STATISTICS I - 2nd Year Management Science BSc - 1st semester – **02/02/2016**

Appeal Season Exam – Theoretical Part V1

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (6.5 marks). This answer sheet will be collected 30 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____Section:_____Number:____

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A and B be events of a sample space S.

	T	F
If A and B are independent events, then $P(A-B) = P(A) \times P(\bar{B})$.		
If A and B are a partition of the sample space, then $P(A) = P(\bar{B})$		
If $A \subset B$ and $P(B) > 0$, then $P(A\|B) \times P(B) = P(A)$		
If event $A \cup B = \Omega$ then the class of events $\{A, B\}$ forms a partition of the sample space		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	V	F
If X is a continuous random variable and ξ_α denotes its quantile of order α, then $F_X(\xi_{0.85}) - F_X(\xi_{0.15}) = 0{,}7$		
Let X be a discrete random variable with probability function $f(x)$. If $F_X(1) = 1$, then $f(x) = 0$ for $x > 1$		
If X is a continuous random variable, then $F_X(x)$ is a strictly increasing function		
If X is a continuous random variable with probability density function $f(x)$, then $0 \leq f(x) \leq 1$ for all x.		

3. Let (X, Y)be a two-dimensional random variable.

	V	F
If X and Y have a common distribution, then $Cov[X, Y] \leq Var(X)$		
$E(X+Y) = E(X) + E(Y) - Cov(X,Y)$		
When (X, Y) is a discrete two-dimensional random variable with joint probability mass function f(x, y), then we can say that for every (x, y), $f(x,y) \leq P(X = x)$		
If X and Y are independent random variables, then X^2 and Y^2 are independent too.		

4.

	V	F
Let X_1 and X_2 be the number of occurrences of a Poisson process with mean rate λ per hour respectively in the interval $\Delta t_1 = (0{,}2]$ and $\Delta t_2 = (1{,}5]$, then $X_1 + X_2 \sim Po\ (\lambda_1\ +\ \lambda_2)$.		
If $(X_1, X_2, \dots, X_n)$ is a random sample of size $n > 2$ of a population X, then $P(X_1 > x) = 1 - P(X_2 \le x)$ for any value of x		
Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X. If $X \sim Po(\lambda)$ and $n = 3$, then the distribution of $\bar{X}$ can be well approximated by a normal distribution.		
If $X \sim N(\mu_X, \sigma_X^2)$ and $a > \mu_X$, then $P(X > a) > 0.5$		

Answer the following questions in the space provided for this purpose below. Justify carefully all the steps. Quotation for each of the questions: 15 marks

5. Let X be a continuous random variable, then

$$\forall x, h \in R, \quad \lim P(x < X \le x + h) = 0 \text{ when } h \to 0^+,\ h > 0.$$

Argue about the truth of the above statement.

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6. Show, using the definition of variance of a random variable, that if $Var(X)$ exists and c is a constant, then $Var(cX) = c^2 Var(X)$.

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Appeal Season Exam – Theoretical Part V2

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (6.5 marks). This answer sheet will be collected 30 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____Section:_____Number:____

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A and B be events of a sample space Ω.

	T	**F**
If A and B are mutually exclusive events, then $P(A) = P(\bar{B})$.		
If the class of events A and B is said to be a sample space partition, then A and B are independent events.		
If $A \subset B$ and $P(B) > 0$, then $P(B\|A) \times P(B) = P(B)$		
If A and B are a partition of the sample space, then $P(A) = P(\bar{B})$.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	V	**F**
If X is a continuous random variable, then $F_X(x)$ is a strictly increasing function		
Let X be a discrete random variable with probability function $f(x)$. If $F_X(2) = 1$, then $f(x) = 0$ for $x > 2$		
If X is a discrete random variable and ξ_α denotes its quantile of order α, then $P(X > \xi_{0.85}) = 0,2$		
If X is a discrete random variable with probability function $f(x)$, then $0 \leq f(x) \leq 1$ for all x.		

3. Let (X, Y)be a two-dimensional random variable.

	V	**F**
If X and Y have a common distribution, then $Cov[X, Y] = Var(X)$		
$E(X - Y) = E(X) + E(Y) - Cov(X, Y)$		
When (X, Y) is a discrete two-dimensional random variable with joint probability function f(x, y), then we can say that for every (x, y), $f(x, y) \leq P(Y = y)$		
If X and Y are independent random variables, then X^2 and $Ln(Y/2)$ are independent too.		

4.

	V	F
Let X_1 and X_2 be the number of occurrences of a Poisson process with mean rate λ per hour respectively in the interval $\Delta t_1 = (0,2]$ and $\Delta t_2 = (3,5]$, then $X_1 + X_2 \sim Po\ (\lambda_1 + \lambda_2)$.		
Let $(X_1, X_2, \dots, X_n)$ be a random sample of size $n > 2$ selected from a population X, then $P(X_1 > x) = 1 - P(X_n \leq x)$ for any value of $x \in \mathbb{R}$		
Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X. If $X \sim b(1, \theta)$ and $n = 3$, then the distribution of $\bar{X}$ can be well approximated by a normal distribution.		
If $X \sim N(\mu_X, \sigma_X^2)$ and $a < \mu_X$, then $P(X > a) > 0.5$		

Answer the following questions in the space provided for this purpose below. Justify carefully all the steps. Quotation for each of the questions: 15 marks

5. Let X be a continuous random variable, then

$$\forall x, h \in R, \quad \lim P(x < X \leq x + h) = 0 \ \text{ when } h \to 0^+,\ h > 0.$$

Argue about the truth of the above statement.

