continuous random variable then $P(a < X < c) \neq F_X(c) - F_X(a)$		
If $\varphi(X)$ is a real function of real variable and X is continuous, $Y = \varphi(X)$ can only be a continuous random variable		
If X is a discrete random variable, then $0 \leq f_X(x) \leq 1$.		
For $h > 0$ $F(x+h) \geq F(x)$ $\forall x, h \in \mathbb{R}$,		

3. Let (X, Y)be a two-dimensional random variable.

	V	F
Let (X, Y) be a two dimensional discrete random variable, then $F_Y(y) = \sum_{y_i \leq y} \sum_{x \in D_X} f_{X,Y}(x, y)$ $(y \in \mathbb{R})$ is the marginal distribution function of random variable Y.		
If X, Y are independent random variables, then $Var(X - Y) = Var(X) - Var(Y)$		
If X is a discrete random variable the expected value of X exists always.		
If $Cov[X, Y] \neq 0$ then X and Y are independent random variables		

Turn please →

4. Let X be a random variable:

	V	F
Let X be the random variable associated to a Bernoulli trial with probability of success θ, then the number of failures in n independent trials follows a Binomial with mean $n\theta$.		
If $X \sim N(\mu, \sigma^2)$, then for any $a > 0 \in \mathbb{R}$ the $P(X \leq \mu + a) < 0.5$		
If X is the daily number of sales in an automobile stand, then X can be well represented by a Binomial distribution..		
If the random variable X follows a uniform distribution in the interval $(a, 2a)$ $a \in \mathbb{R}$, then its median $\mu_e = 3\,a/2$.		

5. Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters.

	V	F
If $\bar{X}_1$ and $\bar{X}_2$ are sample means of two diferente samples selected from population X then $\bar{x}_1 = \bar{x}_2$.		
The larger the sample size, *n,* the smaller is the variance of the sample mean (assuming that the variance exists)		
$T = (X_1 - \mu)(X_2 - \mu)$ is a statistic		
$F_{X_1, X_2, \cdots, X_n}(x_1, x_2, \cdots, x_n) = nF_X(x)$		

Answer the following questions in the space provided for this purpose below. Justify carefully all the steps. Quotation for each of the questions: 15 marks

6. Let A and B be two independent events of a sample space S. Using the Komolgorov probability postulates prove that A' and B' are independent events too.

7. Using the definition of moment generation function of the sum of random variables and its properties prove that if X and Y are independent random variables then $E(X+Y) = E(X) + E(Y)$.

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Época Recurso – Theoretical Part (40 minutes)

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Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A and B be events of a sample space S with positive probability.

	T	**F**
$P[(A \cap B) \cup (A' \cap B)] = P(B)$		
If A and B are independent events, $P(A\|B) = P(B)$		
If $A \subset B$, then $P(A \cup B) = P(B)$		
If the class of events A_1, A_2 and A_3 with positive probabilities is said to be a sample space partition, then A_1, A_2 and A_3 are mutually exclusive events.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	V	**F**
Let a and c $(a < c)$ be integers, if X is discrete, then $P(a < X < c) = F_X(c) - F_X(a)$		
If $\varphi(X)$ is a real function of real variable and X is discrete, $Y = \varphi(X)$ can only be a discrete random variable		
If X is a continuous random variable, and $F_X(x)$ differentiable in $\mathbb{R}$, then $F'_X(x)$ is a non-negative function		
If X is a continuous random variable and $h > 0$, then $\forall x, h \in \mathbb{R}$, $F(x+h) = F(x)$		

3. Let (X, Y)be a two-dimensional random variable.

	V	**F**
If $Cov[X,Y] = 0$ then X and Y are independent random variables		
If X, Y are dependent random variables, then $Var(X - Y) > Var(X) + Var(Y)$		
Let (X,Y) be a continuous random variable with joint probability density function $f_{X,Y}(x,y)$, then $F_X(x) = \int_{-\infty}^{x}\int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dydx$ is the marginal distribution function of random variable X.		
If X is a continuous random variable the expected value of X always exists.		

Turn please →

4. Let X be a random variable:

	V	F
If X is the number of unemployed who come, every day, to a Social Security department, then X can be well represented by a Poisson distribution.		
If the random variable X follows a uniform distribution in the interval $(0, 2a)$ $a \in \mathbb{R}$, then its median $\mu_e = a/2$.		
If $X \sim N(\mu, \sigma^2)$, then for any $a > 0 \in \mathbb{R}$ the $P(X \leq \mu + a) > 0.5$		
Let X be the random variable associated to a Bernoulli trial with probability of success θ, then the number of failures in n independent trials follows a Binomial with mean $n.(1-\theta)$.		

5. Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters.

	V	F
If $\bar{X}_1$ and $\bar{X}_2$ are sample means of two diferente samples from population X then, in general, $\bar{x}_1 \neq \bar{x}_2$.		
The larger the sample size, *n,* the smaller is the expected value of the sample variance (assuming that the variance exists)		
$T = (X_1, X_2)$ is a statistic		
$F_{X_1, X_2, \cdots, X_n}(x_1, x_2, \cdots, x_n) = [F_X(x)]^n$		

Answer the following questions in the space provided for this purpose below. Justify carefully all the steps. Quotation for each of the questions: 15 marks

6. Let A and B be two independent events of a sample space S. Using the Komolgorov probability postulates prove that A' and B' are independent events too.

