LISBOA SCHOOL OF ECONOMICS & MANAGEMENT — SINCE 1911 — ISEG

U LISBOA | UNIVERSIDADE DE LISBOA

STATISTICS I - 2nd Year Management Science BSc – 2nd semester – **29/06/2016**

Appeal Season Exam – Theoretical Part V1

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (70 points). This answer sheet will be collected 30 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____Number:______

Each of the following 4 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 4 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A_1, A_2 e A_3 be events of a sample space $\boldsymbol{S}$.

	V	F
If A_1, A_2 and A_3 are such that the pairwise intersection is empty and $P(A_1) + P(A_2) = P(\overline{A_3})$, then A_1, A_2 and A_3 are a partition of sample space $\boldsymbol{S}$.	X	
A_2 and $\mathrm{A}_3$are independent events if and only if $P[\mathrm{A}_2 \cap \mathrm{A}_3] = 0$.		X
If $P(A_1 \cup A_2) = P(A_2)$ and $P(A_1) < P(A_2)$, then $A_1 \subset A_2$.	X	
If $P(\overline{A_1} \vert A_3) = 1$, then $P(A_1 \cap A_3) = 0$	X	

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is a discrete random variable, then $F_X(x) \geq P(X < x)$ for any $x \in \mathbb{R}$	X	
Let $Y = \varphi(X)$ be a function of X. If X is a continuous random variable, then Y can be a discrete random variable.	X	
If X is a continuous random variable the expected value of X does not always exist.	X	
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a \leq X \leq b) = F_X(b) - F_X(a - 0)$.	X	

3. Let (X, Y)be a two-dimensional random variable.

	T	F
If X and Y are independent random variables, then $f_{X\vert Y=y}(x) * f_Y(y) = f_{Y\vert X=x}(y)$.		X
If $\rho_{X,Y} \in (0,1]$, then if X grows, Y grows too.	X	
If $M_X(s) = M_Y(s)$, then $F_X(x) = F_Y(y)$.	X	
If X is a discrete random variable, then $\forall x, h \in \mathbb{R},\ F(x+h) \geq F(x) \Rightarrow h > 0$	X	

4.

	T	F
Let $X \sim Po(\lambda)$ and $Y \sim Po(\lambda)$, be independent random variables, then $Var(2X - Y) = 3\boldsymbol{\lambda}$.		X
If $X \sim N(\mu, 1)$, then $Y = (X-\mu)^2 \sim \chi^2_{(1)}$	X	
If the random variable X follows a uniform distribution in the interval $(a, 2a)$ $a \in \mathbb{R}$, then $P\left(X < a + \frac{a}{4}\right) = 1/2$.		X
If X is the daily number of sales in an automobile stand, then X can be well represented by a Binomial distribution..		X

The following questions worth 150 points each and should be answered in the space provided. The answers should be duly formalized and justified.

5. Let A, B be events of sample space S with positive probability. **Show** that $P(\bar{A}|B) = 1 - P(A|B)$.

$$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B - A)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)} = 1 - P(A|B)$$

6. Let (X, Y) be a discrete two dimensional random variable. **Prove**, **using the definition** that, if X and Y are independent random variables, $E(XY) = E(X).E(Y)$

$$E(XY) = \overbrace{\sum_{x \in D_x} \sum_{y \in D_Y} x.y.f_{X,Y}(x,y) = \sum_{x \in D_x} \sum_{y \in D_Y} x.y.f_X(x).f_{,Y}(y) = }^{X \text{ and } Y \text{ are independent random variables} \Rightarrow f_{X,Y}(x,y) = f_X(x).f_Y(y)\ \forall (x,y) \in D_{X,Y}} \sum_{x \in D_x} x.f_X(x)\,.\sum_{y \in D_Y} y.f_{,Y}(y) =$$

$$= E(X).E(Y)$$

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Appeal Season Exam – Theoretical Part V1

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (70 points). This answer sheet will be collected 30 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____ Number:____

Each of the following 4 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A_1, A_2 e A_3 be events of a sample space $\boldsymbol{S}$.

