LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

February 5, 2016

Answer to 4 Groups only

2 hours

Group 1

1. Consider a consumer whose utility function is $u(x_1, x_2) = ln(x_1 + 3x_2)$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2.

a) (0.5 marks) Formulate the consumer choice problem.
R: Max $u(x_1, x_2) = \ln(x_1 + 3x_2)$, s.t. $p_1 x_1+p_2 x_2 \leq m$, $x_1 \geq 0$, $x_2 \geq 0$.
b) (2 marks) Find this consumer's demand for goods 1 and 2.
R: The goods are perfect substitutes. $x(p_1, p_2, m) = (m/p_1,0)$ if $p_1 < p_2/3$; $(0, m/p_2)$ if $p_1 > p_2/3$; (x_1, x_2) s. t. $p_1 x_1+p_2 x_2 = m$ if $3p_1 = p_2$
c) (0.5 marks) Determine the indirect utility function.
R: $v(p, m) = \ln(m/p_1)$, if $p_1 < p_2/3$; $v(p, m) = \ln(3m/p_2)$, if $p_1 \geq p_2/3$.
d) (1 mark) Determine the expenditure function.
R: $e(p, u) = p_1e^u$ if $p_1 < p_2/3$; $e(p, u) = p_2 e^u/3$ if $p_1 \geq p_2/3$.

2. (1 mark) Let $\succcurlyeq$ be a preference relation on R^n_+ and suppose $u(\cdot)$ is a utility function that represents it. Show that u(x) is quasiconcave if and only if $\succcurlyeq$ is convex.

R: Immediate from the definition of a quasiconcave function: u() is quasiconcave if and only if the upper contour set of each of its level curves is convex.

Group 2

1. (1.25 marks) The consumer buys bundle x^0 at prices p^0 and bundle x^1 at prices p^1. State whether the following choices staisfy the Weak Axiom of Revealed Preferences (WARP): $p^0 = (1,3)$, $x^0 = (4,2)$, $p^1 = (3,5)$, $x^1 = (3,1)$.

R: Yes. We have $p^0 * x^0 = 10$ and $p^0 * x^1 = 6$, which means that x^0 is revealed preferred to x^1. On the other hand, $p^1 * x^1 = 14$ and $p^1 * x^0 = 22 > 14$, which means that x^1 is not revealed preferred to x^0.

2. (1.25 marks) A consumer's utility function is given by $u(x_1, x_2) = \min\{x_1,x_2\}$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2. The consumer's income is €30 and the price of each unit of good 1 and 2 is €2. Compute the compensating varation associated with a reduction in the price of good 1 to €1.

R: Initially, the uitility maximizer consumer buys the bundle (x_1, x_2) such that $x_1= x_2$ and $p_1 x_1 + p_2 x_2 = m$, i.e., $(x_1, x_2) = (7.5,7.5)$. After the price change, we have $(x_1, x_2) = (10,10)$. At the final prices, the amount of income needed to buy the bundle (7.5,7.5) is 1*7.5 +2*7.5 = 22.5. Therefore, the compensating variation is 22.5 -30 = -7.5.

3. (2.5 marks) An expected utility maximizaer with wealth w may invest B, B < w, in an asset that has a rate of return a > 0 with probability p and a rate of return b < 0 with probability 1 – p (investing B, with probability p he receives (1 + a)B; with probability 1-p he receives (1 + b)B). Show that if the expected rate of return is 0, the agent will invest B = 0 if he is risk averse.

R: Solve the utility maximization problem Max p u(w+aB) + (1-p) u(w+bB) s.t. B ≥ 0. The Kuhn-Tucker condition for B > 0 gives ap u'(w+aB) + b(1-p) u'(w+bB) = 0. Since u'(w+bB) > u'(w+aB) from risk aversion, we must have ap + b(1-p) > 0, for B > 0 to be a solution. Since we have the expected rate of return equal to 0, i.e., ap + b(1-p) =0, B > = is not a solution. Therefore, the solution is B = 0.

