STATISTICS I - 2nd Year Economics\Management Science BSc – 2nd semester – **05/04/2016**

2nd Mid Term Exam– Theoretical Part (15 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (35 points). During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name: ____**nº**______

Each of the following 2 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 2 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cros**s (X)**

1. Let (X,Y) be a two dimensional random variable.

	T	F
Let (X,Y) be a discrete random variable. X and Y are independent random variables if and only if $P(X=x \mid Y=y) = P(X=x)\ \forall (x,y) \in D_{X,Y}$	X	
If (X,Y) is a discrete random variable, the marginal expected value of X always exist.		X
If $M'_X(0)$ and $M''_X(0)$ exist μ_2 may not exist.		X
If the correlation coeficiente $\rho_{X,Y} \neq 0$, then X and Y are dependent random variables.	X	

2.

	T	F
Let $X_1 \sim Po(\lambda_1)$ and $X_2 \sim Po(\lambda_2)$ $\lambda_1 \neq \lambda_2$, be random variables representing the number of events in a time interval $\Delta t_1 = (0,2]$ and $\Delta t_2 = (3,5]$ respectively. Then the random variable $W = X_1 + X_2$ follows a Poisson distribution with mean $\lambda_1 + \lambda_2$.	X	
Let $Y_1 \sim B(n_1, \theta_1)$ and $Y_2 \sim B(n_2, \theta_2)$ $\theta_1 = \theta_2$ be independent random variables, then the random variable $W = Y_1 + Y_2$ follows a binomial distribution with probability of success $\theta = \theta_1 + \theta_2$.		X
Let $X \sim N(\mu, \sigma^2)$, $P[X < a] = 0.3 \Leftrightarrow a < \mu$	X	
If the random variable $X \sim (a, a+2)$ $a \in \mathbb{R}$, then the 3^{rd} quartile $\xi_{0.75} = 3/2$		X

3. Show, **using the definition** of variance of a random variable, that if $Var(X)$ exists and c is a constant, then $Var(cX) = c^2 Var(X)$ [Cotação: 15]

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Name: ____**nº**______

Each of the following 2 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 2 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cros**s (X)**

4. Let (X,Y) be a two dimensional random variable.

	T	F
Let (X,Y) be a discrete random variable. X and Y are independent random variables if and only if $P(X=x\|Y=y)=P(X=x)$ for some $(x,y)\in D_{X,Y}$		X
If (X,Y) is a continuous random variable, the marginal expected value of Y always exist.		X
If $M_X'(0)$ and $M_X''(0)$ exist μ_2 always exist.	X	
If the correlation coeficiente $\rho_{X,Y}=0$, then X and Y are independent random variables		X

5.

	T	F
Let $X_1 \sim Po(\lambda_1)$ and $X_2 \sim Po(\lambda_2)$ $\lambda_1 \neq \lambda_2$, be random variables representing the number of events in a time interval $\Delta t_1 = (0,3]$ and $\Delta t_2 = (2,5]$ respectively. Then the random variable $W = X_1 + X_2$ follows a Poisson distribution with mean $\lambda_1 + \lambda_2$.		X
Let $Y_1 \sim Bi(n_1,\theta_1)$ and $Y_2 \sim Bi(n_2,\theta_2)$ $\theta_1 \neq \theta_2$, be independent random variables, then the random variable $V = Y_1 + Y_2 \sim Bi(n_1+n_2, \theta_1+\theta_2)$		X
Let $X \sim N(\mu,\sigma^2)$, $P[X>a]=0.3 \Leftrightarrow a>\mu$	X	
If the random variable $X \sim (a, a+2)$ $a \in \mathbb{R}$, then the 1st quartile $\xi_{0.25} = \frac{1}{2}$.		X

6. Show, **using the definition** of variance of a random variable, that if $Var(X)$ exists and c is a constant, then $Var(cX) = c^2 Var(X)$ [Cotação: 15]

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!st Mid Term Exam– Practical Part (45 minutes)

This is Part 2: 6.5 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the provided space.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.
Open questions should be duly justified and formalized.

Name:____Nº:_________

Espaço reservado para a classificação

1 a) (10)	**1 c) (15)**	**2 (15)**	**T:**
1 b) (10)	**1 d) (20)**		**P:**
_________	_________	_________	_________

1. A machine that works continuously has on average three breakdowns per 8-hour shift operation. Breakdown occurrences follow a Poisson process.

a) What is the probability that in the 1st half of the shift there are less than 2 breakdowns.
0.2510 ☐ X 0.8088☐ 0.5578 X 0.3347☐

b) Find the probability that in two of the 3 shifts, in a day, there are just one breakdown.

