Revision of Fundamental Concepts

Gestão Financeira I
Gestão Financeira
Corporate Financel
Corporate Finance

Licenciatura
2016-2017

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
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Introduction

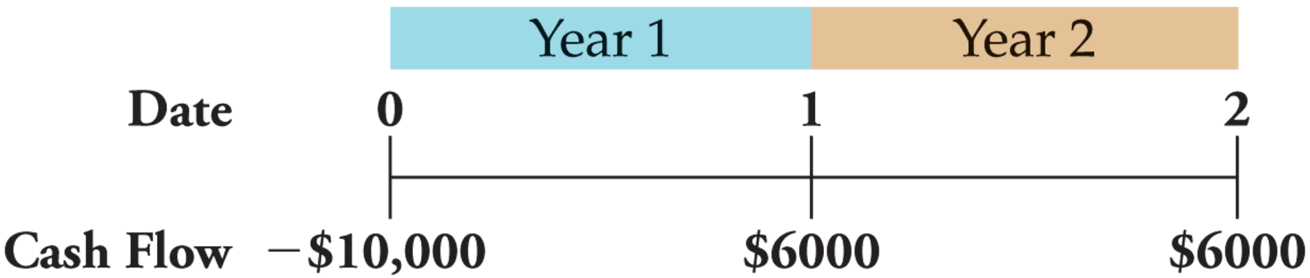
1. Arbitrage and the Law of One Price
2. The Time Value of Money and the Valuation Principle
3. Interest Rates

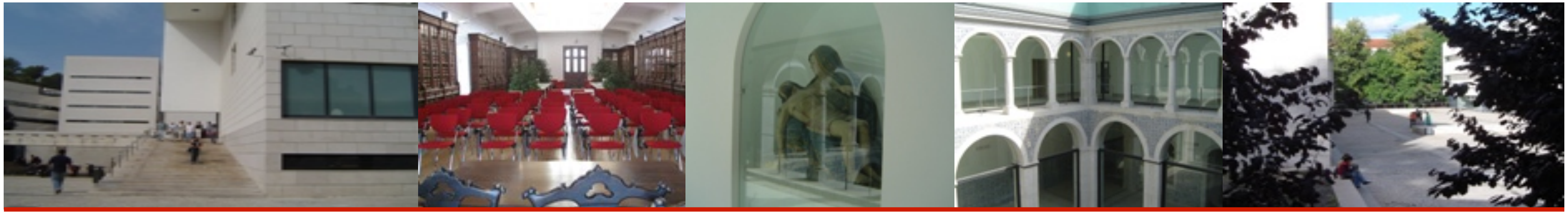
Arbitrage

- Arbitrage
  - The practice of buying and selling equivalent goods in different markets to take advantage of a price difference.
  - An **arbitrage opportunity** occurs when it is possible to make a profit without taking any risk or making any investment.
- Normal Market
  - A competitive market in which there are no arbitrage opportunities.
- Law of One Price
  - If equivalent investment opportunities trade simultaneously in different competitive markets, then they must trade for the same price in both markets.

Time Value of Money

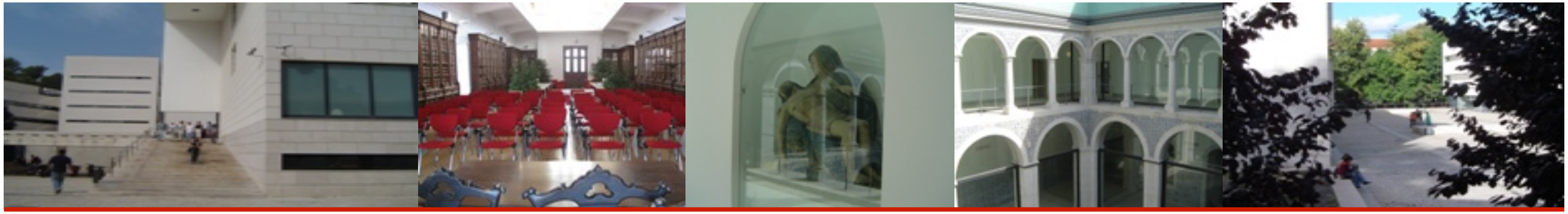
- The Timeline:
  - A timeline is a linear representation of the timing of potential cash flows.
  - Drawing a timeline of the cash flows will help you visualize the financial problem.
  - Example: Assume that you are lending $10,000 today and that the loan will be repaid in two annual $6,000 payments.





- Three Rules of Time Travel:

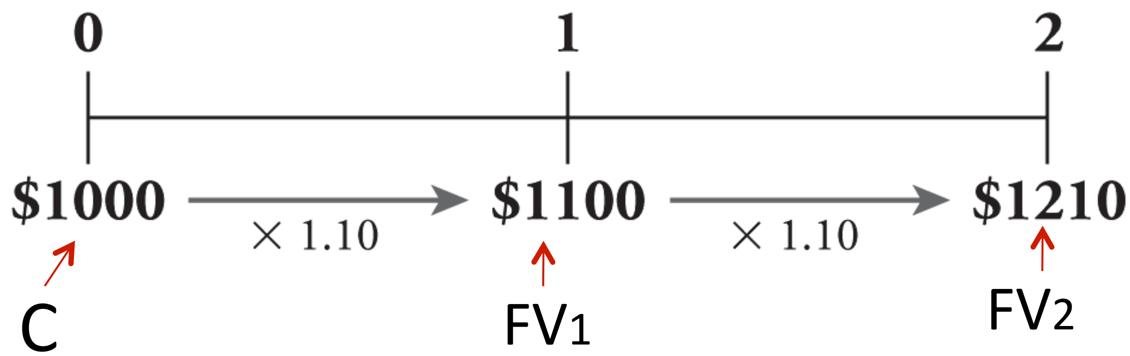
Rule 1	Only values at the same point in time can be compared or combined.	
Rule 2	To move a cash flow forward in time, you must compound it.	Future Value of a Cash Flow $FV_n = C \times (1 + r)^n$
Rule 3	To move a cash flow backward in time, you must discount it.	Present Value of a Cash Flow $PV = C \div (1 + r)^n = \dfrac{C}{(1 + r)^n}$



