STATISTICS I - 2nd Year Management Science BSc - 1st semester – **14/01/2015**

Época Normal – Theoretical Part (40 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (8 marks). This answer sheet will be collected 40 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____nº______

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 5 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by marking the corresponding box with a cros**s (X)**

1. Let A_i $(i = 1,2,3)$, B and C events of a sample space S with positive probability.

	T	F
If $P(A_1) + P(A_2) + P(A_3) = 1$, then A_1, A_2 and A_3 are a partition of the sample space S.		
Se $B \subset A_2$, então $P(B) = P(B \mid A_2) \times P(A_2)$		
$P[(A \cap B) \cup (A \cap B')] = P(A)$		
If $P(B) \neq 0$ and A_1, A_2, A_3 are mutually exclusive events then $P(A_1 \cup A_2 \cup A_3 \vert B) = P(B \vert A_1)P(A_1) + P(B \vert A_2)P(A_2) + P(B \vert A_3)P(A_3)$.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
Let X be a discrete random variable, then $\forall\, x \in \mathbb{N}, F(x+0) - F(x) = 0$.		
Let X be a continuous random variable with p.d.f. $f(x) > 0$ $(x > 0)$ and the random variable $Y = \begin{cases} 0 & X \leq 1 \\ 1 & X > 1 \end{cases}$. Y is a discrete random variable.		
If X is a continuous random variable, its p.d.f. $f(x)$ has range $\Re$ and codomain [0;1]		
Let X be a mixed random variable and **b** a discontinuity point of $F(x)$, then $P(X \leq b) > P(X < b)$		

3. Let (X, Y) be a two-dimensional random variable with cumulative distribution function $F_{X,Y}(x, y)$.

	T	F
If $Cov(X, Y) = 0$, then $E[X \vert Y] = E(X)$.		
If $M_X'(0)$ and $M_X''(0)$ exist, μ_2 may not exist.		
Let (X, Y) be discrete with joint probability function $f_{(X,Y)}(x, y)$, then $P(Y \leq a) = \sum_{y \leq a} \sum_{x \in D_X} f_{(X,Y)}(x, y)$ $a \in D_Y$.		
If $F_{X,Y}(1.5, 2.0) = 1$, then $F_{X,Y}(2.0, 1.5) \leq 1$		

Turn please →

4. Let X be a random variable:

	T	F
Consider a Poisson process with intensity λ in an interval Δt. The time that elapses until the 3rd occurrence has a Gamma distribution with parameters $(3, \lambda/\Delta\mathrm{t})$.		
If $X \sim N(1, \sigma^2) \Rightarrow Y = \left(\frac{X-1}{\sigma}\right)^2 \sim \chi^2_{(1)}$.		
If the random variable X follows a uniform distribution in the interval $(0, 1)$, and $0 < a < b < 1$ then $P(X > b \mid X > a) = (1-b)/(1-a)$.		
Let $Y_1 \sim Bi(n_1, \theta_1)$ and $Y_2 \sim Bi(n_2, \theta_2)$ $\theta_1 \neq \theta_2$ be independent random variables, then the random variable $V = Y_1 + Y_2 \sim Bi(n_1 + n_2, \theta_1 + \theta_2)$		

5. Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters.

	T	F
Let $\bar{X}_1$ and $\bar{X}_2$ be sample means of samples from population X with dimension n_1, n_2 $(n_1 \neq n_2)$ respectively. Then $E(\bar{X}_1) = E(\bar{X}_2)$.		
$Min\{X_i\}$ is a statistic.		
If $T \sim t_{(n)}$, then $P(T < a) < 0.5 \quad (a < 0)$.		
$Max\{X_i\}$ and $Min\{X_i\}$ are independent random variables.		

Each of the following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. Let $A, B, C \in \Omega$ with positive probabilities be mutually independent events.
Prove that $P[A \cap (B \cup C)] = P(A)P(B \cup C)$. [Cotação: 15].

7. Let $(X_1, X_2, \cdots, X_n)$ be a random sample from an exponential population with mean **λ**. Find the distribution of the sample. Justify every step. [Cotação: 15]

STATISTICS I - 2nd Year Management Science BSc - 1st semester – **14/01/2015**

Época Normal – Theoretical Part (40 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (8 marks). This answer sheet will be collected 40 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____**nº**____________

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 5 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by marking the corresponding box with a cros**s (X)**

2. Let A_i $(i = 1,2,3)$, B and C be events of a sample space S with positive probability.

	T	F
If $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3)$, then A_1, A_2 and A_3 are a partition of the sample space S.		
If $B \subset A_1$, then $P(B) = P(B\|A_1).P(A_1)$		
$P[(A \cap B) \cap (A \cap B')] = P(A)$		
If $P(B) \neq 0$ and A_1, A_2, A_3 are mutually exclusive events, then $P(A_1 \cup A_2 \cup A_3\|B) = P(A_1\|B)P(B) + P(A_2\|B)P(B) + P(A_3\|B)P(B)$.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
Let X be a continuous random variable, then $\forall\, x \in \mathbb{R},\ F(x+0) - F(x) = 0$.		
Let X be a continuous random variable with p.d.f. $f(x) > 0$ $(x > 0)$ and the random variable $Y = \begin{cases} 0 & X \leq 1 \\ X & X > 1 \end{cases}$. Y is a continuous random variable.		
If X is a discrete random variable, its p.d.f. $f(x)$ has range $\Re$ and codomain [0;1]		
Let X be a mixed random variable and **b** a point where $F(x)$ is continuous, then $P(X \leq b) > P(X < b)$		