	T	F
If A_1, A_2 and A_3 are such that the pairwise intersection is empty and $P(A_1) = P(\overline{A_2 \cup A_3})$, then A_1, A_2 and A_3 are a partition of sample space $\boldsymbol{S}$.	X	
If $P[A_2 \cap A_3] = 0$, then A_2 and $A_3$are independente events.		X
If $P(A_1 \cap A_2) = P(A_2)$ and $P(A_2) < P(A_1)$, then $A_2 \subset A_1$	X	
If $P(A_3 \vert \overline{A_1}) = 1$, then $P(A_1 \cap A_3) \neq 0$	X	

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is a discrete random variable, then , $a, c \in D_X,\ a < c \Rightarrow P(a < X < c) = F(c) - F(a)$		X
If X is a continuous random variable, then its probability density function has range and co-domain $\mathbb{R}$.	X	
If X is a discrete random variable the expected value of X should always exist.		X
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a < X < b) = F_X(b) - F_X(a - 0)$.		X

3. Let (X, Y)be a two-dimensional random variable.

	T	F
If X and Y are independent random variables, then $f_{Y\vert X=x}(y) * f_X(x) = f_{X\vert Y=y}(x)$.		X
If $\rho_{X,Y} \in [-1,0)$, then if X grows, Y decreases.	X	
If $F_X(x) = F_Y(y)$ then, $M_X(s) = M_Y(s)$,	X	
If X is a continuous random variable, then $\forall x, h \in \mathbb{R},\ F(x+h) > F(x) \Rightarrow h > 0$	X	

4.

	T	F
Let $X \sim Po(\lambda)$ and $Y \sim Po(\lambda)$, be independent random variables, then $Var(X+2Y) = 5\boldsymbol{\lambda}$.	X	
If $X \sim N(\mu, 1)$, then $Y = X^2/\sigma^2 \sim N(\mu^2, 1)$		X
If the random variable X follows a uniform distribution in the interval $(a, 2a)$ $a \in \mathbb{R}$, then $P\left(X < a + \frac{a}{2}\right) = 1/2$	X	
If X is the daily number of automobiles in a stand that cost more than 40000 euros, then X can be well represented by a Binomial distribution.	X	

The following questions worth 150 points each and should be answered in the space provided. The answers should be duly formalized and justified.

5. Let A, B be events of sample space S with positive probability. **Show** that $P(\bar{A}|B) = 1 - P(A|B)$.

6. Let (X, Y) be a discrete two dimensional random variable. **Prove**, **using the definition** that, if X and Y are independent random variables, $E(XY) = E(X).E(Y)$

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STATISTICS I - 2nd Year Management Science BSc - 2nd semester – **29/06/2016**

Appeal Season Exam – Practical Part - 2134

(practical part duration – 90 minutes)

This is Part 2: 130 points. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____ Nº:______________

Don't write here

1a.(10)	2a.(20)	2c.(15)	3 a. (10)	4. (20)	T:
1b.(20)	2b.(15)		3 b. (20)		P:

1. At **Drinks, Ltd**, the percentage of bottles of Water, Juice and Beer sold are respectively 30%, 20%, and 50%. Accordind to quality control tests, 70% of the Water bottles has a good quality, and from the bottles with good quality, 20% are Juice bottles. It is also known that 70% of the bottles sold by this firm are of good quality.

a) If 10 bottles from the firm production are randomly selected find the probability that more than half of them are Juice bottles (signal with an X the right answer,)

(i) 0.0328 ☐ (ii) 0.9120 ☐ (iii) 0.0064 X (iv) 0.9736 ☐

a) If 10 bottles from the firm production are randomly selected find the probability that more than half of them are Water bottles (signal with an X the right answer,)

(i) 0.0473 X (ii) 0.8971 ☐ (iii) 0.1503 ☐ (iv) 0.7999 ☐

a) If 10 bottles from the firm production are randomly selected find the probability that more than half of them are Beer bottles (signal with an X the right answer,)

(i) 0.7949 ☐ (ii) 0.3770 X (iii) 0.7539 ☐ (iv) 0.6230 ☐

a) If 16 bottles from the firm production are randomly selected find the probability that more than half of them are Water bottles (signal with an X the right answer,)

(i) 0.8254 ☐ (ii) 0.5982 ☐ (iii) 0.0257 X (iv) 0.8036 ☐

b) For quality control of Juice bottles, one Juice bottle is randomly selected. Compute the probability that it is of good quality?

$$P(W) = 0.3;\ P(J) = 0.2;\ P(B) = 0.5;\ P(GQ|W) = 0.7;\ P(J|GQ) = 0.2;\ P(GQ) = 0.7$$

$$P(GQ|J) = \frac{P(GQ \cap J)}{P(J)} = \frac{0.14}{0.2} = 0.7$$

$$P(J|GQ) = \frac{P(J \cap GQ)}{P(GQ)} = \frac{P(J \cap GQ)}{0.7} = 0.2 \Leftrightarrow P(J \cap GQ) = 0.2 * 0.7 = 0.14 = P(GQ \cap J)$$