0.992 ☐ 0.003 0.057 X 0.096 ☐ 0.997 ☐

c) Considering the first shift, compute the probability that the first breakdown occurs after 6 hours knowing that there were no breakdowns in the first 2. Comment your result.

Y- time, in hours, until the 1st breakdown$\sim Ex(3/8)$ (5)

$$\underbrace{P(Y>6|Y>2)}_{2.5}=\underbrace{\frac{exp\left(-\frac{3}{8}*6\right)}{exp\left(-\frac{3}{8}*2\right)}=exp\left(-\frac{3}{8}*6+\frac{3}{8}*2\right)=exp\left(-\frac{3}{8}*4\right)=0.2231}_{2.5}$$

$$=\underbrace{P(Y>4)}_{2.5}$$

o que ilustra a propriedade de falta de memória da distribuição exponencial.(2.5)

d) Determine the probability that in a day, the 6th breakdown occurred before the end of the 2nd shift.
W- time, in hours, until the 6th breakdown $\sim G(6,3/8)$ (5)

$$\underbrace{P(W\geq 16)}_{2.5}=\underbrace{1-P(W<16)}_{2.5}=1-\underbrace{P\left(2*\lambda*W<2*\frac{3}{8}*16\right)}_{2.5}=1-\underbrace{P\left(\chi^2_{(2*6)}<12\right)}_{2.5}$$

$$=1-\underbrace{0.554}_{2.5}=\underbrace{0.446}_{2.5}$$

2. A quality standard requires that the weight in grams of a certain type of grain contained in a package differs from the mean weight, in **absolute value**, by less than ***k*** grams. The weight of cereals contained in a package is well represented by a normal distribution with mean 350 grams and variance 25. Determine the value of ***k*** so that the package meets the quality standard with a probability of 90%.

X- weight of cereals contained in a package$\sim N(350, 25)$

$$\underbrace{a, b: P(a < X < b) = 0.9}_{5} \Leftrightarrow \begin{cases} \underbrace{P(X < a) = 0.05}_{2.5} \Leftrightarrow \underbrace{a = invnorm(0.05, 350{,}5) = 341.776}_{2.5} \\ \underbrace{P(X < b) = 0.95}_{2.5} \Leftrightarrow \underbrace{b = invnorm(0.95, 350{,}5) = 358.224}_{2.5} \end{cases}$$

Ou

$$a, b: P(a < X < b) = 0.9 \Leftrightarrow \underbrace{P\left(\frac{a-350}{5} < \frac{X-\mu}{\sigma} < \frac{b-350}{5}\right) = 0.9}_{2.5}$$

$$\Leftrightarrow \underbrace{P\left(-z_{\varepsilon/2} < Z < z_{\varepsilon/2}\right) = 0.9}_{2.5}$$

A tabela 5 dá para $\varepsilon = 0.1$ o valor $\underbrace{z_{\varepsilon/2} = 1.645}_{2.5}$, pelo que:

a: $\frac{a-350}{5} = -1.645 \Leftrightarrow a = 350 - 1.645 * 5 = 341.775$ (1.25) e

b: $\frac{b-350}{5} = 1.645 \Leftrightarrow a = 350 + 1.645 * 5 = 358.225$ (1.25)

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Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.
Open questions should be duly justified and formalized.

Name:____Nº:_________

Espaço reservado para a classificação

1 a) (10)	**1** c) (10)	**2 (15)**	**T:**
b) (10)	d) (20)		**P:**
________	________	________	________

1. A machine that works continuously has on average three breakdowns per 8-hour shift operation.

a) What is the probability that in the 1st half of the shift there are less than 3 breakdowns.

0.2510 ☐ 0.8088 X 0.1255 ☐ 0.9344 ☐

b) Find the probability that in every shift, in a day, there are just one breakdown.

0.003 X 0.057 ☐ 1 ☐ 0.997 ☐

c) Considering the first shift, compute the probability that the first breakdown occurs after 6 hours knowing that there were no breakdowns in the first 2. Comment your result.

d) Determine the probability that in a day, the 6th breakdown occurred before the end of the 2nd shift.

2. A quality standard requires that the weight in grams of a certain type of grain contained in a package differs from the mean weight, in **absolute value**, by less than 5 grams. The weight of cereals contained in a package is well represented by a normal distribution with mean 350 grams and variance 25. Determine the interval of variation of the weight of cereals contained in a package so that it meets the quality standard with a probability of 90%.