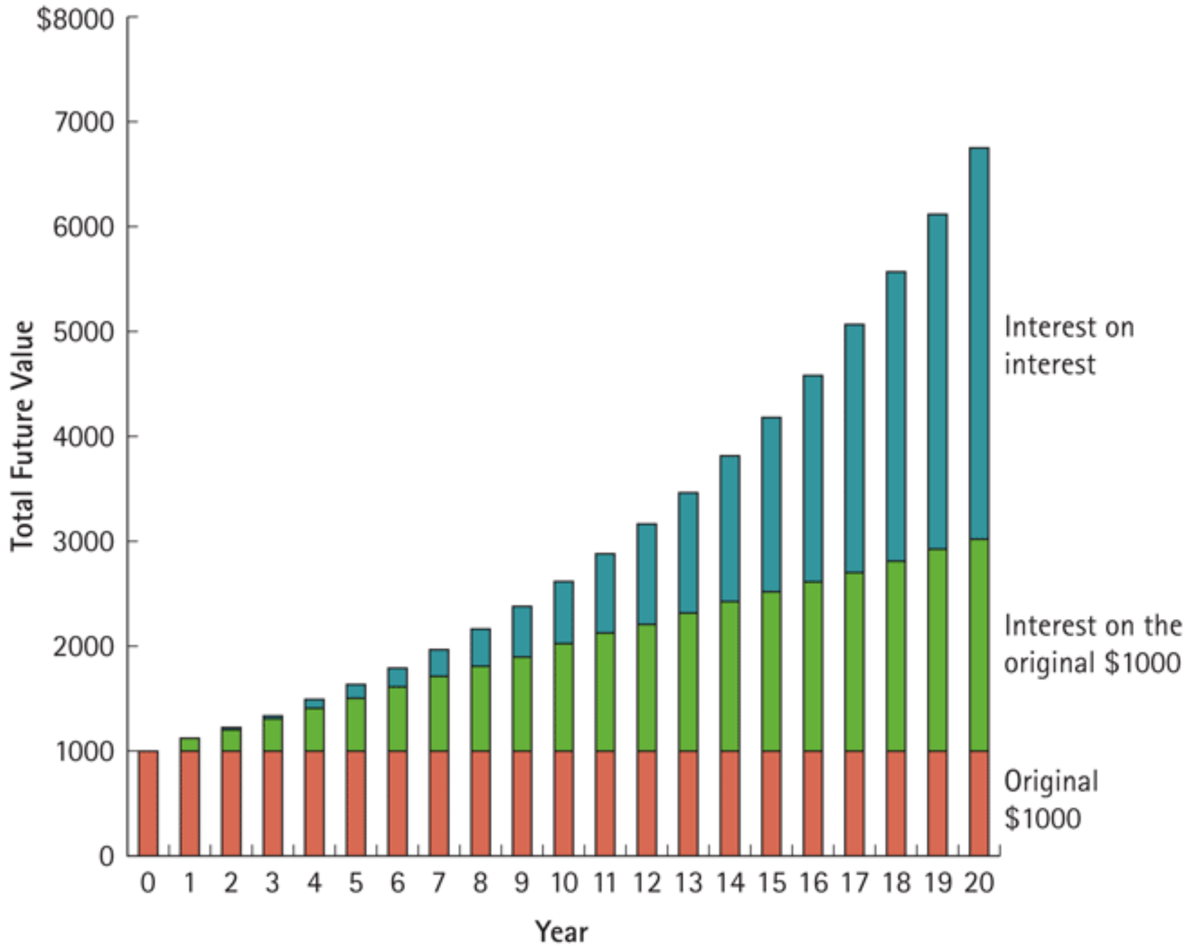
- Future Value of today's Cash Flow C, after n periods, at interest rate r (Compounding):

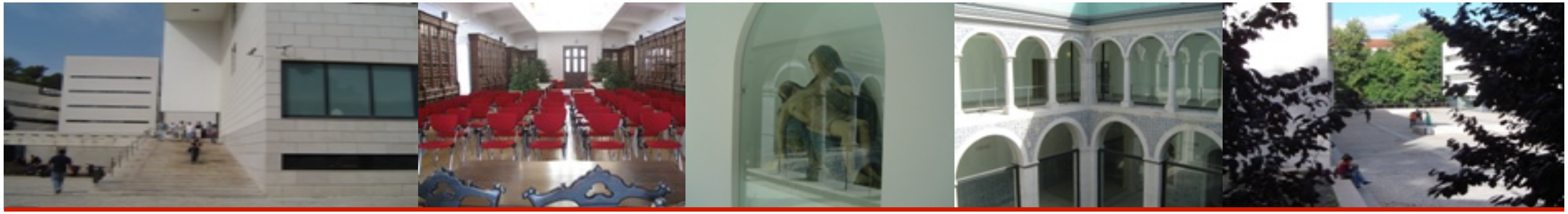
$$FV_n = C \times \underbrace{(1+r)\times(1+r)\times\cdots\times(1+r)}_{n \text{ times}} = C \times (1+r)^n$$

- Example: You believe you can earn 10% on the $1,000 you have today, but want to know what the $1,000 will be worth in two years. The time line looks like this:



The composition of interest over time

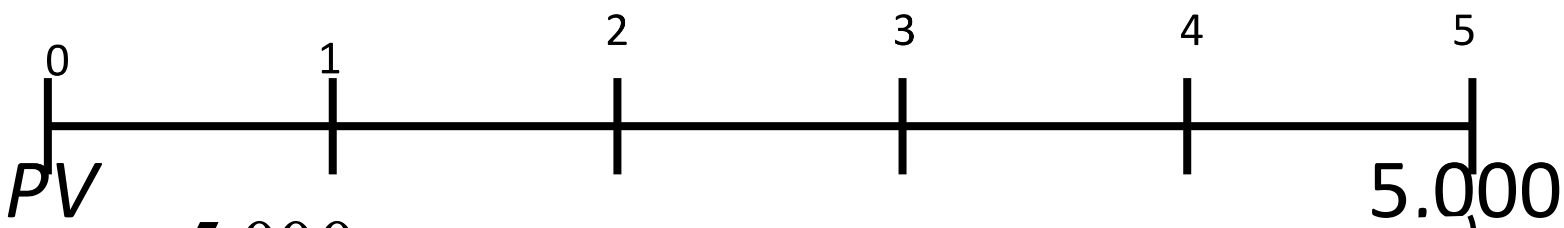




- Present Value today of a Cash Flow C (to be realized n periods from now), assuming interest rate r (Discounting):

$$PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n}$$

- Example: How much does an investor have to set aside today in order to have $5,000 in 5 years, at 10% per year?



