STATISTICS I - 2nd Year Management Science BSc - 1st semester – **23/01/2015**

Appeal Exam – Theoretical Part (40 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (8 marks). This answer sheet will be collected 40 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____**nº**______

Each of the following 5 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 5 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by marking the corresponding box with a cros**s (X)**

1. Let A_i $(i = 1,2,3)$ be a partition of the sample space S, events $B, C \subset S$ with positive probability.

	T	F
$P(B\|C) = P(C\|B) * P(B)/[P(C\|A_1)P(A_1) + P(C\|A_1')P(A_1')]$		
$A_1 \cup A_2 \cup A_3 = S;\ A_i \cap A_j = \emptyset \quad i \neq j \quad i,j = 1,2,3$		
$P(A_1\|A_3) = 0$		
A_1, A_2, A_3 are independent events		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
Let X be a discrete random variable, then $\forall\, h \in \mathbb{R}. h > 0, F(x + h) - F(x) < 0$.		
Let X be a continuous random variable and a random variable $Y = \psi(X)$. Y can be a discrete random variable.		
If X is a mixed random variable, there is at least one point of discontinuity of $F_X(x)$ with positive probability.		
If $Y = 2X + 1$, then $Var(Y) = 4Var(X) + 1$		

3. Let (X, Y)be a two-dimensional random variable with cumulative distribution function $F_{X,Y}(x, y)$.

	T	F
If $Cov(X, Y) = 0$, then X, Y are independent random variables.		
If X, Y are independent random variables, then $M_{X+Y}(s) = M_X(s) \times M_Y(s)$		
Let (X, Y) be discrete with joint probability function $f_{(X,Y)}(x, y)$, then $P(X \leq x\|Y \leq y) = F_{X,Y}(x, y)/F_Y(y)$		
Let (X, Y) be continuous then $f_{(X)}(x) = \int_{-\infty}^{+\infty} f_{(X,Y)}(x, y)dx$		

Turn please →

4. Let X be a random variable:

	T	F
Consider a Poisson process with intensity λ in an interval Δt. The time that elapses until the 3rd occurrence has a Gamma distribution with parameters $(3, \lambda/\Delta t)$.		
If $X \sim N(\mu, \sigma^2) \Rightarrow P(X > \mu) = 0.5$ for any values of μ, σ^2.		
If the random variable X follows a uniform distribution in the interval $(0, a)$, then its median equals $a/2$.		
Let $X \sim Bi(n, \theta)$, $n - X \sim Bi(n, 1-\theta)$.		

5. Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters.

	T	F
$Cov(X_i, X_j) = 0 \quad i \neq j$.		
$(X_1 + X_n)/\sigma$ is a statistic.		
If $T \sim t_{(n)}$, then $P(T < a) < 0.5 \quad (a < 0)$.		
$P(X_i \leq x) = F_X(x) \quad (i = 1, \cdots, n)$		

Each of the following questions is worth 15 points and should be answered in the space provided. Justify all your steps.

6. Let $A, B \in S$ be events with positive probabilities. Using the postulates of the measure of probability and the property that if $A \subset B \Rightarrow P(A) \leq P(B)$, prove that $P(A) \leq 1$. [15].

7. Prove, using the definition of second moment about the mean that $Var(X) = E(X^2) - \mu_X^2$. Justify every step. [15]

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Appeal Exam – Practical Part (80 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by marking the corresponding square with an **X** . The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____Nº:_________

Espaço reservado para a classificação

1.a) (10)	**2 a)(15)**	**3.a) (10)**	**4 a) (10)**	**T:**
1.b) (20)	**2 b)(15)**	**3.b) (20)**	**4 b) (20)**	**P:**
______	______	______	______	________

1. At a certain region, 60% of all families spend their holidays at the seaside and 42% of these families have at least two children, 32% spend their holidays at the countryside and 18% of these have two or more children. All other families spend their holidays abroad and 5% of them have at least two children.

a) The Silva family has three children. How probable is it that they spend holidays abroad?

b) In a group of 20 of the families in this region, compute the probability that more than half of them don't spend their holidays at the seaside.

0.8403 ☐ 0.2447 ☐ 0.1275 ☐ 0.8829 ☐

2. Let (X, Y) be a two dimensional continuous random variable with joint probability density function given by:

$$f_{X,Y}(x,y) = \frac{3}{8}x^3\sqrt{y} \qquad 0 < x < 2\ ,\ \ 0 < y < 1$$

a) Are X, Y independent random variables?

b) Compute $E\left(X|Y = \frac{1}{2}\right)$. Justify your answer thoroughly.

3. It is known that the duration (in minutes) of advertising programs of a television channel is a random variable with uniform distribution in the interval (10,20). On a day are issued 40 of these advertising programs.

a) Compute the probability that one of these advertising programs lasts more than 12 minutes.

0.8 ☐ 0.2 ☐ 0.1 ☐ 0. 9 ☐

b) If 30 of these programs are randomly selected, determine the probability that the difference, in absolute value, between the sample mean and the population mean is lower than 1 minute.

4. Plains arrive at a certain airport following a Poisson process with mean rate of 6 per hour.

a) What is the probability of a 10-minute period have less than two arrivals?

0.3679 ☐ 0.7358 ☐ 0.9197 ☐ 0. 1839 ☐

b) What is the probability that the time between the third and sixth consecutive arrivals do not exceed 10 minutes?