Group 3

1. In a perfectly competitive market, let a firm's production function be given by $f(k,l) = k^2l$, where k denotes the quantity of capital and l denotes the quantity of labour used in the production process.

a) (2 marks) Compute the conditional input demand function and the cost function.
R: Solve the cost minimization problem, i.e., find l, k ≥ 0 that solve Min wl + rk s. t. $k^2l \geq y$, to obtain: l(y,w,r) = $\sqrt[3]{4y^5w^2/r^2}$ and k(y,w,r) = $\sqrt[3]{r/2wy}$ The cost function is c(y,w,r) = $\sqrt[3]{4y^5w^5/r^2} + \sqrt[3]{r^4/2wy}$.

b) (0.5 marks) Evaluate this technology's returns to scale.
R: Since $f(tk,tl) = (tk)^2(tl) = t^3f(k,l)$, we have f(tk,tl) > tf(k,l), for all t >1, so that returns to scale are increasing.

c) (1 marks) Now assume that, in the short run, the firm has k = 1. Find the conditional demand of labour and the short run cost function
R: $l(y,w,r,k) = y/ k^2 = y$ and $c^s(y,w,r,k) = wy + r$.

2. (1,5 marks) A technology has non-decreasing returns to scale. For some prices it is posible to obtain positive profits. At these prices, does the profit maximization problem have a finite solution? And does the cost minimization problem to produce a given amount have a finite solution? Explain.
R: The profit maximization problem does not have a finite solution because the returns to scale are non-decreasing. However, we can always solve the cost minimization.

Group 4

1. (2,5 marks) Duopolists producing substitute goods q_1 and q_2 face inverse demand schedules:

$$p_1 = 20 + p_2/2 - q_1 \text{ and } p_2 = 20 + p_1/2 - q_2,$$

respectively. Each firm has constant marginal costs of 20 and no fixed costs. Each firm is a Cournot competitor in price (not in quantity!). Compute the Cournot equilibrium in this market, giving equilibrium price and output for each good.
R: Firm i finds p_i such that Max $(p_i - 20)(20 + p_j/2 - p_i)$, i =1,2. The solution is $p_1 = p_2 = 30$, so that $q_1 = q_2 = 5$.

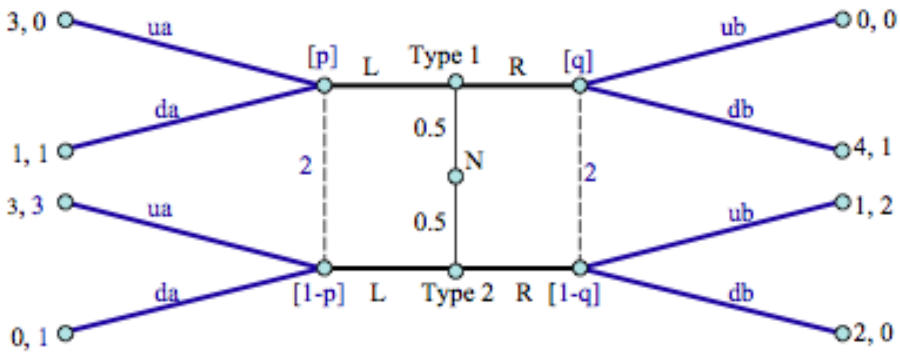
2. A monopolist faces linear demand p = a -bq and has cost C = cq + F, where all parameters are positive, a > c, and $(a - c)^2 > 4bF$.

a) (1,25 marks) Solve for the monopolist's output, price, and profits.
R: The monopolist finds q such that Max q(a -bq) - (cq+ F). Therefore, q =(a-c)/2b, p = (a+c)/2, and Profit = $(a-c)^2/4b$ -F.

b) (1,25 marks) Calculate the deadweight loss.
R: in a perfectly competitive market, p = c and q=(a-c)/b. Then, DWL = $(a-c)^2/8b$.

Group 5

1. (5 marks) Compute the weak perfect Bayesian Nash equilibria of the following game.



R: WPBN = {[(L,L),(ua,ub),p=0.5, q ≥3/4], [(R,R), (da,db), p ≥2/3, q=0.5]}.

Group 6

1. (2.5 marks) Comment on the following statement: "A mixed strategy can strictly dominate a pure strategy, but a mixed strategy cannot be strictly dominant."
R: True. A mixed strategy can strictly dominate a pure strategy. However, since the payoffs of a mixed strategy are a convex combination of the payoffs of pure strategies, a mixed strategy cannot be strictly dominant.

2. (2.5 marks) Players 1 and 2 face an incomplete information game. Player 1 does not know the type of player 2, believing that he is type I with probability 1/3 and type II with probability 2/3. Compute all Bayes-Nash equilibria in pure strategies.

Type I

	L	R
U	1,2	1,5
D	-1,3	2,0

Type II

	L	R
U	1,3	1,4
D	-1,2	2,3

R: BNE = {[U,(L,R)],[D, (R,R)]}.