$$PV = \frac{5{,}000}{(1.1)^5} = 3{,}104.61$$

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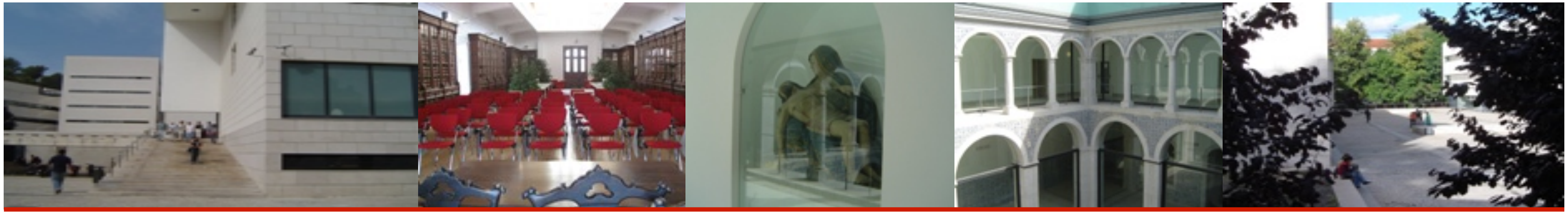
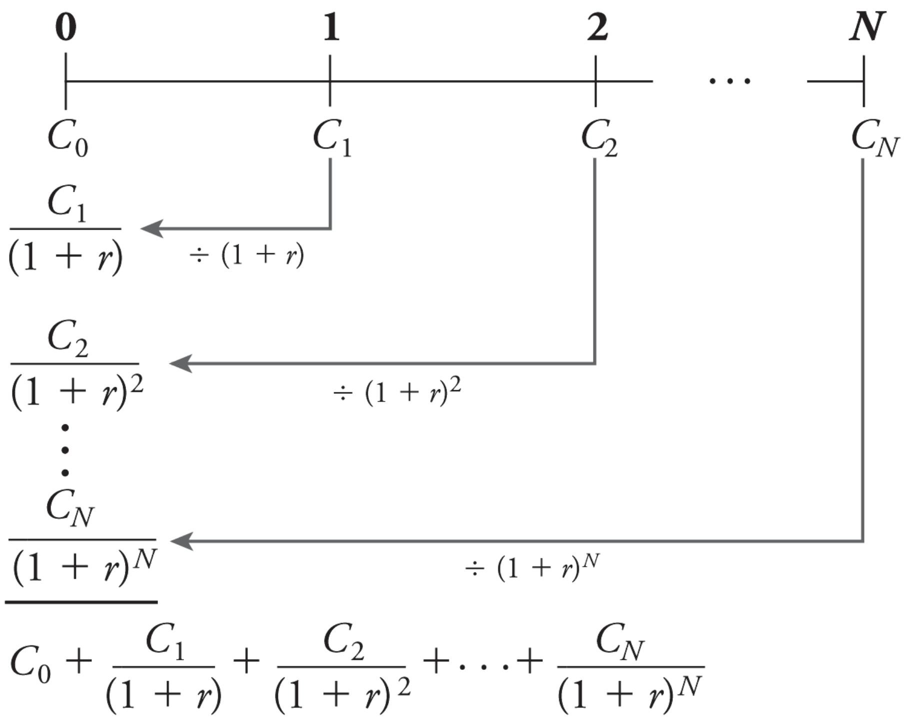


• Present Value of a Stream of Cash Flows:

$$PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n}$$



Example: Suppose you are promised the following stream of annual cash flows:

C1=€5,000

C2=€5,000

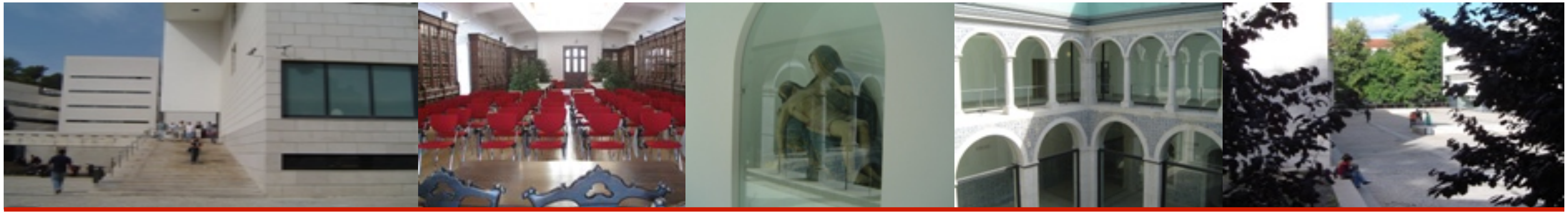
C3=€8,000

The interest rate is 10%. What is the Present Value of the cash flow stream?

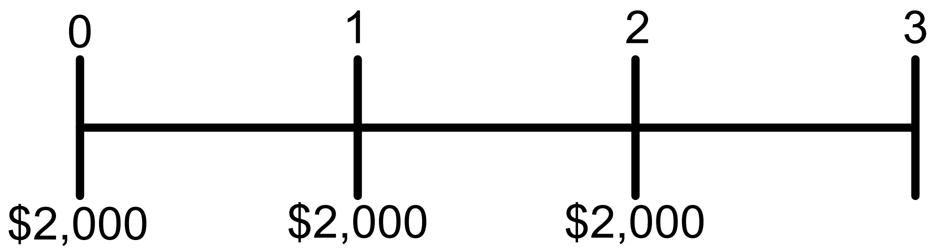
$$PV_0 = \frac{5{,}000}{(1+0.1)^1} + \frac{5{,}000}{(1+0.1)^2} + \frac{8{,}000}{(1+0.1)^3} =$$

$$= €14{,}668.20$$

•PV=€14,668.20

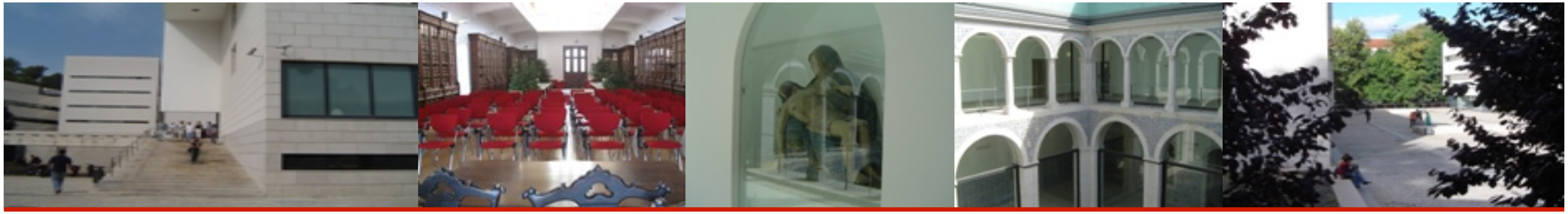


- **Future Value of a Stream of Cash Flows** with present value PV, after n periods, with interest rate r: $FV_n = PV \times (1 + r)^n$
- **Example**: What is the future value in three years of the following cash flows if the compounding rate is 10%?