.

6. Show, using the definition of variance of a random variable, that if $Var(X)$ exists and c is a constant, then $Var(cX) = c^2 Var(X)$.

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Appeal Season Exam – Practical Part

(practical part duration – 90 minutes)

This is Part 2: 13.5 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:___________

Don't write here

1a.(10)	**2a.(20)**	**3a.(10)**	**4a.(10)**	**4c.(10)**	**T:**
1b.(20)	**2b.(20)**	**3b.(20)**	**4b.(15)**		**P:**
______	______	______	______	______	______

1. During his daily motorcycle trip to ISEG, John goes through two traffic lights operating independently. The probability of each of the traffic lights being green is equal to 0.4 for the first and to 0.5 for the second. Consider that this student arrives in time for the beginning of the first morning class if and only if at least one of the two traffic lights is green.

a) In three trips to ISEG, compute the probability that John finds the first traffic light green in his first trip but not in the other two.

(i) 0,0518 ☐ (ii) 0,1440 ☐ (iii) 0,4320 ☐ (iv) 0,6480 ☐

b) John has arrived in time for his first morning class. Find the probability that the first traffic light was green.

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2. Let (X,Y) be a two dimensional random variable with joint density function:

$$f(x,y) = 9x^2y^2 \quad 0 < x < 1 \quad 0 < y < 1$$

a) Compute the median of X.

b) Let $W = X - 3$. If a random sample of size 3 is selected from population W, find the probability that the minimum value in the sample is lower than -2.6.

3. In a particular counter of a bank, customers are served at an average rate of 2 every 10 minutes. Customers are served according to a Poisson Process.

a) In the first hour of operation of the counter, what is the probability that less than 11 customers have been served?

(i) 0,4616 ☐ (ii) 0,1048 ☐ (iii) 0,3472 ☐ (iv) 0,1144 ☐

b) If in a specific day there is a queue of 10 persons to be served at this particular counter, how likely it is that the 10^{th} customer have waited more than an hour to be served.

4. Admit that the duration, in minutes, of a pop song is well modelled by a Normal distribution with mean 3.5 and standard deviation 0.5.

a) What is the maximum duration of a pop song with a probability of 90%?

(i) 4.480 ☐ (ii) 4.322 ☐ (iii) 4.663 ☐ (iv) 4.141 ☐

b) A sample of 4 pop songs were randomly selected. Compute the probability that the sample standard deviation more than doubles the population one.

c) Compute the probability that in ten pop songs randomly selected, less than 3 has a duration lower than 2.68 minutes.

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Appeal Season Exam – Practical Part

(practical part duration – 90 minutes)

This is Part 2: 13.5 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:__________

Don't write here

1a.(10)	**2a.(15)**	**3a.(10)**	**4a.(10)**	**4c.(10)**	**T:**
1b.(20)	**2b.(20)**	**3b.(15)**	**4b.(15)**		**P:**
_______	________	________	________	__________	________

1. During his daily motorcycle trip to ISEG, John goes through two traffic lights operating independently. The probability of each of the traffic lights being green is equal to 0.4 for the first and to 0.5 for the second. Consider that this student arrives in time for the beginning of the first morning class if and only if at least one of the two traffic lights is green.

a) In three trips to ISEG, compute the probability that the first traffic light was green in the two first trips but not in the third one.

(i) 0,096 ☐ (ii) 0,1440 ☐ (iii) 0,2880 ☐ (iv) 0,4320 ☐

b) John has arrived in time for his first morning class. Find the probability that the first traffic light was green.

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2. Let (X, Y) be a two dimensional random variable with joint density function:

$$f(x,y) = 9x^2y^2 \quad 0 < x < 1 \quad 0 < y < 1$$

a) Compute the median of X.

b) Let $W = X - 3$. If a random sample of size 3 is selected from population W, find the probability that the maximum value in the sample is lower than -2.6.

3. In a particular counter of a bank, customers are served at an average rate of 2 every 10 minutes. Customers are served according to a Poisson Process.

a) In the first hour of operation of the counter, what is the probability that less than 9 customers have been served?

(i) 0,1550 ☐ (ii) 0,0655 ☐ (iii) 0,2424 ☐ (iv) 0,0874 ☐

b) If in a specific day there is a queue of 10 persons to be served at this particular counter, how likely it is that the 10^{th} customer have waited more than an hour to be served.

4. Admit that the duration, in minutes, of a pop song is well modelled by a Normal distribution with mean 3.5 and standard deviation 0.5.

a) What is the maximum duration of a pop song with a probability of 95%?

(i) 4.480 ☐ (ii) 4.322 ☐ (iii) 4.663 ☐ (iv) 4.141 ☐

b) A sample of 4 pop songs were randomly selected. Compute the probability that the sample standard deviation more than doubles the population one.

c) Compute the probability that in ten pop songs, less than 3 has a duration lower than 2.86 minutes.