LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

Microeconomics

January 30, 2015

Solution Topics

Group 1

1. A consumer of two goods faces positive prices and has a positive income. His utility function is $u(x_1, x_2) = x_1$.

a) (1.5 marks) Derive the Marshallian demand functions.

R: Solve the UMP to obtain $x_1^*(\text{m}, p_1, p_2)=\text{m}/ p_1$ and $x_2^*(\text{m}, p_1, p_2)=0$.

b) (0.5 marks) Compute the indirect utility function.

R: $\text{v}(\text{m}, p_1, p_2)= \text{m}/ p_1$.

c) (0.5 marks) Determine the expenditure function.

R: $\text{e}(\text{u}, p_1, p_2)= \text{u}p_1$.

d) (1 mark) Using Shephard's lemma, derive the compensated (or Hicksian) demand functions.

R: Computing the derivative of the expenditure function with respect to p_1, we obtain $x_1^h(\text{m}, p_1, p_2)=\text{u}$.

2. (1.5 marks) In a world of two goods, let the preferences of a consumer be given by the following:

(x_1, x_2) is "at least as good as" (y_1, y_2) if and only if $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$.

Can these preferences be represented by a utility function? Explain.

R: No. There is a result that says that if preferences are complete, transitive, and continuous, they can be represented by a continuous utility function. Since these preferences are not continuous, we cannot apply the result. However, this does not mean that there exists no utility function that represents these preferences. Still, given the definition of the preferences, each indifference curve is a single point. Therefore, each consumption vector must have a value, different from all others, i.e., no two points in R^2 can have the same value. But, R^2 has many more points than R from which the numerical values must be assigned. Concluding, there are not enough real numbers to assign to all consumption vectors, so that there exists no utility function that represents the preferences.

Group 2

1. Marc is risk averse. He has initial wealth of w and suffers a loss $D < w$ with probability p. With probability $1 - p$ Marc's wealth does not suffer any change. Marc can buy an amount of insurance A (receiving A in case the loss occurs), paying a price qA. Show that:

a) (1.5 marks) Marc wants to buy full insurance (i.e., $A = D$) when $p = q$.

R: Marc finds A to solve Max pu(w-D+A-qA) + (1-p)u(w-qA) s.t. A ≥0. When p = q, the FOC gives u'(w-D+A-qA) = u'(w-qA), which, since u''<0, implies w-D+A-qA = w-qA or A = D.

b) (1.5 marks) Marc wishes to buy insurance $A < D$ if $q > p$.

R: From the FOC computed above, when q>p, we have u'(w-D+A-qA) > u'(w-qA), which implies w-D+A-qA < w-qA or A < D.

c) (1 mark) Explain intuitively.

R: Since Marc is risk averse, when insurance is actuarilly fair (p=q), he wants to buy full insurance (A=D). But, for p<q, he only insures partially.

2. (1 mark) Explain the Weak Axiom of Revealed Preference (WARP).
R: According to WARP, if bundle 1 is revealed preferred to bundle 2, then we cannot have bundle 2 being revealed preferred to bundle 1.

Group 3

1. Consider a firm whose technology is:

$$f(x_1, x_2) = (x^a{}_1 + x^a{}_2)^{1/a}$$

a) (2 marks) Determine the conditional input demands.

R: Solve the cost minimization problem, i.e., find $x_1, x_2 \geq 0$ that solve Min $p_1 x_1 + p_2 x_2$ s. t. $(x^a{}_1 + x^a{}_2)^{1/a} \geq y$, to obtain: $x_i = y w_i^{1/(a-1)} (w_1^{a/(a-1)} + w_2^{a/(a-1)})^{-1/a}$, for $i = 1,2$.

b) (1 mark) Compute the cost function.

R: $c(w,y) = y\, (w_1^{a/(a-1)} + w_2^{a/(a-1)})^{(a-1)/a}$

c) (1 mark) Show that the cost function is homogeneous of degree one in the prices of inputs.

R: $c(kw,y) = y\, [(kw_1)^{a/(a-1)} + (kw_2)^{a/(a-1)}]^{(a-1)/a} = kc(w,y)$.

