Master in Actuarial Science, Exam 2, 03/02/2016. **2h30m**
Ratemaking and Experience Rating, 2nd year, 3rd term

1. Consider that an individual risk can produce in one year a total claim amount of one of the following sizes, 10 000, 5 000 or 0 (claim free year) only. This individual risk is taken from a collective, which contains three types of risks: Good (65% of the portfolio), Medium (30%), Bad (5%). The conditional probabilities are given in the following table (in %):

Total claim	Conditional probabilities		
amount	Good	Medium	Bad
0	92%	85%	75%
5000	5%	10%	15%
10000	3%	5%	10%

Risks has been observed for two consecutive years, let X be the annual total amount for a risk taken out at random from the portfolio, $S = X_1 + X_2$ and let θ denote the risk characteristic. Consider that the usual hypothesis (H_1 and H_2) in credibility theory are applicable to the risk group under study (with the addition to H_1 given by Bühlmann's model, where appropriate).

Bühlmann's credibility formula for a given risk X in a homogeneous portfolio, for a coming year of exposure, is given by

$$P_c = z\bar{X} + (1 - z)\mu_X, \qquad (1)$$

where $z = n/(n + k)$, $k = \upsilon/a)$, $\mu = E[\mu(\theta)]$, $\upsilon = E[\upsilon(\theta)]$, $a = Var[\mu(\theta)]$, $\mu(\theta)$ and $\upsilon(\theta)$ are the risk mean and variance, respectively, n is the number of years in force of that risk, and $\bar{X}$ is its sample mean.

(a) Calculate the risk premia and the collective premium.

(b) Determine the conditional probability function $f_{S|\theta}(x|\theta)$ for the total amount for the two years of a given risk.

(c) Calculate the probability of having a total amount of 10 000 for the two observed years of a risk taken out at random from the portfolio. [In the following consider the observed value of $S = 10\,000$, where appropriate]

(d) Calculate the posterior distribution $\pi_{\Theta|S}(\theta|10\,000)$.

(e) Determine the conditional distribution $f_{X_3|S}(x|10\,000)$ of next year total amount X_3 given that $S = 10\,000$ was observed in the previous years.

(f) Determine the Bayesian premium for year 3.

(g) Compute the structural parameters $\mu = E(\mu(\Theta))$, $\upsilon = E(\upsilon(\Theta))$ and $a = V(\mu(\Theta))$.

(h) Compute Bühlmann's credibility premium.

(i) On what condition(s) can we talk on *exact credibility model*? Comment appropriately.

(j) Let $P_{c,n+1}$ be Bühlmann's credibility premium for year $n + 1$, $n = 1, 2, \ldots,$ based on the n previous annual observations [it is given by Formula (1)]. Show that $P_{c,n+1}$ can be recursively calculated as

$$P_{c,n+1} = \alpha_n X_n + (1 - \alpha_n)P_{c,n}\,,$$

where weight $\alpha_n = z/n$ [z is given in (1)] depend on the n previous observations and on parameter k.

2. A certain insurer is considering a *bonus-malus* system (BMS) based on the individual's annual claims record to rate each individual risk in a given motor insurance portfolio.

(a) Determine the value of α such that the transition probability matrix $\mathbf{P}$ has vector $(5/12, 7/12)$ as its steady state vector, if is given by

$$\mathbf{P} = \begin{pmatrix} \alpha & 1-\alpha \\ 1/2 & 1/2 \end{pmatrix}.$$

(b) Consider a *bonus* system that evolves according to what shown in Table 1. Determine the percentage of the basic premium to be paid by a driver who originally entered the scale at level 100%, drove without claim for seven years, then reported one claim during the eighth policy year, and has been driving claim-free for the three years since then. Would the total of premia he paid have been different if his one only claim occurred in the second policy year? Show with calculation.

Step	%	New step after claims			
		0	1	2	3+
14	30	14	9	5	1
13	32.5	14	8	4	1
12	35	13	8	4	1
11	37.5	12	7	3	1
10	40	11	7	3	1
9	45	10	6	2	1
8	50	9	5	1	1
7	55	8	4	1	1
6	60	7	3	1	1
5	70	6	2	1	1
4	80	5	1	1	1
3	90	4	1	1	1
2	100	3	1	1	1
1	120	2	1	1	1

Table 1: Rules and premium percentages

(c) "The ultimate goal of a BMS is to make everyone pay a premium which is near as possible the expected value of his yearly claims. In practice, changes on the number of claims should be compensated by a change in the expected value of the risk, accordingly". Discuss briefly and appropriately the statement, considering that you are facing a BMS based on number of claims Poisson(λ) distributed.

(d) Suppose that for some 3-state *bonus* system and some given θ, the steady state premium distribution is given by vector $\big((1-\theta), \theta^2, \theta(1-\theta)\big)$, where θ is the probability of not getting any claim in one year. Number of claims is mixed Poisson distributed with parameter taking values according to a mean one exponential distribution.

Compute the resulting limiting distribution.

3. A working party is modelling a tariff for a given large motor insurance portfolio. The study group is re-evaluating an existing tariff, evaluating a wide variety of existing and non-existing risk factors supposed to bring impact in both the claim frequency and the claim size means.

(a) For each risk category considered, a future premium is estimated in two ways, e.g. *ratemaking and experience rating.* Explain briefly.

(b) For the estimation of the pure premium, we could use GLM's to model the pure premium altogether or we could model claim counts and claim sizes separately. Discuss brief and appropriately.

(c) Working key ratios and relativities is not only very common but appropriate. Discuss briefly.

(d) Suppose that the group is using a GLM Gamma model for the loss ratio, with "log link", and three rating factors with two, four and three levels, labeled F11, F12, F21, F22, F23, F24, F31, F32 and F33. Suppose that the claim frequency mean for a given policy was given calculated as k, write the estimated pure premium for the risk for the three rating factors with levels 1, 1 and 2, respectively.

Marks (out of 200):

1.*a*)	*b*)	*c*)	*d*)	*e*)	*f*)	*g*)	*h*)	*i*)	*j*)	2.	3.
10	25	7.5	20	10	7.5	10	10	5	15	50	30
10	35	42.5	62.5	72.5	80	90	100	105	120	170	200