

EXERCISES FOR CHAPTER FOUR

1. A soft drinks manufacturer decided to launch an advertising campaign, offering prizes printed on the bottle caps. During the campaign, 5% of the bottles that were distributed have a prize. If you purchase 15 bottles, what is the probability of receiving at least one prize?
2. From the daily output of a machine are randomly taken out 10 pieces, for control purposes. From past experience it is concluded that 80% of the parts can be considered good. Calculate the probability that in 10 parts no more than 8 are good.
3. It is known from previous experience that the probability that a machine needs to be adjusted in each work period of 30 minutes is 0.05 (periods independent). Determine
 - a. The mean and variance of the number of tunings in a week in which the machine runs 20 hours.
 - b. The probability that, within 8 hours of labor, at least one tuning has been done. Two to five tunings have been done.
4. The production of screws in a certain plant is provided by two independently operating machines M_1 and M_2 . From past experience, it is known that the proportion of defective screws on each of the machines is 5%. Given the capability of each machine, and for purposes of quality control, a sample of four screws from M_1 and of eight screws from M_2 are daily randomly collected.
 - a. Calculate the probability of finding two defective screws in the set of two samples.
 - b. Screws are sold in packs of 20, and the manufacturer ensures that 90% are of good quality. Calculate the probability that this guarantee is violated.
5. In a batch of 500 pieces there are 50 defective. For the purposes of acceptance of the batch, a sample is randomly picked up. The batch is rejected if the sample has more than 2 defective pieces.
 - a. If the sample size is 10 what is the probability that the batch is rejected.
 - b. Determine the maximum dimension of the sample so that the batch is rejected with a probability lower than 0.05.

6. An electronics store decided to reject a delivery of electronic games if in a random sample of 20 it finds any defective game.
 - a. What is the probability of accepting the delivery if 10% of games are defective?
 - b. If the delivery is accepted if it is found at the most one broken game, what is the probability of rejecting the provision, in the case of a).

7. A supplier of locks for armored doors sends the pieces in batches of 100 to 50 independent factories of armored doors. Due to manufacturing flaws a series of defective locks was produced. To avoid damage the vendor decided to include in each batch five faulty locks, in the hope that they pass unnoticed by the quality control of their customers. This control consists of randomly picking two locks on each lot. The lot is returned if any of the two locks has some defect.
 - a. Calculate the probability of a lot being accepted by the factory that received it.
 - b. What is the probability that in the quality control of a batch at least one lock has no defect?

8. Let X be a random variable that follows a Poisson distribution with mean equal to 5. Compute the following probabilities:
 - a. $P(X = 5)$; b. $P(X \geq 1)$; c. $P(1 \leq X \leq 3)$; d. $P(4 < X < 8)$

9. Let X be a random variable that follows a Poisson distribution with parameter λ .
 - a. For a fixed value the value k find the value of λ that maximizes $P(X = k)$, $k = 1, 2, \dots$
 - b. λ being integer, show that $P(X = \lambda - 1) = P(X = \lambda)$.
 - c. Find the value k that maximizes $P(X = k)$.
Sugestion: Use the ratio $P(X = k)/P(X = k - 1)$.

10. A factory produces tiles that are packed in boxes of 20 units. It is known that the number of defects per tile follows a Poisson process with mean rate equal to 0.1.
- What is the percentage of defective tiles?
 - Calculate the probability that in a box there are more than two defective tiles.
 - If a supply of tiles is accepted on the condition that the total number of defects found on a sample of 100 tiles does not exceed 15, what is the probability that the supply is rejected?
11. The number of spelling errors that a student gives in a page of a written test of Statistics, follows a Poisson process with an average rate of 1.5 errors.
- What percentage of two pages tests have no spelling errors?
 - If a student wrote four pages, what is the probability of committing more than 8 errors?
 - If five tests of four pages each are randomly selected, what is the probability that only one of them has no spelling errors.
 - In a test with six pages where 10 errors were found, what is the probability that half of the errors are in the first two pages
12. The number of students entering or leaving the room after the first 10 minutes of a 90 minute class follows a Poisson process with an average rate of 1 per every ten minutes. The disturbance is evident and it is considered intolerable if it exceeds the 10 movements per lesson (80 minutes working).
- What is the probability that no movement (inbound or outbound) is registered in the last half hour of class?
 - If the normal situation would be a maximum of three movements per class, find the percentage of classes under these conditions?
 - In a semester the program will hardly be fulfilled if the number of intolerable classes exceeds a quarter of the programmed ones. If 20 lessons are programmed, calculate the probability that the program is fulfilled.

13. The number of people arriving in a certain public service department follows a Poisson process with an average rate of 15 per day. This service operates from 10 to 16 hours and serves up to 25 people per day.
- What is the probability that less than 5 people arrive between 10 and 12 hours?
 - What is the probability that in a day the first person arrives after 12 hours?
 - What proportion of days have more than 25 people arriving to this public service department ?
14. The number of eggs laid per second on certain poultry house follows a Poisson process with an average rate of 1 per second.
- Determine the probability that the number of eggs laid per second is greater than twice the variance.
 - What is the probability that in 5 seconds three eggs have been laid?
 - If within 10 seconds 12 eggs were laid, what is the probability that 10 eggs have been laid in the first 8 seconds?
15. Admit that the occurrence of fouls committed by a team in a football game follows a Poisson process with an average rate of 18 per game. Assume that the game is 90 minutes divided in two parts of 45 minutes each. Do not consider in this exercise the discounts offered by the referee at the end of each part.
- What is the probability that the team commits less than 6 fouls in the first quarter of an hour?
 - Knowing that the team committed four fouls in the first quarter of an hour, what is the probability of committing less than 10 in the first half?
 - In a year in which the team will carry 20 games, what is the probability that the limit of 21 fouls per game is only exceeded in no more than 4 games?

16. An integrated Computer circuit has 1000 transistors. The probability of a transistor be defective is 0.0012.
- Compute the probability of an integrated Computer circuit having at most two defective transistors?
 - On 20 integrated circuits, chosen at random, find the probability that half of them are not defective?
17. Medical statistics revealed that a certain disease, whose treatment is extremely costly, affects one in five thousand people. An insurer, after studying the matter, decided to create an insurance to cover the cost of treatment. In a given year the insurance company has a portfolio of 3000 such policies.
- Determine the probability of none of the insured persons contracting the disease.
 - Knowing that in that year a claim to the insurer was already made, calculate the probability that more than three claims will be made until the end of the year.
18. A land with 10 square meters on a side is divided into 100 equal squares (numbered 1 to 100). Four hundred seeds are launched randomly and independently on this land. It is assumed that the probability of a seed to fall into a square is 0.01. Compute the probability that square number 1 runs out of seeds? How likely it is that at least 6 seeds fall in square n°50 ?