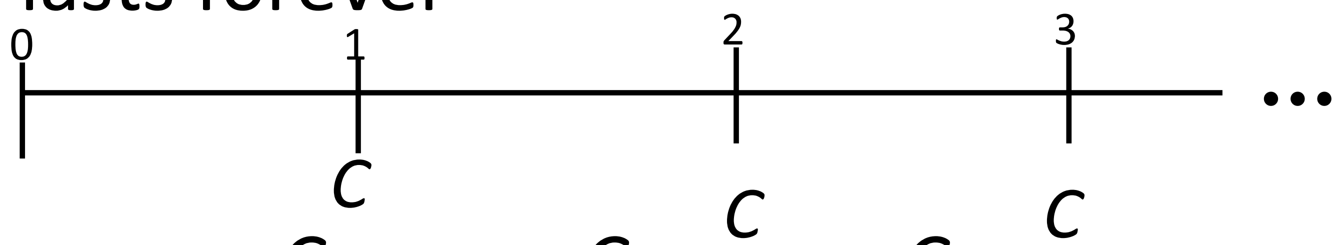


$$PV_0 = \frac{2{,}000}{(1+0.1)^0} + \frac{2{,}000}{(1+0.1)^1} + \frac{2{,}000}{(1+0.1)^2} =$$
$$= €5{,}471.07$$

$$FV_3 = €5{,}471.07 \times (1+0.1)^3 =$$
$$= €7{,}282$$



- Perpetuity: A constant stream of cash flows that lasts forever



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$$

$$PV = \frac{C}{r}$$

Example: What is the present value of a perpetuity of $15 if the discount rate is 5%?

$$PV = \frac{15}{0.05} = 300$$

•The PV is $300.

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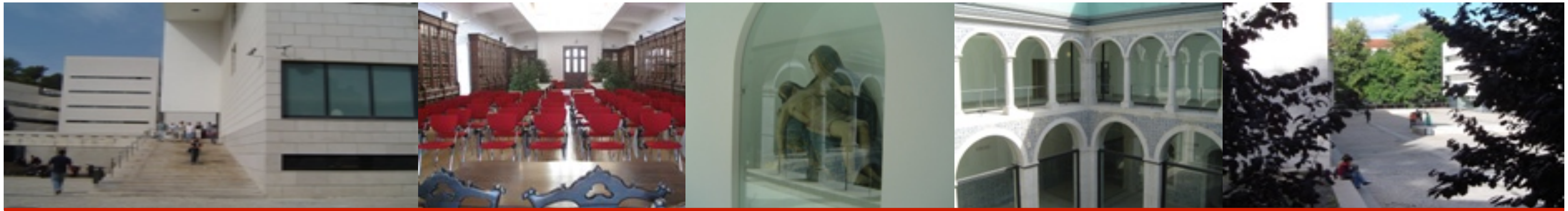
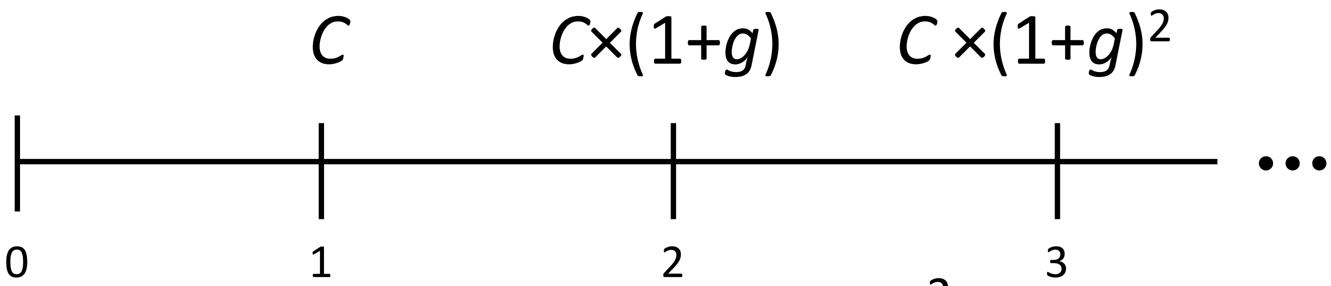


- A Growing Perpetuity is a stream of cash flows that grows at the same rate g , and lasts forever.



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \ldots$$

$$PV = \frac{C}{r-g}$$

- Example: What is the present value of a perpetuity of \$25 that starts in one year's time, and grows forever at 5%? Consider the discount rate is 10%

$$PV = \frac{25}{0.1-0.05} = 500$$

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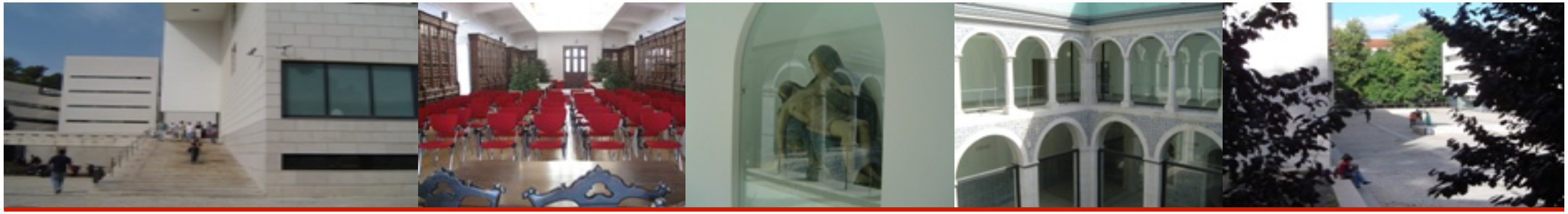
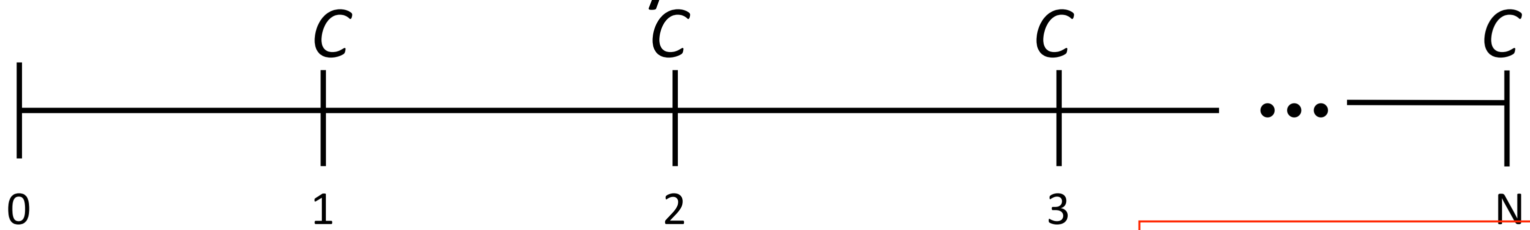