2. The monthly revenue (in $million$ euros) from the sale of two products X and Y are well modelled by random variables X and Y with joint probability density functions given by:

$$f_{X,Y}(x,y) = \frac{xy}{9} \qquad (0 < x < 3;\ 0 < y < 2)$$

a) Find the percentage of months in which the revenue from the sale of product X is lower than the revenue from the sale of product Y?

$$P(X < Y) = \int_0^2 \int_0^y \frac{xy}{9} dxdy = \frac{1}{9}\int_0^2 y \int_0^y x\, dxdy = \frac{1}{9}\int_0^2 y\, \frac{x^2}{2}\Big]_0^y dy = \frac{1}{9}\int_0^2 y\, \frac{y^2}{2} dy = \frac{1}{9}\int_0^2 \frac{y^3}{2} dy =$$

$$= \frac{1}{9}\frac{y^4}{8}\Big]_0^2 = \frac{1}{9}.2 = \frac{2}{9}$$

b) Are X and Y independent random variables?

$$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\, dy = \int_0^2 \frac{xy}{9}\, dy = \frac{x}{9}\frac{y^2}{2}\Big]_0^2 = \frac{x}{9}.2 = \frac{2x}{9} \quad (0 < x < 3)$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\, dx = \int_0^3 \frac{xy}{9}\, dx = \frac{y}{9}\frac{x^2}{2}\Big]_0^3 = \frac{y}{9}.\frac{9}{2} = \frac{y}{2} \quad (0 < y < 2)$$

$$f_X(x) x f_Y(y) = \frac{2x}{9}.\frac{y}{2} = \frac{xy}{9} = \frac{xy}{9} = f_{X,Y}(x,y) \Rightarrow X \text{ and } Y \text{ are independent random variables}$$

c) Compute the mean revenue of the sale of product Y, in the months when the revenue from the sale of product X is of 2 million euros..

$$E(Y|X=2) = \int_{-\infty}^{+\infty} y.f_{Y|X=2}(y)\, dy = \int_{-\infty}^{+\infty} y.\frac{f_{X,Y}(x,y)}{f_X(x)}\, dy = \int_0^2 y\,.\frac{xy/9}{2x/9}\, dy =$$

$$= \int_0^2 \frac{y^2}{2}\, .dy = \frac{y^3}{6}\Big]_0^2 = \frac{8}{6}$$

3. Visits to a website per minute occur according to a Poisson process with mean 0.5.

a) Determine the probability that the number of visits in 10 minutes is lower than 5.

(i) 0.1755 ☐ (ii) 0.6160 ☐ (iii) 0.5714 ☐ (iv) 0.4405 X

a) Determine the probability that the number of visits in 5 minutes is lower than 4.

(i) 0.2138 ☐ (ii) 0.8912 ☐ (iii) 0.1336 ☐ (iv) 0.7576 X

a) Determine the probability that the number of visits in 15 minutes is lower than 7.

(i) 0.1465 ☐ (ii) 0.3782 X (iii) 0.1367 ☐ (iv) 0.5246 ☐

a) Determine the probability that the number of visits in 5 minutes is lower than 6.

(i) 0.9580 X (ii) 0.0668 ☐ (iii) 0.9858 ☐ (iv) 0.0278 ☐

b) Determine the probability that the time between three consecutive visits is bigger than 7 minutes.

$X-$ nº of visits to a website per minute$\sim Po(0{,}5) \Rightarrow$

Y- time between consecutive visits$\sim Ex(0.5)$

W- time between three consecutive visits$\sim G(2, 0.5)$

$$P(W > 7) = P(2\lambda W > 2\,.\,0.5\,.\,7) = P\left(\chi^2_{(4)} > 7\right) = 0.1359$$

Or

$Y-$ nº of visits to a website in 7 minutes$\sim Po(0{,}5 * 7)$

$$P(Y < 2) = P(Y \leq 1) = F_Y(1) = 0.1359$$

4. The duration of small ads (between 5 and 10 **seconds**) in a private TV channel is well modeled by a uniform distribution. If before the start of a football EURO 2016 game there are 50 of these ads, determine the **approximate** probability that the total duration of advertising last less than six **minutes**?

X- duration, in seconds, of small ads $\sim U(5, 10) \Rightarrow \mu_X = \frac{10+5}{2} = 7.5;$

$$\sigma_X^2 = \frac{(10-5)^2}{12} = \frac{25}{12}$$

$Y = \sum_{i=1}^{50} X_i$- duration, in seconds, of 50 small ads$\dot{\sim} N\left(50 * 7.5, \sqrt{50\,.\frac{25}{12}}\right)$

$$P(Y < 6 * 60) = normalcdf\left(-10000, 360, 50 * 7.5, \sqrt{50 * \frac{25}{12}}\right) = 0{,}0708$$