Master in Actuarial Science, Exam 1, 08/01/2016. **2h30m**
Ratemaking and Experience Rating, 2nd year, 3rd term

1. The following table shows data generated by a household insurance portfolio in some year corresponding to a "regular" year of exposure.

No. of Claims	0	1	2	3	4	5	Total
No. of policies	143 991	11 588	1 005	85	6	2	156 677

Let N be the number of claims per year for a given risk in the portfolio and suppose that $N \frown \text{Poisson}(\lambda)$. The parameter λ is unknown. Consider that the usual hypothesis in credibility theory may be applicable to the risk portfolio under study, λ is the associated risk parameter, it does not depend on the sum insured, claim number and claim sizes are independent and the expected value of the claim size is proportional to the sum insured. Observations from the past n years for the risk are available.

Bühlmann's credibility (pure) premium for a given risk X in a homogeneous portfolio, for a coming year of exposure, is given by formula

$$P_c = z\bar{X} + (1-z)\mu_X,$$

where $z = n/(n+\upsilon/a)$, $\mu = E[\mu(\theta)]$, $\upsilon = E[\upsilon(\theta)]$, $a = Var[\mu(\theta)]$, $\mu(\theta)$ and $\upsilon(\theta)$ are the risk mean and variance, respectively, n is the number of years in force of that risk, and $\bar{X}$ is its sample mean.

(a) Consider the following statement: *Homogeneous risks are grouped in portfolios so that premia should be not only better estimated but also be equal for every policyholder, proportional to each risk exposure. Differences on annual realizations of risks should be considered the result of natural randomness*. Comment and argue on possible contradictions with credibility estimation.

(b) Consider two risks taken at random from the portfolio. Doing appropriate calculation, would you consider those risks to be positively correlated?

(c) Consider the data above. Would you consider to come from a Poisson distributed population? Do a quick calculation to support your answer.

(d) From now onwards admit that the parameter λ is a realization of a random variable Λ, not observable, following a Gamma distribution with mean $E[\Lambda] = \alpha/\beta$ and variance $V[\Lambda] = E[\Lambda]/\beta$.

Show that the posterior distribution is of the same family of the prior, i.e. it follows a Gamma distribution with parameters $\alpha_* = \alpha + N_*$, $\beta_* = (\beta + n)$ and mean α_*/β_*, $N_* = \sum_{j=1}^{n} N_j$.

(e) In addition, suppose that from now onwards a given risk from the portfolio has produced one claim in last four years.

Compute the credibility premium as well as the Bayes premium for next rating year for the given risk.

(f) Although the risk parameter λ is not observable you can estimate parameters of the prior using collective data above in the table. Comment and calculate estimates $\hat{\alpha}$ and $\hat{\beta}$.

(g) Using the above estimates calculate estimates for structural parameters and Bühlmann's credibility premium for the given risk.

(h) The insurer wants to introduce a "no-claims discount" of 15% after three consecutive claim-free years. Then, the referred given insured will be claiming that discount.

i. Calculate an estimate for the probability of the 15% discount and the impact on the pure premium. (For simplicity reasons consider that all policies have three or more observation periods)
[**Remark**: if you haven't answered the parameters estimate question use $\hat{\alpha} = 0.878$ and $\hat{\beta} = 9.911$]

ii. Is the discount rate of 15% reasonable? Compare it with the value obtained for the corresponding credibility estimator.

(i) Consider now that you needed to estimate not only the expected number of claims per year but also the expected claim amounts using credibility, and the credibility estimator should reflect the risk exposure. Considering the models you learned in class which would you use and how would you define the risk random variables.

2. A certain insurer is considering a *bonus-malus* system based on the individual's annual claims record to rate each individual risk in a given motor insurance portfolio.

Step	%	New step after claims			
		0	1	2	3+
14	30	14	9	5	1
13	32.5	14	8	4	1
12	35	13	8	4	1
11	37.5	12	7	3	1
10	40	11	7	3	1
9	45	10	6	2	1
8	50	9	5	1	1
7	55	8	4	1	1
6	60	7	3	1	1
5	70	6	2	1	1
4	80	5	1	1	1
3	90	4	1	1	1
2	100	3	1	1	1
1	120	2	1	1	1

Table 1: Rules and premium percentages

(a) Can you give him some advise on avantages and disavantages of a system not based on severities? Discuss briefly and appropriately.

(b) Bonus systems are usually based on Markov chain analysis where premiums are computed and adjusted annually. Suppose you have a 3-state system with an entry class, a sole bonus class and a remainder penalty class. From any class, you could reach a next year bonus by having no claims in last two consecutive years only. The system is not directly Markovian. How would you solve the problem under a Markov chain framework, yearly based, as you learned in class?

(c) Consider a *bonus* system that evolves according to what shown in Table 1. Considering a Poisson($\lambda = 0.1$) distribution for the claim counts build the associated transition probability matrix.

(d) Suppose that for some 3-state *bonus* system and some given θ, the steady state premium distribution is given by vector $(\theta^2, \theta(1-\theta), 1-\theta)$, where θ is the probability of not getting any claim in one year. Number of claims is Poisson distributed and parameter can take values $\lambda = 0.1, 0.125, 0.2$ with probabilities 0.7, 0.2, 0.1, respectively.

3. In a study of the tariffs for a large motor insurance portfolio, a study group assessed the impact of different factors on both the claim frequency and the claim size means.

(a) Explain/discuss briefly what consists of "prior" and "posterior" ratemaking. Give application examples.

(b) For the estimation of the pure premium, should we use additive or multiplicative models. Discuss brief and appropriately.

(c) Should the study group actuaries consider modeling claim counts and claim sizes separately? Explain briefly.

(d) Suppose that the group is using a GLM Poisson model for the claim frequency key ratio, with three rating factors with two, three and four levels, labeled F11, F12, F21, F22, F23, F31, F32, F33 and F34, respectively. Suppose that the claim frequency mean for a given policy was calculated as k, write the estimated pure premium for the risk for the three rating factors with levels 1, 2 and 3, respectively.

Marks (out of 200):

1.*a*)	*b*)	*c*)	*d*)	*e*)	*f*)	*g*)	*h*)*i*.	*ii*.	*i*)	2.	3.
10	15	10	15	15	10	10	15	10	10	50	30
10	25	35	50	65	75	85	100	110	120	170	200