3. Let (X, Y)be a two-dimensional random variable with cumulative distribution function $F_{X,Y}(x, y)$.

	T	F
If $Cov(X, Y) = 0$, then $E[X\|y_i]$ varies with y_i.		
If $M_X^{(4)}(0)$ exists then $M_X''(0)$ exist.		
Let (X, Y) be continuous with joint probability density function $f_{(X,Y)}(x, y)$, then $P(X \leq a) = \int_{-\infty}^{a}\int_{-\infty}^{+\infty} f_{(X,Y)}(x,y)dydx \quad a \epsilon D_X$		
If $F_{X,Y}(2.0, 1.5) = 1$, then $F_{X,Y}(1.5, 2.0) \leq 1$		

Turn please →

4. Let X be a random variable:

	T	F
Consider a Poisson process with intensity λ in an interval Δt. The time that elapses until the 3rd occurrence has a Gamma distribution with parameters $(3, \Delta t/\lambda)$.		
If $X \sim N(\mu, 16) \Rightarrow Y = \left(\frac{X-\mu}{\sigma 4}\right)^2 \sim \chi^2_{(1)}$.		
If the random variable X follows a uniform distribution in the interval $(0,1)$, and $0 < a < b < 1$ then $P(X < a \| X < b) = a/b$.		
Let $X_1 \sim Bi(n_1, \theta_1)$ and $X_2 \sim Bi(n_2, \theta_2)$ $\theta_1 = \theta_2 = \theta$, be independent random variables, then the random variable $V = X_1 + X_2 \sim Bi(n_1 + n_2, 2\theta)$		

5. Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with finite mean.

	T	F
Let $\bar{X}_1$ and $\bar{X}_2$ be sample means of samples from population X with dimension n_1, n_2 $(n_1 \neq n_2)$ respectively. Then $Var(\bar{X}_1) = Var(\bar{X}_2)$.		
If $T \sim t_{(n)}$, then $P(T < a) > 0.5 \quad (a > 0)$		
$Max\{X_i\}$ is a statistic.		
$Max\{X_i\}$ and $Min\{X_i\}$ are dependent random variables		

Each of the following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. Let $A, B, C \in \Omega$ with positive probabilities be mutually independent events.
Prove that $P[A \cap (B \cup C)] = P(A)P(B \cup C)$. [Cotação: 15]

7. Let $(X_1, X_2, \cdots, X_n)$ be a random sample from an exponential population with mean **λ**. Find the distribution of the sample. Justify every step. [Cotação: 15]

STATISTICS I - 2nd Year Management Science BSc - 1st semester – **14/01/2015**

E N – Practical Part (80 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by marking the corresponding square with an **X** . The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:__________

Espaço reservado para a classificação

1.a) (10)	**2 a)(15)**	**3.a) (10)**	**4 a) (10)**	**T:**
1.b) (20)	**2 b)(15)**	**3.b) (20)**	**4 b) (20)**	**P:**
_______	_______	_______	_______	_________

1. The manager of a service station is planning to install new sections of car wash . To determine the number of sections to install, he decided that in the period of most intensive use, the likelihood of a car not being served immediately should not exceed 10 %. It is known that during the period in question, the number of car arrivals to be washed is a random variable with a Poisson distribution with mean 5. It is also known that each wash section can only serve one car in that period.

a) What is the minimum number of sections that should operate during this period? (Mark the correct answer box with a cros**s (X)**)

6 ☐ 7 ☐ 8 ☐ 9 ☐

b) If the period of most intensive use lasts 3 hours and at the beginning of this period there is a row of six cars, determine the probability that it will take more than one hour for the 6th vehicle to be attended.

2. The difference between the scheduled time of arrival of a bus at a certain stop and the effective time it arrives at that stop, in minutes, is a random variable whose probability density is given by:

$$f_X(x) = \begin{cases} k & 0 < x < 1 \\ \dfrac{6}{5x^2} & 1 \le x < 5 \end{cases}$$

a) Find the value of k and determine the cumulative distribution function of the random variable X.

b) Consider a random sample of 9 users of this bus at this stop. Compute the probability that, in the sample, the lowest difference between the scheduled time of arrival and the effective time of arrival, has lasted less than 2 minutes.

3. Let (X, Y) be a two dimensional random variable, where X is the number of sports cars and Y is the number of vans sold at a certain car stand each month, with joint probability function given by:

$X \setminus Y$	0	1	2	3
0	0.05	0.10	0.10	0.05
1	0.10	0.10	0.05	0.03
2	0.10	0.08	0.05	0.03
3	0.05	0.05	0.04	0.02

a) Knowing that one sports car has been sold in December, find the expected value of van sales in the same month. (Mark the correct answer box with a cross **(X)**)

1.035 ☐ 1.50 ☐ 1.66 ☐ 1.242 ☐

b) Compute the probability that, in a certain month, the total number of sports cars and vans sold at the stand is at least three.

4. Based on information collected in previous years, it is known that the probability that an individual resident in a given region will join the annual vaccination campaign against flu is 0.25.

a) From a group of 12 residents determine the probability that more than 7 have joined the annual vaccination campaign against flu.

0.9885 ☐ 0.0027 ☐ 0.9599 ☐ 0.0143 ☐

b) Knowing that this region has a population of 123,000 people, find the minimum number of vaccines that medical services should have to meet the vaccination requirements with a probability of at least 95%.