- An Annuity is a constant stream of cash flows C with a fixed maturity N.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^N}$$

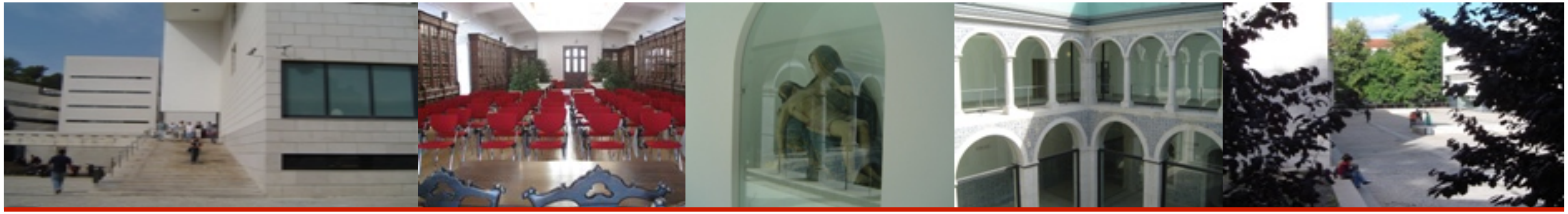
$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

- The Future Value of an Annuity is:

$$FV \text{ (annuity)} = PV \times (1 + r)^N$$

$$= \frac{C}{r}\left(1 - \frac{1}{(1 + r)^N}\right) \times (1 + r)^N$$

$$= C \times \frac{1}{r}\left((1 + r)^N - 1\right)$$



- Example: You are the lucky winner of the $30 million state lottery. You can take your prize as 30 payments of $1 million per year (starting today). What is the present value of this lottery prize, considering a discount rate of 8%?

C_0

29-year annuity starting in year 1

$$PV_0 = \$1{,}000{,}000 + \$1{,}000{,}000 \times \frac{1}{0{,}08}\left[1 - \frac{1}{(1+0.08)^{29}}\right] =$$

$$= \$1{,}000{,}000 + \$1{,}000{,}000 * 11.15841 =$$

$$= \$1{,}000{,}000 + \$11{,}158{,}406 = \$12{,}158{,}406$$

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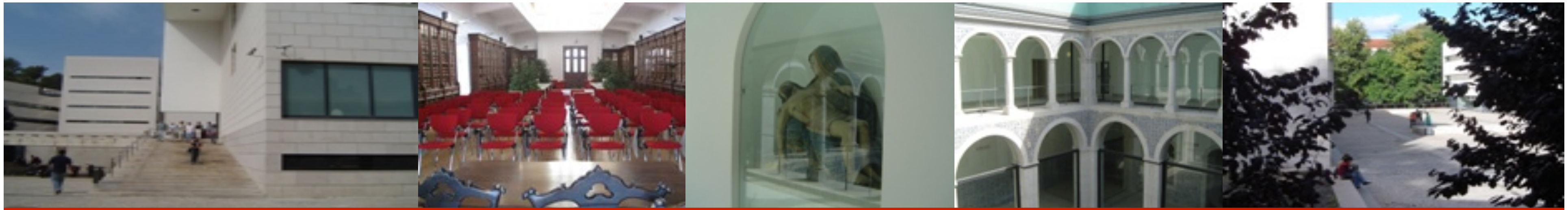
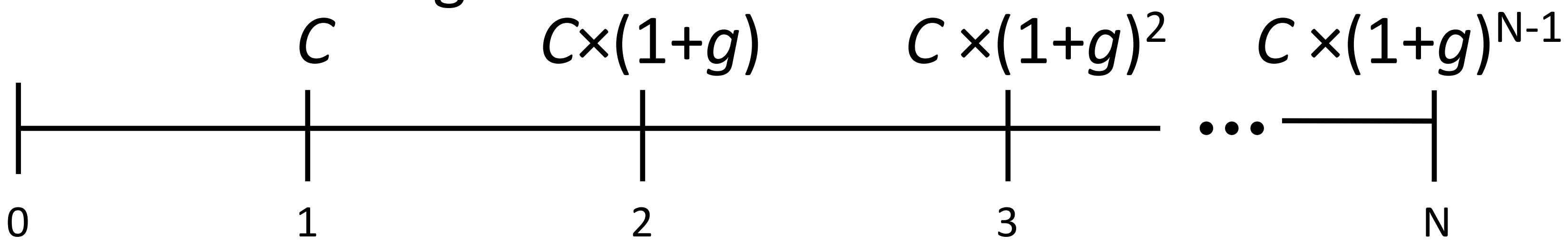


- A Growing Annuity is a stream of N cash flows that grow at a constant rate g.

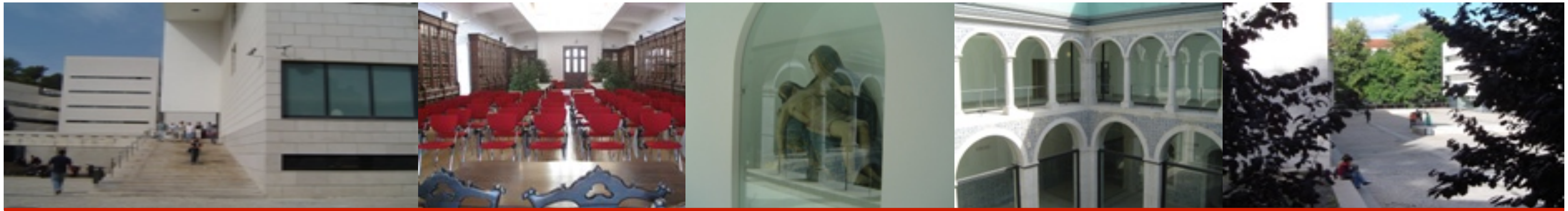


$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \ldots + \frac{C\times(1+g)^{N-1}}{(1+r)^N}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^N\right]$$

Interest Rates

- The Effective Annual Rate (EAR):
  - Indicates the total amount of interest that will be earned at the end of one year. Considers the effect of compounding
    - Also referred to as the effective annual yield (EAY) or annual percentage yield (APY)
    - It's the kind of rate we used in the previous slides.



