Some Exercises for Lisbon 2016

1. Assume that W^1 and W^2 are independent Wiener processes. Prove that W is Wiener, where W is defined by

$$W_t = \frac{1}{\sqrt{a^2+b^2}}\left\{aW_t^1 + bW_t^2\right\}$$

   and a, b,are real numbers.

2. Consider a B-S model with continuous dividend yield q. Prove that G^Z is a Q martingale directly by using the Qdynamics.

3. Same model as above. Prove that if $f(t,s)$ is the arbitrage free price of a derivative, then $f(t,S_t)/B_t$ is a Q-martingale, by using the Q dynamics and the relevant PDE.

4. Same model as above. Using classical arguments, derive the hedging portfolio for a claim of the form $\Phi(S_T)$.

5. Consider an asset S with general dividend process D. Show that the forward price $f(t,T,S_T)$ i.e. the forward price contracted at t for delivery of one unit of S at time T, is given by the cost of carry formula

$$f(t,T,S_T) = \frac{1}{p(t,T)}\left(S_t - E_{t,s}^Q\left[\int_t^T e^{-\int_t^s r(u)du} dD_s\right]\right).$$

   Hint: Use the cost of carry formula for dividend paying assets.

6. Consider a Black-Scholes model, and compute the price of an asset-or-nothing option where the payoff is given by

$$X = S_T \cdot I\left\{S_T \le K\right\}$$

7. Let the stock prices S^1 and S^2 be given as the solutions to the following system of SDE:s.

$$\begin{aligned} dS_t^1 &= \alpha_1 S_t^1 dt + \sigma_1 S_t^1 dW_t^1, \quad S_0^1 = s_1, \\ dS_t^2 &= \alpha_2 S_t^2 dt + \sigma_2 S_t^2 dW_t^2, \quad S_0^2 = s_2, \end{aligned}$$

   The Wiener processes W^1 and W^2 are assumed to be independent. The parameters are assumed to be known and constant. There is also a bank account with constant short rate r. Your task is to price the T-claim X defined by

$$X = S_T^1 \cdot I\left\{S_T^2 \leq K\right\}$$

where I denotes the indicator.

8. Compute the price of an exchange option on the two assets in the previous exercise so

$$X = \max\left[S_T^2 - S_T^1, 0\right]$$

This was basically done in the OH slides so the only thing that has to be computed is the relevant volatility.

9. Compute the price of a maximum option on the two assets in exercise 7, so

$$X = \max\left[S_T^2, S_T^1\right]$$