2. (1 mark) Let $y = f(x_1, x_2)$ be a constant returns-to-scale production function. Show that if the average product of x_1 is rising, the marginal product of x_2 is negative.
R: The average product of x_1 (AP_1) is rising if and only if the derivative of AP_1 with respect to x_1 is positive. If we compute the derivative of AP1 and check when it is greater than 0, we obtain $f_1 > f/x_1$, where f_1 is the derivative of f with respect to x_1. Now, constant returns to scale implies $f = f_1 x_1 + f_2 x_2$ (see the solution to the previous exam). Then, $f_2 x_2 = f - f_1 x_1 < f - f = 0$. Thus, $f_2 x_2 < 0$, which implies $f_2 < 0$ as $x_2 > 0$.

Group 4

1. A monopolist faces linear demand $p = a - bq$ and has cost $C = cq + F$, where all parameters are positive, $a > c$, and $(a - c)^2 > 4bF$.

a) (2 marks) Solve for the monopolist's output, price, and profits.

R: Find q ≥0 that solves Max (a - bq)q - cq - F. The FOC give q*=(a-c)/2b. Substituting in the demand, we obtain p*=(a+c)/2. Profits are $(a-c)^2/4b$ - F.

b) (1.5 marks) Calculate the deadweight loss and show that it is positive.

R: In a perfectly competitive market, we have p = c. When p=c, total surplus is $(a-c)^2/2b$. In the monopoly solution, consumer surplus is $(a-c)^2/8b$ and producer surplus is $(a-c)^2/4b$, so that total welfare is $3(a-c)^2/8b$. Welfare loss is thus $(a-c)^2/2b - 3(a-c)^2/8b = (a-c)^2/8b$, which is positive given that a >c by assumption.

2. (1.5 marks) "Consumer surplus is an exact measure of consumer welfare." Under which conditions is this statement true? Explain.
R: Consumer surplus is an exact measure of welfare if and only if the income effect is zero. This happens, for example, when preferences are quasilinear.

Group 5

Consider the Battle of Sexes with incomplete information, where player 2 (the column-player) may have two types:

Type I

	F	O
F	3,1	0,0
O	0,0	1,3

Type II

	F	O
F	3,0	0,1
O	0,3	1,0

a) (2.5 marks) Compute all Nash equilibria (in pure and in mixed strategies) when player 1 (the row-player) knows that player 2 is Type I.

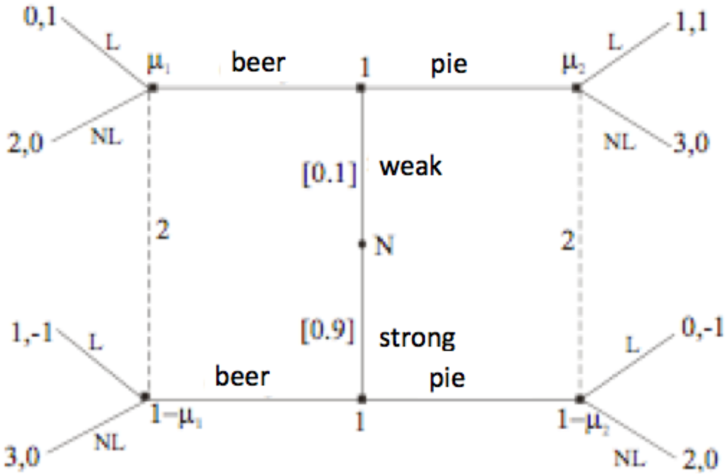
R: S*={(1,1), (0,0), (1/4,3/4)}

b) (2.5 marks) Now assume player 1 does not know the type of player 2. Compute the Bayes Nash equilibria in pure strategies of the incomplete information game. (Let p be the probability with which player 1 believes player 2 is of Type I.)

R: The Bayes Nash equilibria (BNE) in pure strategies are the following: For p ≥ 1/4, there is a single BNE: [F,(F,O)]; for p ≥ 3/4, there are two BNE equilibria: [F,(F,O)] and [O,(O,F)].

Group 6

Compute all weak Bayesian perfect equilibria of the following signaling game.



R: {[(beer, beer), (NL,L), u_1=0.1, $u_2 \geq 0.5$], {[(pie, pie), (L,NL), $u_1 \geq 0.5$, u_2=0.1] }