7. Using the definition of moment generation function of the sum of random variables and its properties prove that if X and Y are independent random variables then $E(X+Y) = E(X) + E(Y)$.

STATISTICS I - 2nd Year Management Science BSc - 1st semester – **28/01/2013**

E R – Practical Part (80 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:__________

Espaço reservado para a classificação

1. a) (20) b) (10) ______	**2.** a) (20) b) (15) ______	**3** a) (10) b) (20) ______	**4** a) (10) b) (15) ______	**T:**

1. To select a candidate for a job, a company uses a written examination followed by a possible interview. To be selected a candidate must be aproved in the interview. It is known that:

- The probability of having less than 50% in the written test is 0.4
- The probability of having between 50% and 90% in the written test was 0.35
- The likelihood of getting approved in the interview for those who have more than 90% in the written test is 0.6
- The likelihood of getting approved in the interview for those between 50% and 90% in the written test is 0.4
- An applicant with less than 50% in the written test will not be accepted for an interview.

a) Knowing that the candidate has been aproved in the interview, what is the probability that he has had between 50% and 90% in the written test ?

b) If ten candidates apply for the job compute the probability that at least half of them had less than 50% in the written test.

(i) 0,7492 ☐ (ii) 0,1662 ☐ (iii) 0,3669 ☐ (iv) 0,7993 ☐

2. A study links the following random variables:

X - Tobacco consumption during pregnancy ($X = 1$, if mother smoked during pregnancy, $X = 0$ otherwise);
Y - Weight of the newborn ($Y = 1$ if weight is less than 3 kg, $Y = 2$ if weight is between 3 kg and 3.5 kg, $Y = 3$ if weight is greater than 3.5 kg).

It is known that a non-smoking mother has a probability of 0.15 that her child has a weight less than 3 kg, and that if a newborn weights more than 3.5 kgs the probability that the mother didn't smoke during pregnancy was 0.8.

The joint probability function is given by:

$X \setminus Y$	1	2	3	
0	0.09	a	0.2	
1	b	0.1	c	

a) Fill the empty cells in the table above and determine the marginal probability functions of the random variables X e Y.

b) Compute the $E(Y|X)$. What can you say about the independence of X and Y?

3. The number of goals scored by the National Football Champion follows a Poisson process with mean 2.7 goals per game (it is considered that a game has precisely 90 minutes) ..

a) Compute the probability that the team score at least 10 goals in the first 3 games.

0,8745 ☐ 0,2959 ☐ 0,1942 ☐ 0,8983 ☐

b) Find the probability that the total time needed for this team to score 35 goals does not exceed 900 minutes.

4. Assume that the duration, in minutes, of a pop song is well modelled by a Normal distribution with mean 3.5 and standard deviation 0.5.

a) What percentage of pop songs have a duration greater than 2.25 minutes?

(i) 96,49% ☐ (ii) 97,73% ☐ (iii) 99,38% ☐ (iv) 84,13% ☐

b) A sample of 3 pop songs was selected. Compute the probability that the longer one lasted less than 4 minutes.

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Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:__________

Espaço reservado para a classificação

1.	**c) (20)** **d) (10)**	**2.**	**a) (20)** **b) (15)**	**3**	**a) (10)** **b) (20)**	**4**	**a) (10)** **b) (15)**	**T:**
	________		________		________		________	

1. To select a candidate for a job, a company uses a written examination followed by a possible interview. To be selected a candidate must be aproved in the interview. It is known that:

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- The likelihood of getting approved in the interview for those who have more than 90% in the written test is 0.6
- The likelihood of getting approved in the interview for those between 50% and 90% in the written test is 0.4
- An applicant with less than 50% in the written test will not be accepted for an interview.

a) Knowing that the candidate has been aproved in the interview, what is the probability that he has had between 50% and 90% in the written test ?

b) If ten candidates apply for the job compute the probability that at least half of them had between 50% and 90% in the written test.

(i) 0,2485 ☐ (ii) 0,7623 ☐ (iii) 0,0949 ☐ (iv) 0,8464 ☐

2. A study links the following random variables:

X - Tobacco consumption during pregnancy ($X = 1$, if mother smoked during pregnancy, $X = 0$ otherwise);
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The joint probability function is given by:

$X \setminus Y$	**1**	**2**	**3**	
0	0.09	a	0.2	
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a) Fill the empty cells in the table above and determine the marginal probability functions of the random variables X e Y.

b) Compute the $E(Y|X)$. What can you say about the independence of X and Y?

3. The number of goals scored by the National Football Champion follows a Poisson process with mean 2.7 goals per game (it is considered that a game has precisely 90 minutes) ..

a) Compute the probability that the team score less than 10 goals in the first 3 games.

0,8058 ☐ 0,1017 ☐ 0,1256 ☐ 0,7041 ☐

b) Find the probability that the total time needed for this team to score 35 goals does not exceed 900 minutes.

c) Assume that the duration, in minutes, of a pop song is well modelled by a Normal distribution with mean 3.5 and standard deviation 0.5.

c) What percentage of pop songs have a duration greater than 3 minutes?

(i) 96,49% ☐ (ii) 97,73% ☐ (iii) 99,38% ☐ (iv) 84,13% ☐

d) A sample of 3 pop songs was selected. Compute the probability that the longer one lasted less than 4 minutes.