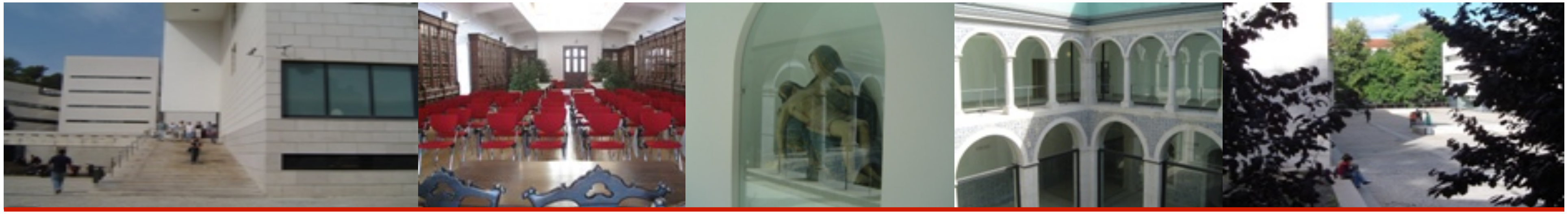
- It is necessary to adjust the EAR to Different Time Periods.
- General Equation for Discount Rate Period Conversion:

$$\text{Equivalent n - period Discount Rate} = (1 + EAR)^n - 1$$

- Example: Earning a 5% return annually is **not** the same as earning 2.5% every six months. The Equivalent Semi-annual discount rate would be:

$(1.05)^{0.5} - 1 = 1.0247 - 1 = .0247 = 2.47\%$

•Note: $n = 0.5$ since we are solving for the six month (or 1/2 year) rate

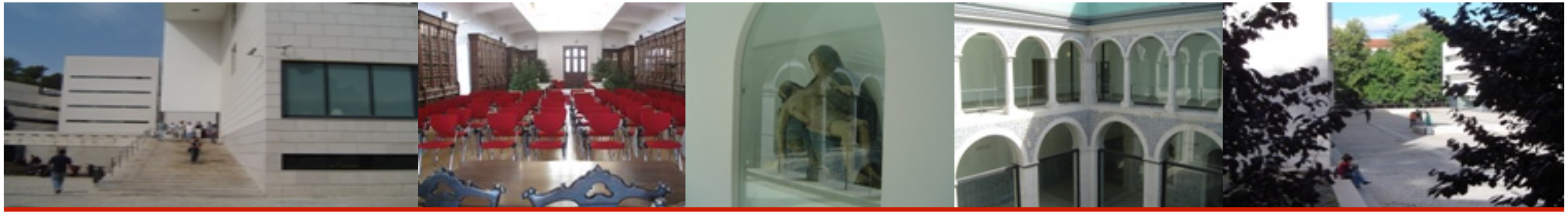


- The **Annual Percentage Rate (APR)**, indicates the amount of simple interest earned in one year.
  - **Simple interest** is the amount of interest earned *without* the effect of compounding.
  - The APR is typically less than the effective annual rate (EAR).

•*The APR itself cannot be used as a discount rate.*

•The APR with *k* compounding periods is a way of quoting the actual interest earned each compounding period:

$$\text{Interest Rate per Compounding Period} = \frac{\text{APR}}{k \text{ periods / year}}$$



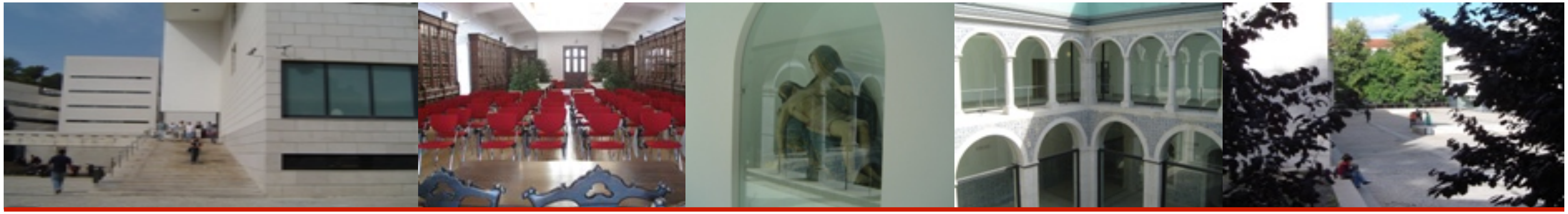
- Converting an APR to an EAR

$$1 + EAR = \left(1 + \frac{APR}{k}\right)^{k}$$

– The EAR increases with the frequency of compounding. Example:

Table 5.1 Effective Annual Rates for a 6% APR with Different Compounding Periods

Compounding Interval	Effective Annual Rate
Annual	$(1 + 0.06/1)^1 - 1 = 6\%$
Semiannual	$(1 + 0.06/2)^2 - 1 = 6.09\%$
Monthly	$(1 + 0.06/12)^{12} - 1 = 6.1678\%$
Daily	$(1 + 0.06/365)^{365} - 1 = 6.1831\%$



- Inflation and Real Versus Nominal Rates

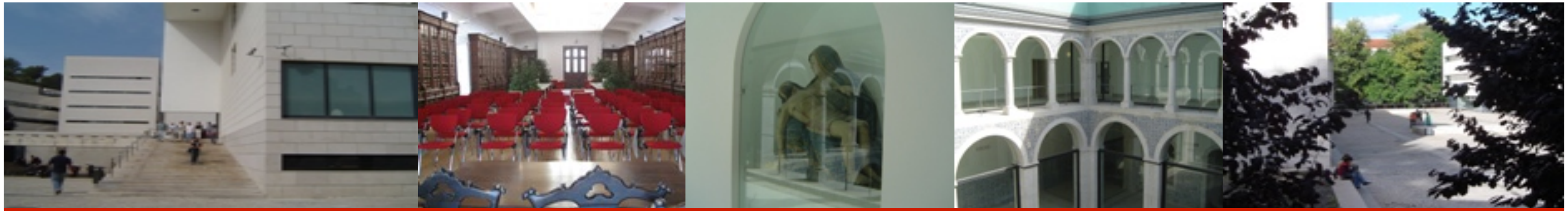
  - **Nominal Interest Rate:** The rates quoted by financial institutions and used for discounting or compounding cash flows

  - **Real Interest Rate:** The rate of growth of your purchasing power, after adjusting for inflation

$$\text{Growth in Purchasing Power} = 1 + r_r = \frac{1 + r}{1 + i} = \frac{\text{Growth of Money}}{\text{Growth of Prices}}$$

  - The Real Interest Rate is:

$$r_r = \frac{r - i}{1 + i} \approx r - i$$



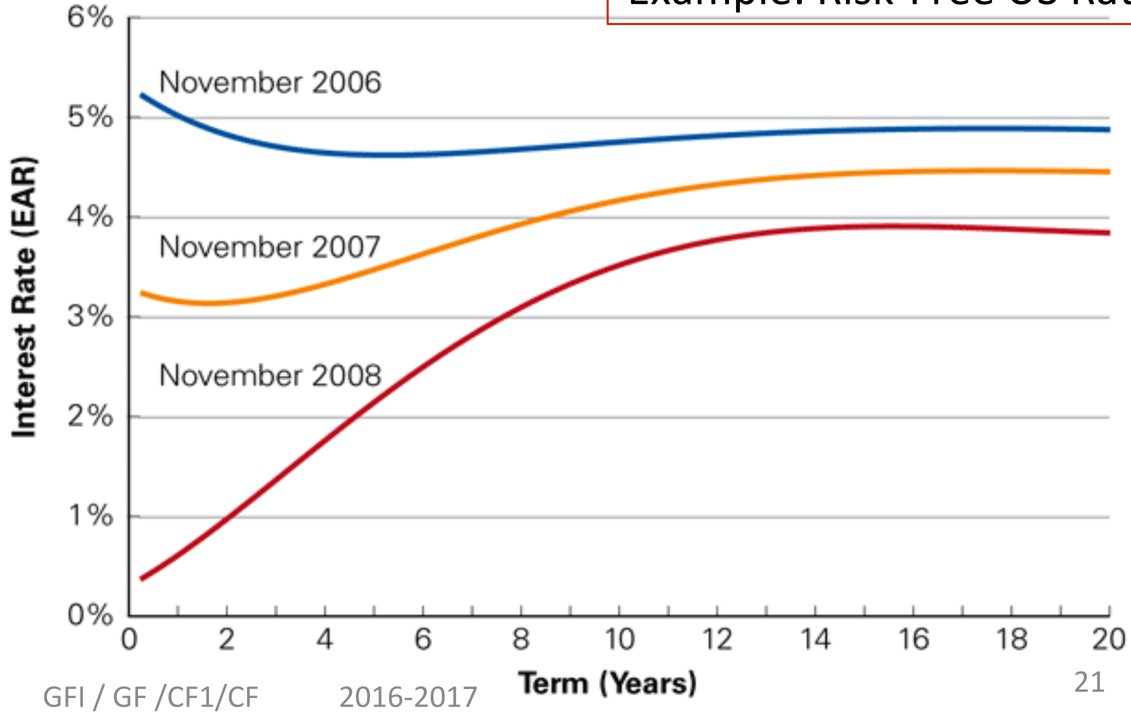
• Term Structure and the Yield Curve:

– **Term Structure:** The relationship between the investment term and the interest rate

– **Yield Curve:** A graph of the term structure

Example: Risk-Free US Rates

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.15%	3.20%	0.44%
1	5.02%	3.15%	0.60%
2	4.83%	3.14%	0.96%
3	4.71%	3.20%	1.35%
4	4.64%	3.32%	1.75%
5	4.62%	3.47%	2.13%
6	4.62%	3.63%	2.49%
7	4.65%	3.78%	2.81%
8	4.68%	3.93%	3.09%
9	4.71%	4.06%	3.32%
10	4.75%	4.17%	3.51%
15	4.87%	4.44%	3.90%
20	4.88%	4.45%	3.84%



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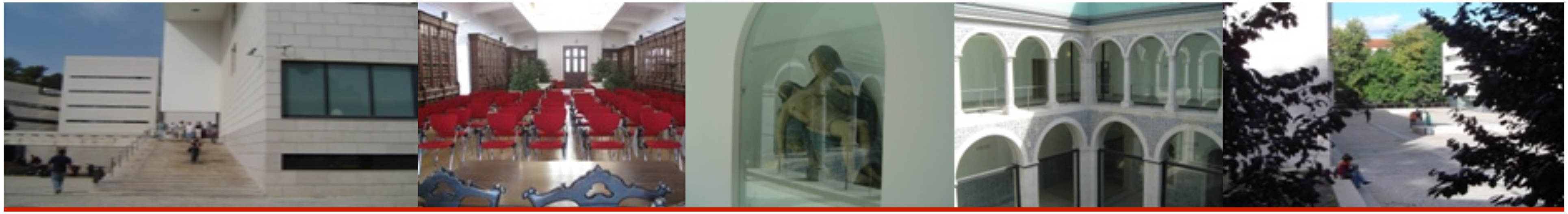
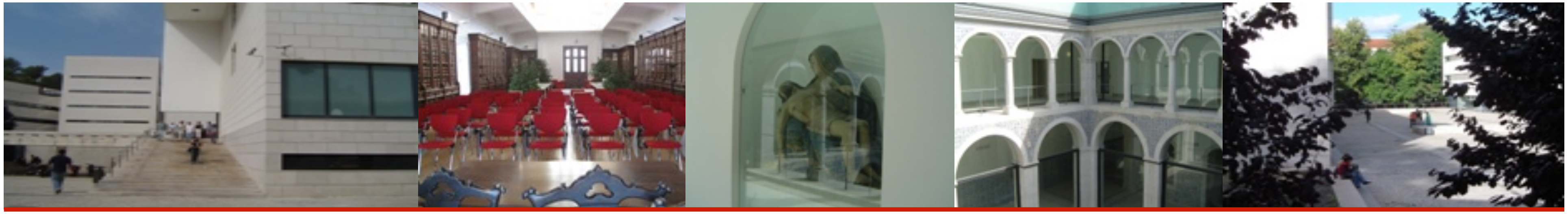


- Note: The term structure can be used to compute the present and future values of a risk-free cash flow over different investment horizons.

$$PV = \frac{C_n}{(1 + r_n)^n}$$

- Present Value of a risk-free Cash Flow Stream Using a Term Structure of Discount Rates:

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1 + r_n)^n}$$



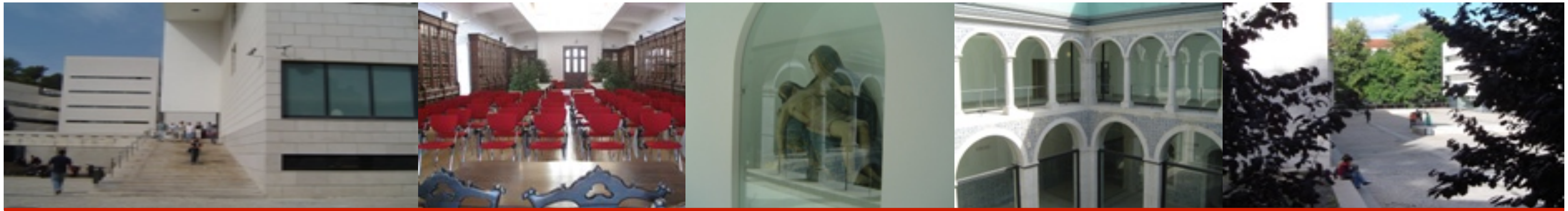
- Example: Compute the present value of a risk-free three-year annuity of $500 per year, given the following yield curve:

November-09

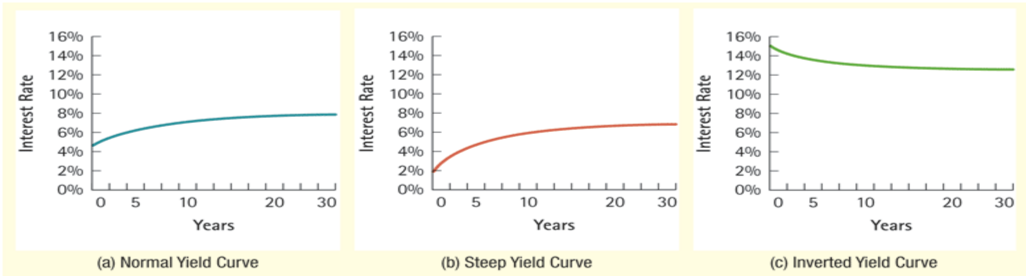
Term (Years)	Rate
1	0.261%
2	0.723%
3	1.244%

$$PV = \frac{\$500}{1.00261} + \frac{\$500}{1.00723^2} + \frac{\$500}{1.01244^3}$$

$$PV = \$498.70 + \$492.85 + 481.79 = \$1,473.34$$

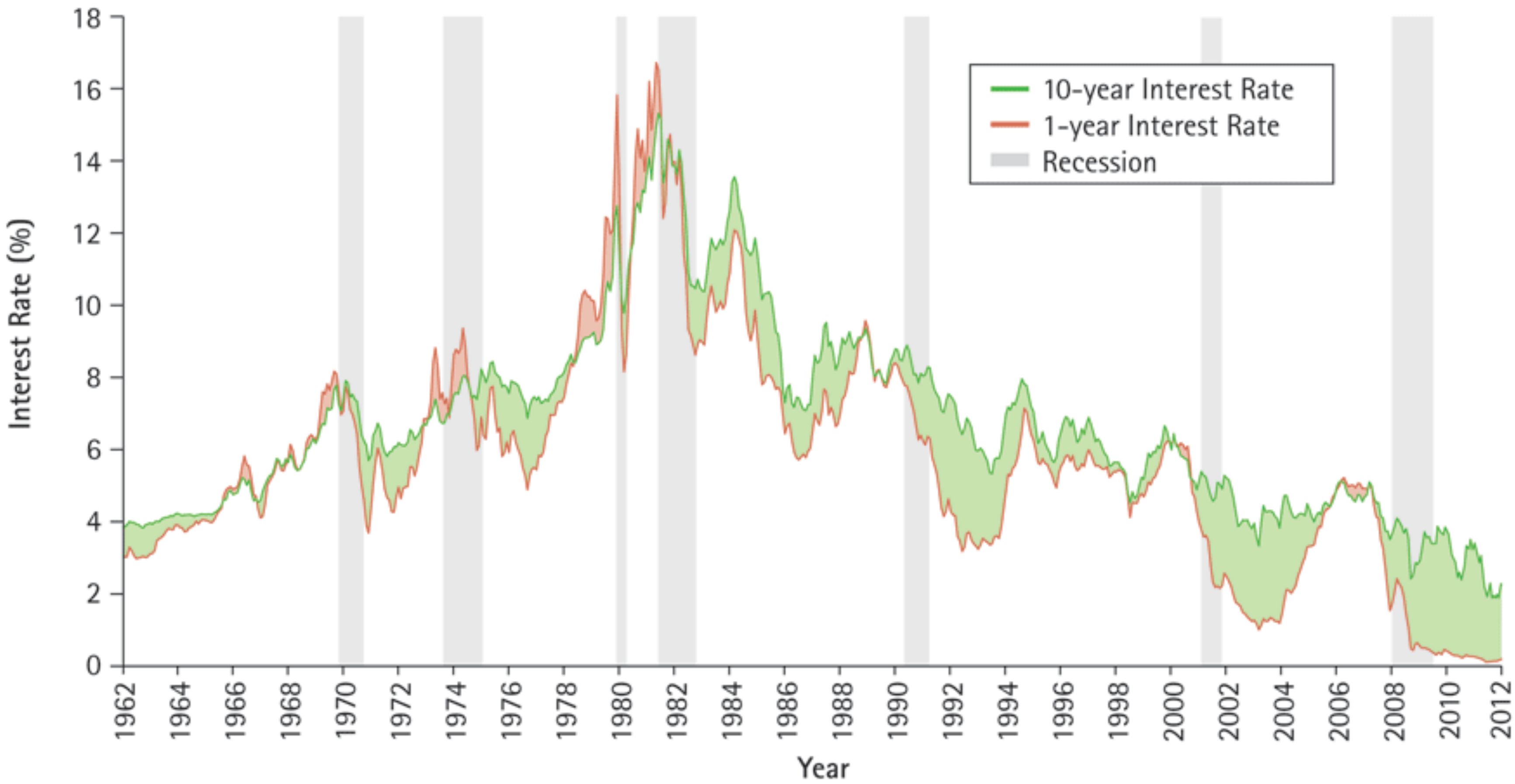


- Interest Rate Expectations
  - The shape of the yield curve is influenced by interest rate expectations.
    - An inverted yield curve indicates that interest rates are expected to decline in the future.
      - Because interest rates tend to fall in response to an economic slowdown, an inverted yield curve is often interpreted as a negative forecast for economic growth.



- Risk and Interest Rates
  - U.S. Treasury securities are considered “risk-free.” All other borrowers have some risk of default, so investors require a higher rate of return.

Short-Term Versus Long-Term U.S. Interest Rates and Recessions



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Dicionário Essencial

EN	PT
Discounting	Atualizar
Compounding	Capitalizar
Perpetuity	Perpetuidade/Renda perpétua
Annuity	Anuidade/Renda
Growing Perpetuity	Renda perpétua em progressão geométrica
Effective Annual Rate (EAR)	Taxa Annual Equivalente (TAE)
Annual Percentage Rate (APR)	Taxa Anual Nominal (TAN)